Differentially Private State Estimation in Distribution Networks with Smart Meters

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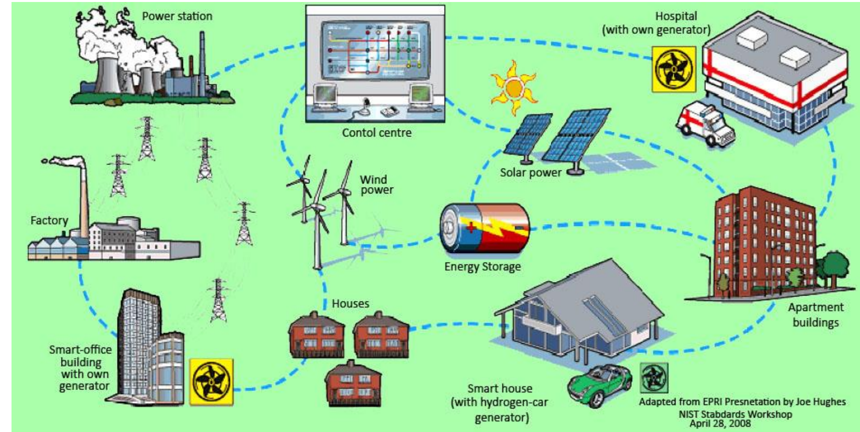
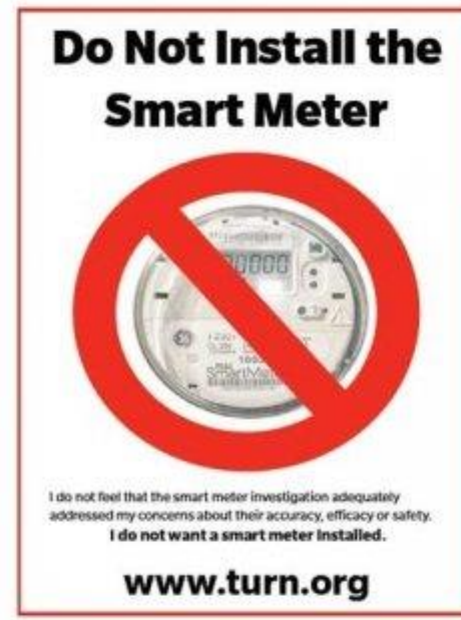
Motivation



- **The promise:** Smart meters help in demand response, billing, etc.

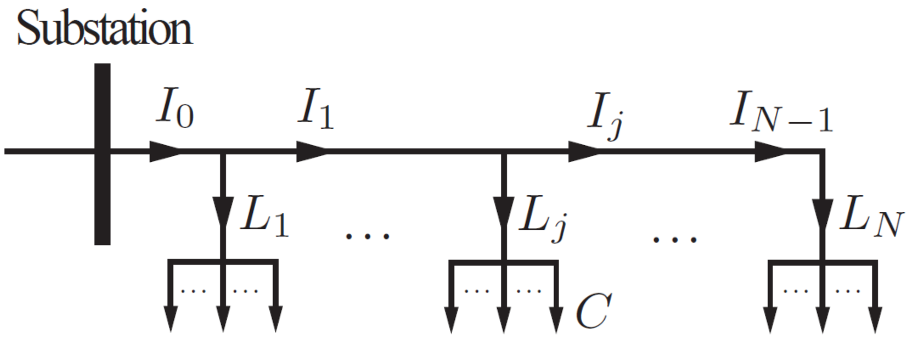
- Few real-time measurements in today's distribution networks → *Enabler for state estimation*?



- **The threat:** Customers' *privacy concerns* (among others)

- **The opportunity:** Privacy-preserving monitoring and control techniques

Distribution Network Model

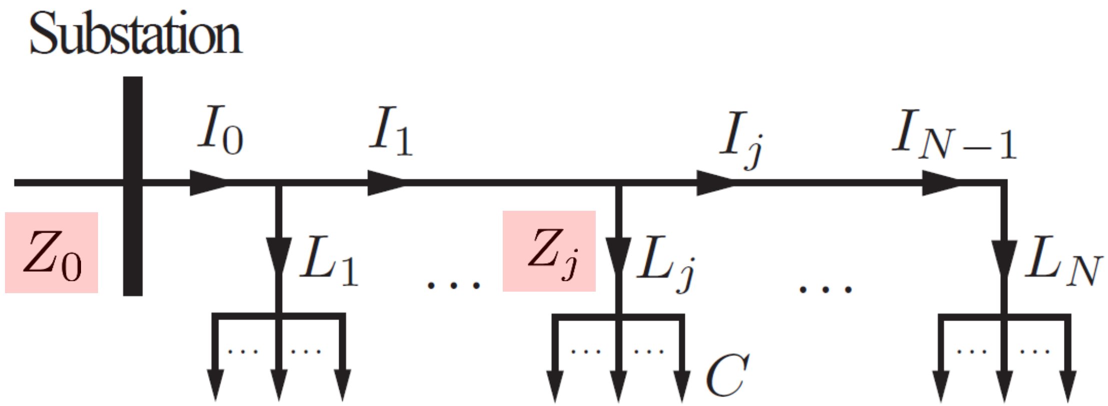


- Conservation of currents:

$$I_j = \sum_{k>j} L_k, \quad j = 0, 1, \ldots, N-1$$

- Large resistance in distribution grids → use currents
- Little dynamics in (current) distribution grids → study steady-state
- **Operator:** Desires to estimate load L_j
- **Customer C:** Desires to keep his/her real-time load private

Measurement Model



The Base Scenario – Total current with "physical meter noise":

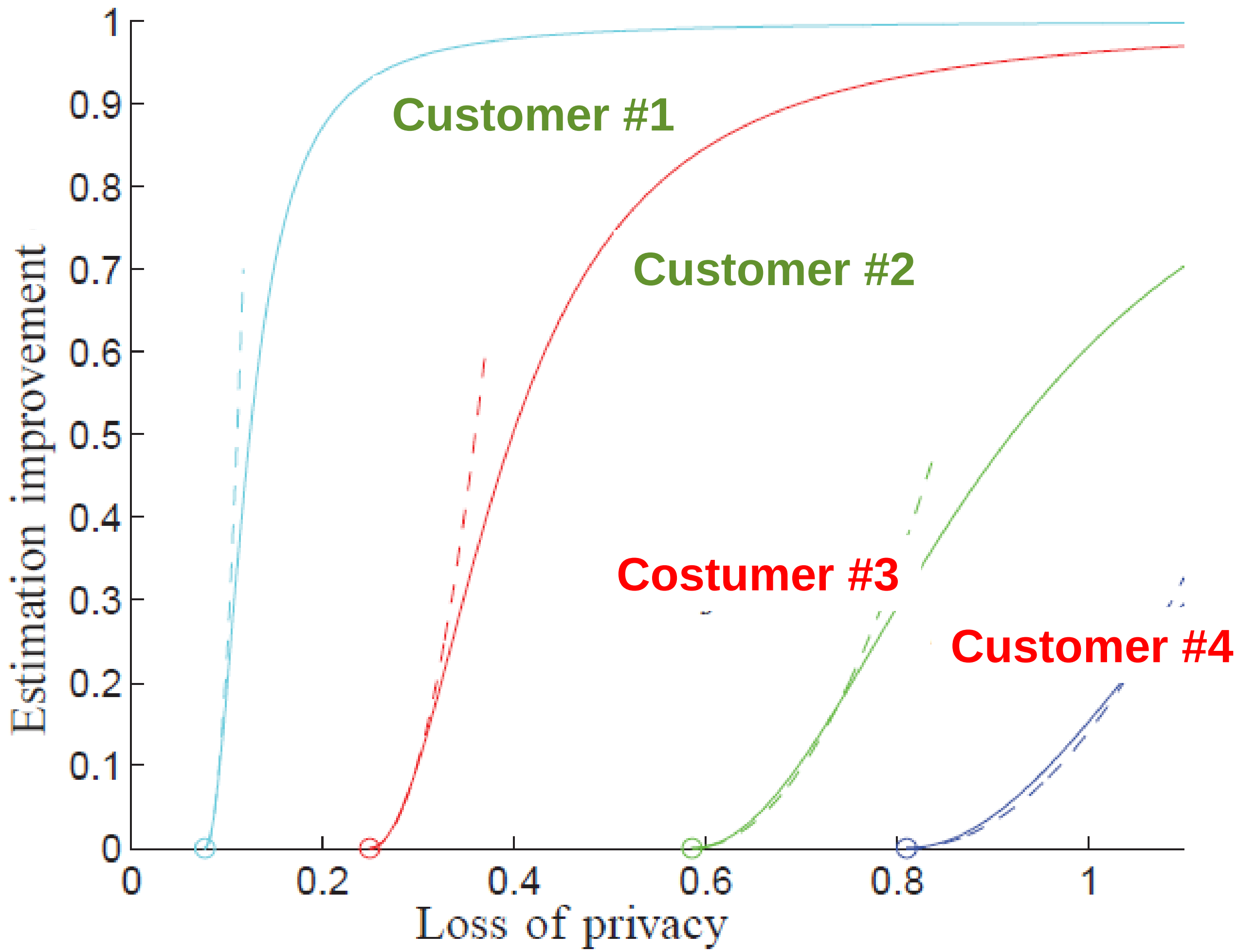
$$Z_0 = I_0 + W_0 \qquad W_0 \sim \mathcal{N}(0, R_0)$$

The Smart Meter Scenario – Load current with "privacy noise":

$$Z_j = L_j + W_j \qquad W_j \sim \mathrm{Lap}(b_j)$$

$$p_{W_j}(w) = \frac{1}{2b_j} e^{-|w|/b_j}, \; R_j = 2b_j^2$$

Problem Formulation:
Characterize Estimation vs. Privacy Trade-Offs



Related Work

- **Differential privacy:**
  - Dwork, McSherry, Nissim, Smith, 2006
- **Differential privacy in control:**
  - Le Ny, Pappas, 2014
  - Huang, Wang, Mitra, Dullerud, 2014
- **Privacy for Smart Meters:**
  - Ács, Castelluccia, 2011
  - Tan, Gunduz, Poor, 2013

Differential Privacy [Dwork *et al., 2006*]

- Two adjacent data vectors:

$$l = \begin{pmatrix} l_1 & l_2 & \dots & l_i & \dots & l_{m-1} & l_m \end{pmatrix}^T$$

$$l' = \begin{pmatrix} l_1 & l_2 & \dots & l_i \pm \Delta & \dots & l_{m-1} & l_m \end{pmatrix}^T$$

- Measurement policy (q deterministic, W stoch. noise)

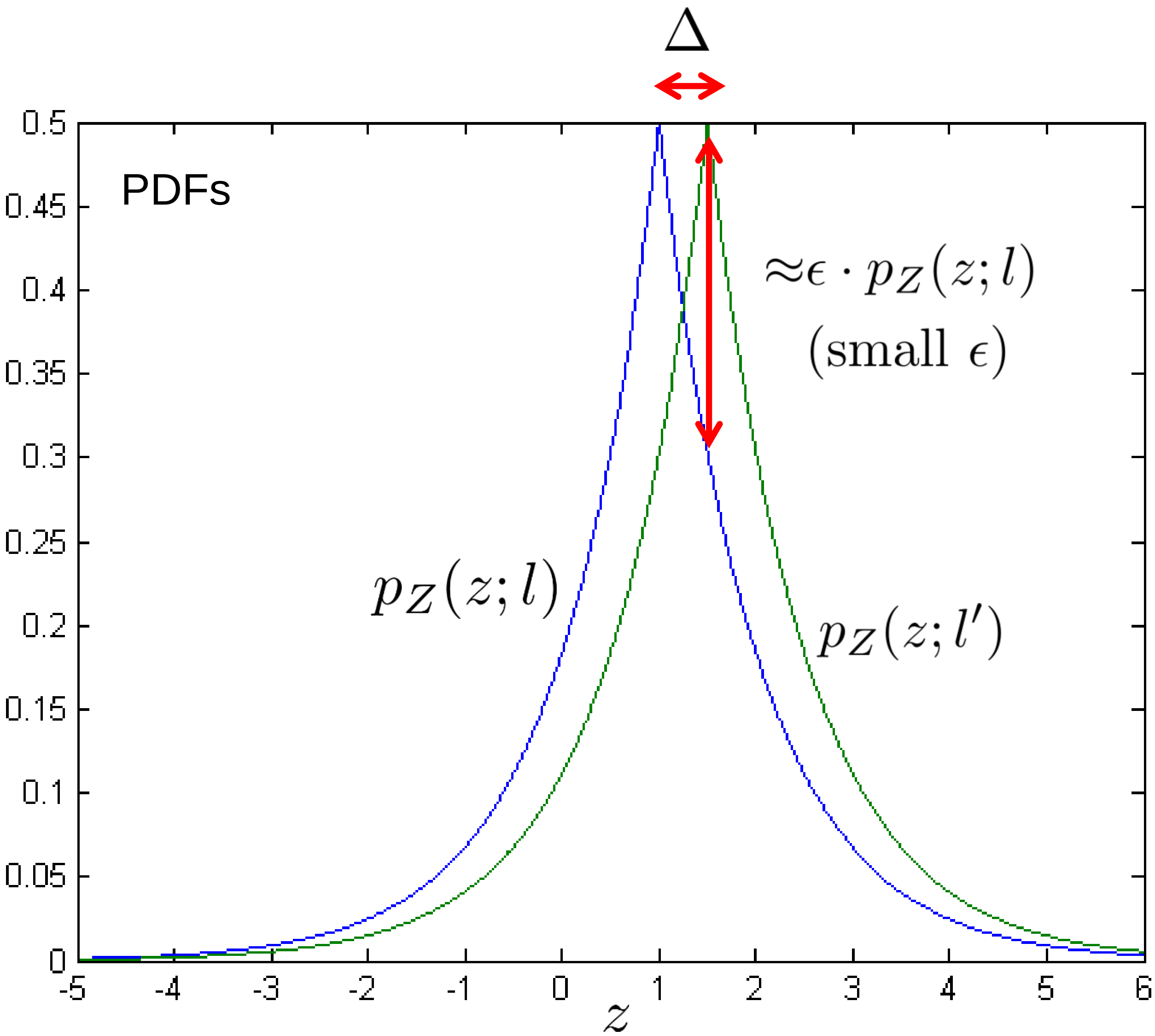
$$Z(l, W) = q(l) + W$$

(We will use $q(l) = \sum_k l_k$)

Definition: Measurement Z is (ϵ, δ)-differentially private if for all events E:

$$\Pr[Z(l, W) \in E] \leq e^{\epsilon} \Pr[Z(l', W) \in E] + \delta$$

Example: ϵ-Differential Privacy with Laplacian Noise

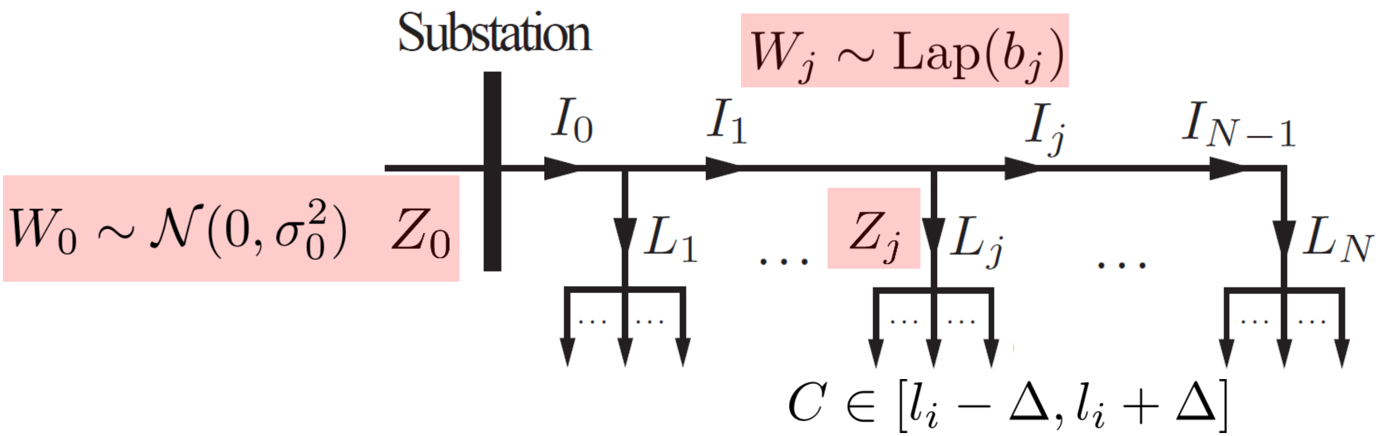


$$W \sim \mathrm{Lap}(b)$$

$$\epsilon = \frac{\Delta}{b} = \frac{\Delta}{\sqrt{R/2}}$$

Measurements of adjacent data vectors virtually indistinguishable for small ϵ

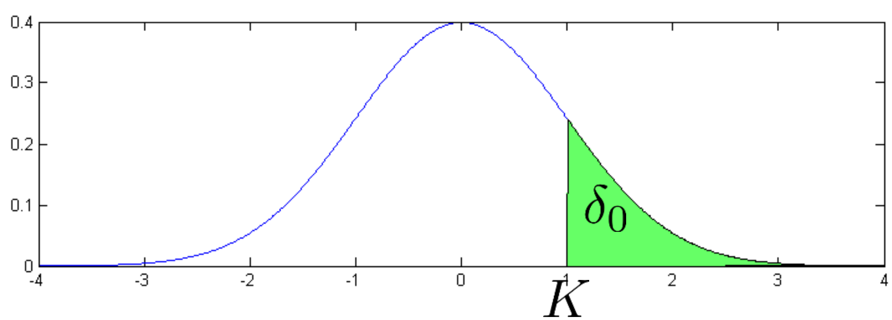
Differential Privacy in the Distribution Network



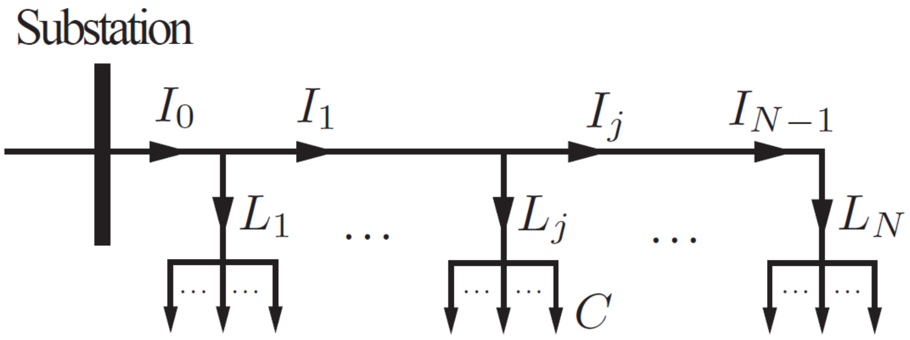
The Base Scenario (Z_0)**:** Customer C has (ϵ_0, δ_0)-differential privacy where

$$\epsilon_0 = \frac{\Delta K}{\sigma_0} + \frac{\Delta^2}{2\sigma_0^2}, \quad K = K(\delta_0) = Q^{-1}(\delta_0)$$

Smart Meter -differential privacy where



Optimal Estimate: Load Model

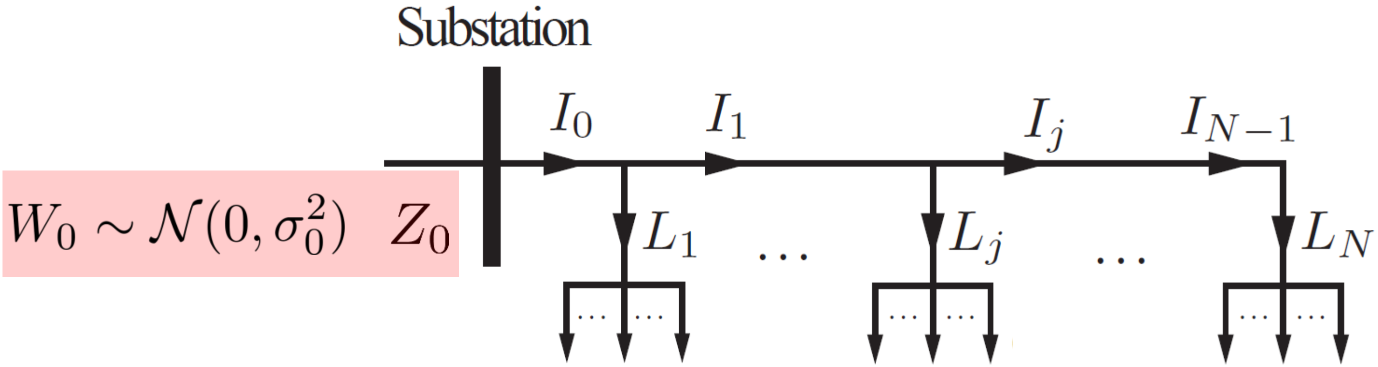


Suppose loads have a known normal distribution:

$$L \sim \mathcal{N}(m, P)$$

$$m = \begin{pmatrix} m_1 \\ \vdots \\ m_N \end{pmatrix} \quad P = \begin{pmatrix} P_{11} & \dots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \dots & P_{NN} \end{pmatrix} \quad \begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} := \begin{pmatrix} P_{11} + \ldots + P_{1N} \\ \vdots \\ P_{N1} + \ldots + P_{NN} \end{pmatrix} = P\mathbf{1}$$

Optimal Estimate: Base Scenario



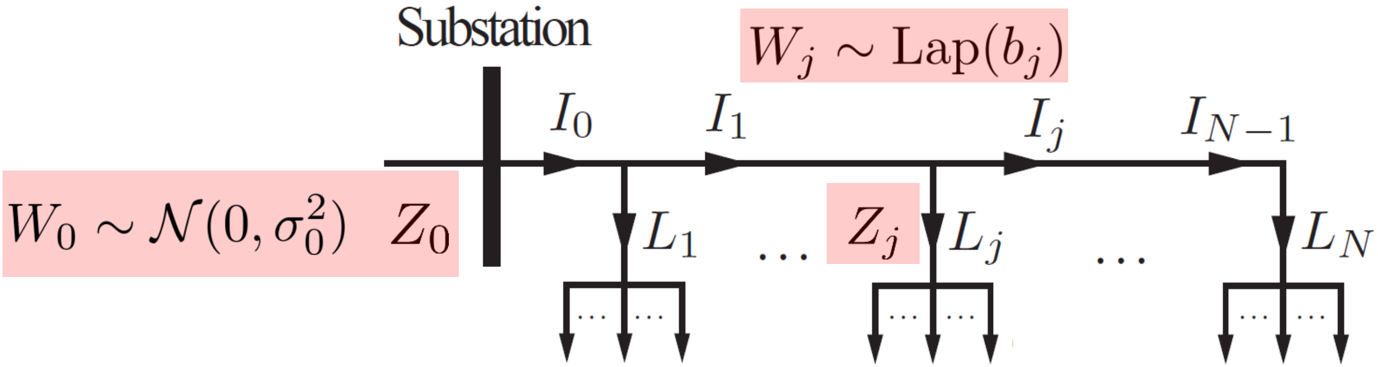
MMSE estimate:

$$\hat{L}_j^0 := \mathbf{E}[L_j|Z_0] = m_j + \frac{P_j}{P_0 + R_0}(Z_0 - m_0)$$

MMSE error:

$$Q_j^0 := \mathbf{E}[(\hat{L}_j^0 - L_j)^2] = P_{jj} - \frac{P_j^2}{P_0 + R_0}$$

Optimal Estimate: Smart Meter Scenario



LMMSE estimate:

$$\hat{L}_j^{0,j} := \mathbf{E}^{\mathrm{lin}}[L_j | Z_0, Z_j]$$

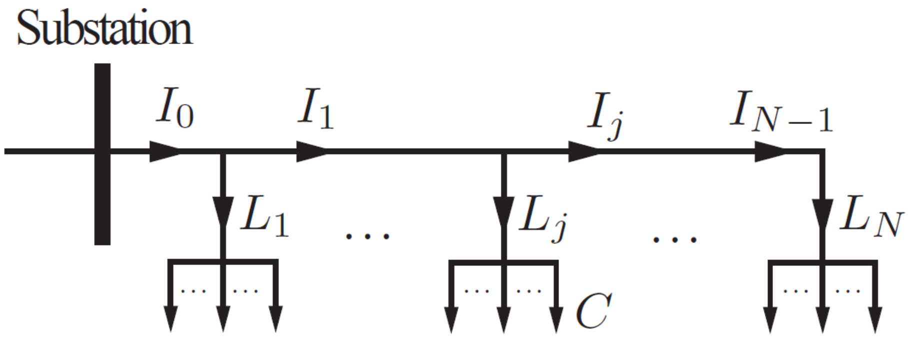
$$= \hat{L}_j^0 + K_j \left[(Z_j - m_j) - \frac{P_j}{R_0 + P_0}(Z_0 - m_0) \right]$$

LMMSE e

$$K_j := \frac{(R_0 + P_0)P_{jj} - P_j^2}{\cdots} \in [0, 1]$$

$$Q_j^{0,j} := \mathbf{E}[(L_j - \hat{L}_j^{0,j})^2] = Q_j^0(1 - K_j) \le Q_j^0$$

Trade-Off: Estimation Quality vs. Privacy



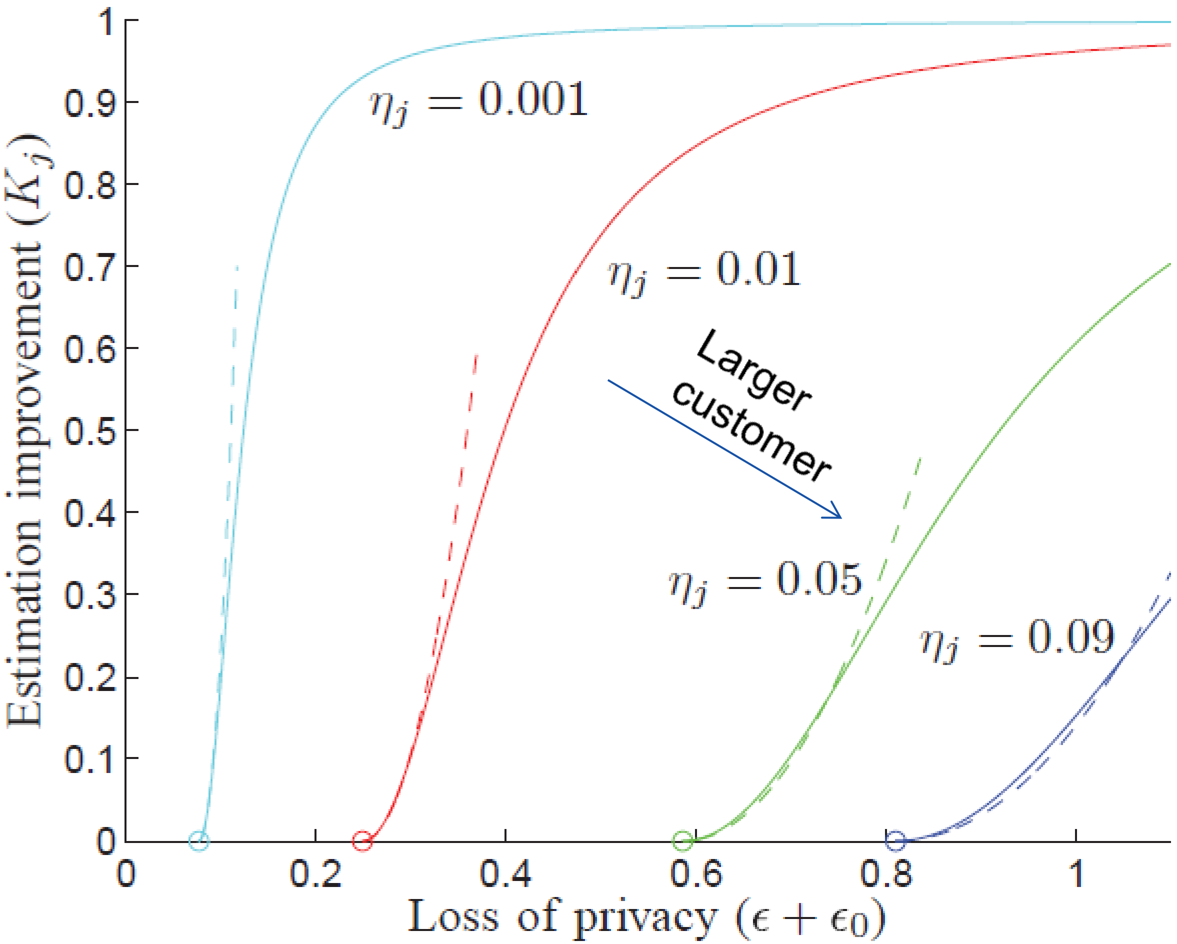
$$C \in [l_i - \Delta, l_i + \Delta]$$

$$L_j \sim \mathcal{N}(m_j, P_{jj})$$

Dimensionless quantities:

- Customers' relative importance at site *j*: $\eta_j := \dfrac{\Delta^2}{P_{jj}}$
- Site *j*'s relative importance on the line: $\zeta_j := \dfrac{P_{jj}}{P_0 + R_0}$

Trade-Off: Estimation Quality vs. Privacy



Baseline privacy:

$$\epsilon_0^2 \approx \frac{\Delta^2 K^2}{R_0}$$

$$= \eta_j \zeta_j K^2 \left(1 + \frac{P_0}{R_0}\right)$$

Est. improvement:

$$K_j = \frac{1}{1 + \frac{2\eta_j}{\epsilon^2(1-\zeta_j)}}$$

$$\approx \frac{\epsilon^2(1-\zeta_j)}{2\eta_j}$$

Summary

Est. improvement with Smart Meters: $\geq \dfrac{1}{1+\frac{2\eta_j}{\epsilon^2(1-\zeta_j)}} \approx \dfrac{K^2}{2}\underbrace{\left(1+\dfrac{P_0}{R_0}\right)}_{\text{SNR at substation}} \underbrace{\zeta_j(1-\zeta_j)}_{\text{Site size}} \underbrace{\left(\dfrac{\epsilon}{\epsilon_0}\right)^2}_{\text{Norm. privacy loss}}$

- Simple analytical treatment of trade-off between state estimation quality and customers' privacy loss ϵ
- Estimation gain $\sim \left(\frac{\epsilon}{\epsilon_0}\right)^2$ → Customers with high baseline privacy can make a large difference!
- Possible extensions: Dynamics, general topologies, active/reactive power flows