Modelling and Control of Series HEVs Including Resistive Losses and Varying Engine Efficiency

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ABSTRACT

A model of a series HEV (Hybrid Electric Vehicle) including resistive losses in the energy storage and varying engine and generator efficiency is presented. The purpose is to study how system properties affect the optimal control of series HEVs. We show that there is an inherent trade-off in the control of such a system, based on the characteristics of the efficiency curve of the EGU (Engine and Generator Unit) together with the energy storage internal resistance, capacitance and energy level. The relation between these properties determines the optimal control of the EGU, and this is thus the mechanism behind all optimal control problems using a similar model setup. We illustrate our reasoning using deterministic dynamic programming where system properties are varied and the optimal control is studied.

INTRODUCTION

Hybrid Electric Vehicles (HEVs) are gaining importance as a means of reducing fuel consumption, emissions and noise. The key idea is to brake electrically thus reusing energy that otherwise would have been lost as heat. The electrical braking is done by an electric machine that is mechanically connected to the vehicle wheel axle. HEVs also allow for optimal operation of the combustion engine, since the electric machine can supply power during acceleration and cruising thereby altering the power demand of the combustion engine. That is, the electric machine works as propulsion motor. In a series HEV there is no mechanical connection between the propulsion motor and the combustion engine. Instead the combustion engine is directly coupled to a generator supplying the system with electrical power. In this way the combustion engine operation point is independent of vehicle velocity thus further increasing the possibilities of optimal operation. If there is a mechanical connection between the driven axle and the combustion engine it is termed a parallel HEV. A HEV is equipped with an energy storage such as supercapacitors or batteries that supplies or absorbs energy to or from the system. This work focuses on series HEVs with supercapacitors as energy storage.

Previous work on control of HEVs focus mainly on finding the optimal control. Methods in use include deterministic [1]-[3] and stochastic dynamic programming [4], [5], Pontryagin's minimum principle [6], [7] and game theory [8]. The optimal control generally depends on knowledge about the probability distribution of factors such as power load over a specific driving cycle, and transferring the optimal solution to a feasible real-time strategy has proved an awkward process. In a previous work we proposed a real-time strategy for a heavy series HEV based on the optimal control over standard city bus driving cycles [9]. Control of series HEVs has also been treated in [10]-[13], but none of them adopt the approach of this work.

In the present work we investigate how fundamental system properties such as energy storage internal resistance and capacity together with efficiency characteristics of the Engine and Generator Unit (EGU) affect the optimal control of series HEVs. Previous works generally use a setup of system parameters without studying how changing the parameters can affect their solution. Our purpose is thus to investigate the mechanism behind the optimal control of this kind of system and map how it is related to system properties. In this way we establish analytically the intuitive insight that the relation between energy storage and EGU efficiency determines the optimal control. This investigation also validates the model we present since it is tested if it can reproduce the behaviour that is intuitively expected.

DISPOSITION OF THE PAPER

In the next section a model of a series HEV is described. We then derive expressions to show how the relation between power losses in the energy storage and the EGU together with the power levels on the driving cycle affect the optimal control. Finally, deterministic dynamic programming is employed to illustrate our result.

MODELLING A SERIES HEV

The series HEV is modelled using a power balance between the propulsion motor power P_{pm}, the generator power P_{gen} and the supercapactitor power P_{sc}. This is expressed as

$$0 = P_{sc}(t) - P_{gen}(t) + P_{pm}(t) \qquad (1)$$

where $P_{pm} < 0$ when energy is input to the system.

While the driver behaviour determines P_{pm} the quantity that can be controlled is P_{gen}, and the supercapacitors supply or absorb power according to (1). Fig. 1 shows a schematic illustration of the model setup. We neglect all effects of powertrain inertia in this analysis.

ENERGY STORAGE MODEL

The supercapacitors are modelled as a resistance connected in series with a capacitance. The change in energy content E of the supercapacitors then becomes

$$\dot{E}(t) = \frac{E(t)}{RC}\left(-1+\sqrt{1+\frac{2RCP_{sc}(t)}{E(t)}}\right) \qquad (2)$$
$$= P_{sc}(t) - d_e(E(t), P_{sc}(t))$$

where we have defined $d_e(E(t), P_{sc}(t))$ as the dissipation rate of the electrical system, adopting the notation of Willems in the classical paper on dissipative systems [14]. R and C are the supercapacitor resistance and capacitance. We thus have

$$d_e(E(t), P_{sc}(t)) = P_{sc}(t) -$$
$$\frac{E(t)}{RC}\left(-1+\sqrt{1+\frac{2RCP_{sc}(t)}{E(t)}}\right) \qquad (3)$$
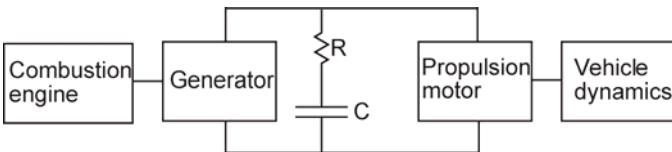
as an expression for the dissipated power.

Figure 1: A schematic picture of the series HEV model. R and C are supercapacitor resistance and capacitance.

ENGINE AND GENERATOR MODEL

We use a simple static model of the EGU,

$$P_{gen}(t) = P_{fuel}(t) - d_c(P_{fuel}(t)), \qquad (4)$$

where P_{fuel} denotes the power corresponding to the fuel used. The difference between input fuel power and the output generator power is the dissipation rate d_c of the EGU. We assume it has the form

$$d_c(P_{fuel}(t)) = P_{fuel}(1 - \eta_{EGU})$$
$$= P_{fuel}\left[1 - \eta_1(1 - e^{-\beta_1(P_{fuel} - P_{idling})}) - \eta_2 e^{-\beta_2(P_{fuel} - P'_{fuel})^2}\right], \qquad (5)$$

where

$$\eta_{EGU} = \eta_1(1 - e^{-\beta_1(P_{fuel} - P_{idling})}) + \eta_2 e^{-\beta_2(P_{fuel} - P'_{fuel})^2} \qquad (6)$$

is the EGU efficiency. The parameters $\eta_1, \eta_2, \beta_1, \beta_2$ determine the shape of the efficiency curve, while P'_{fuel} positions the efficiency maximum of the EGU. Increasing β_2 gives a sharper efficiency peak centered around P'_{fuel}. In the low power region the efficiency approaches zero since the EGU internal friction dominates. The rate at which the efficiency approaches zero is determined by β_1. The efficiency becomes zero at P_{idling} which is the idling power loss. The parameters η_1 and η_2 determine the magnitude of the EGU efficiency.

This way of modelling the EGU captures the essentials of an EGU, that is, an optimal operation point and low efficiency at low powers, and it is possible to alter the characteristics of the EGU using the model. It compares reasonably to manufacturer data though the characteristics of the efficiency peak can differ, but still it serves the purpose of the present work. In Fig. 2 our model with realistic choices of parameters is shown together with typical values for the efficiency of a diesel-EGU based on manufacturer data. We see that both the model and the actual EGU have an optimal operation point giving the maximum efficiency.

MODEL ANALYSIS

In this section we derive expressions for the instantaneous efficiency of the series hybrid powertrain. To improve readability, the time dependence of variables is not always stated explicitly.

ENERGY STORAGE DISSIPATION

In order to be able to do some analysis we simplify our model. We Taylor expand the dissipation rate of the energy storage as

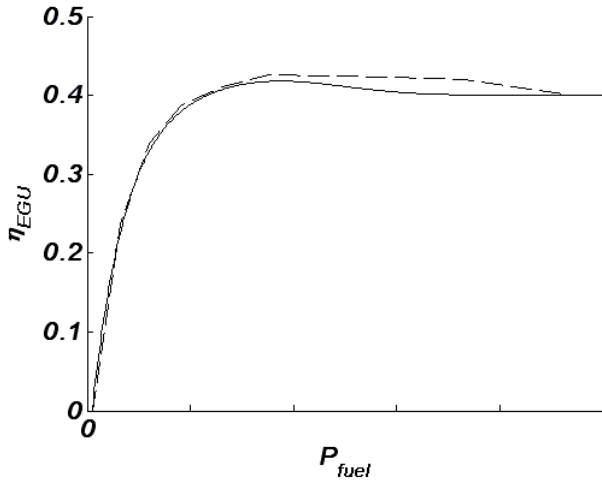$$d_e(E(t), P_{sc}(t)) = \frac{RCP_{sc}^2(t)}{2E} + O(P_{sc}^3 / E^2), \quad (7)$$

Figure 2: Efficiency of the engine and generator unit (EGU) as a function of input fuel power. The model presented here corresponds to the solid line while the dashed line is based on manufacturer data. In both cases there is an optimal operation point giving the maximum efficiency.

and we immediately see that the dissipation rate increases as P_{sc}^2 and that it decreases as E/RC increases. Hence, the dissipation rate depends on the magnitude of the power (independent of sign), and is smaller when there are large amounts of energy in the capacitor relative to the magnitude of RC.

Note that it holds that

$$d_e(E_1, P_{sc}) \geq d_e(E_2, P_{sc}), \quad E_2 \geq E_1$$
$$d_e(E, P_{sc,1}) \geq d_e(E, P_{sc,2}), \quad P_{sc,1} \geq P_{sc,2} \geq 0 \quad (8)$$
$$d_e(E, P_{sc,1}) \geq d_e(E, P_{sc,2}), \quad P_{sc,1} \leq P_{sc,2} \leq 0$$

$\forall E, P_{sc}$. This means that more energy is dissipated at low energy levels and high power levels.

DISCHARGING THE ENERGY STORAGE

The instantaneous efficiency when discharging the energy storage is

$$\eta_{sc,disch.} = \frac{-P_{sc}}{-P_{sc} + d_e} \approx \frac{2E}{2E - RCP_{sc}} \quad (9)$$

using the expression in (7). Note that $P_{sc} < 0$ during discharge. This also indicates that we should have high energy levels, low RC and low P_{sc}.

CHARGING THE ENERGY STORAGE

During normal operation when charging, or not using, the energy storage the total instantaneous efficiency is given by

$$\eta_{total} = \frac{P_{fuel} - P_{pm} - d_c(P_{fuel}) - d_e(P_{gen} - P_{pm})}{P_{fuel} - P_{pm}}. (10)$$

The objective is to maximize the total efficiency η_{total}. Since P_{pm} is electrical power and we neglect the losses in the power electronics, the only effect of P_{pm} is to alter the power level in the system. For the purpose of this analysis we can then put $P_{pm} = 0$. We thus obtain

$$\eta_{total} = \eta_{EGU}(P_{fuel}) - \frac{d_e(P_{fuel} - d_c(P_{fuel}))}{P_{fuel}}$$

$$\approx \eta_{EGU}(P_{fuel}) - \frac{RC(P_{fuel} - d_c(P_{fuel}))^2}{2EP_{fuel}} \quad (11)$$

$$= \eta_{EGU}(P_{fuel}) - \frac{RC}{2E}P_{fuel}\eta_{EGU}(P_{fuel})^2$$

as an expression for the total efficiency. In the second approximate equality (7) is used. Hence, the total efficiency η_{total} deviates most from η_{EGU} when the product $P_{fuel}\eta_{EGU}(P_{fuel})^2$ is large. We see that there is an interesting trade-off between the terms in (11). The first term typically tends to increase P_{fuel}, whereas the second term typically tends to decrease P_{fuel}. Using some further approximations, we will analyze this behavior next.

Assume that it is optimal to run the EGU by itself at some power $P_{fuel} = P_{fuel}^*$. Since P_{fuel}^* is assumed to be a local maximum of η_{EGU} at η_{EGU}^*, in a neighborhood around P_{fuel}^*, we have

$$\eta_{EGU}(P_{fuel}) \approx \eta_{EGU}^* - \beta\Delta P_{fuel}^2,$$
$$\Delta P_{fuel} = P_{fuel} - P_{fuel}^* \quad (12)$$

for some $\beta \geq 0$. Inserting (12) in (11) gives

$$\eta_{total} \approx \eta_{EGU}^*(1 - \eta_{EGU}^* RCP_{fuel}^* / 2E)$$
$$- (\eta_{EGU}^*)^2 (RC/2E)\Delta P_{fuel} \quad (13)$$
$$- \beta(1 - \eta_{EGU}^* RCP_{fuel}^* / E)\Delta P_{fuel}^2$$

where only up to quadratic terms in ΔP_{fuel} are kept. For realistic parameters $\beta(1 - \eta_{EGU}^* RCP_{fuel}^* / E) > 0$ holds, and there is a local maximum of (13) corresponding to

$$P_{fuel,new}^* = P_{fuel}^* - \frac{(\eta_{EGU}^*)^2}{4\beta(E/RC - \eta_{EGU}^* P_{fuel}^*)} . \quad (14)$$

Equation (14) thus gives the new optimal operation point $P_{fuel,new}^*$ of the EGU when both the efficiency of the energy storage and the EGU are taken into account. We see that the deviation from P_{fuel}^* depends on β, which captures the characteristics of the maximum of η_{EGU}, in a way similar to β_2 in (5). If β is increased the efficiency peak becomes more distinct and it is optimal to stay at P_{fuel}^*, since the second term in (14) becomes small. Decreasing β evens out the EGU efficiency curve, and it is optimal to run the EGU at other powers than P_{fuel}^*, since the second term in (14) becomes significant. At the same time, the fraction E/RC influences the optimal operation point in the same way. That is, if E/RC is large the optimal operation point is mainly determined by P_{fuel}^*, but if E/RC decreases it is optimal to lower the EGU power. Including the time dependence, this fraction is $\dfrac{E(t)/C}{R}$. Hence, it is small if the energy level is normally low during operation and/or the internal resistance is high, and vice versa.

Naturally, (11) also tells us that when not using the energy storage the total efficiency is equal to the EGU efficiency.

DEPENDENCE OF THE OPTIMAL CONTROL ON VEHICLE PARAMETERS

To summarize the results so far, we have:

- Keep RC low, P_{sc} low and E high to minimize losses in the energy storage.
- If β is large, i.e., the EGU has a distinct efficiency peak, run the EGU in P_{fuel}^* if $E(t)/RC$ is large enough.
- For small β it can be optimal not to run the EGU in P_{fuel}^* if $E(t)/RC$ is small enough.

These results express the trade-off inherent in the optimal control of this kind of systems. It is based on the relation between β and E/RC, which expresses efficiency characteristics of the EGU and energy storage internal resistance, capacitance and energy level. Calculating the optimal control is simply determining

what values of β and E/RC that should be considered. This trade-off is the mechanism behind all optimal control problems based on a model involving resistive losses and varying engine efficiency.

To clarify the consequences of these results we investigate what happens at extreme values of the parameters. A possible powertrain setup would be a small vehicle with large energy storage with relatively small internal resistance, i.e., E/RC is typically large during operation. If the EGU has a distinct efficiency peak, β is large. Equation (14) then tells us that the EGU should be operated in P_{fuel}^*. The optimal control strategy in this case is thus a thermostat or on-off strategy running the EGU in the optimal operation point. Since the vehicle is small, the power levels involved are low during operation. Equation (11) then tells us that the total efficiency η_{total} is mainly determined by η_{EGU}, which also indicates that the EGU should be operated in P_{fuel}^*.

On the other extreme, we have a heavy vehicle with relatively small energy storage with high internal resistance together with small variations of EGU efficiency over a wide power range. That is, E/RC and β are small while the power levels are high. In this case it is less obvious what determines the total efficiency since the term $(\eta_{EGU}^*)^2 / [4\beta(E/RC - \eta_{EGU}^* P_{fuel}^*)]$ becomes significant. However, we can conclude that P_{fuel} must be carefully controlled since $P_{fuel,new}^*$ can change significantly during operation. Also, since the power levels are high, choosing the wrong P_{fuel} has large consequences on the total efficiency. Since E will vary significantly in this powertrain setup, it is wise to monitor E and base the control on this measurement, in this way assuring that we operate the EGU as close to $P_{fuel,new}^*$ as possible. This also leads us to the conclusion that the control of heavy HEVs using small energy storages with high internal resistance is not as straightforward as the control of small vehicles using large energy storages with low internal resistance. In Fig. 3 this result is shown schematically.

So far only the instantaneous efficiency has been treated. If we consider a whole driving cycle, what is instantaneously optimal is not necessarily optimal globally. To investigate if our conclusions hold when we consider a time history we employ dynamic programming.
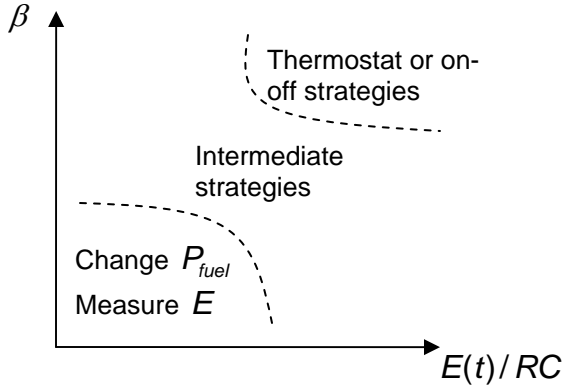
Figure 3: The dependence of the optimal control strategy on vehicle configuration, represented as β and $E(t)/RC$. Large energy storages with high internal resistance together with a distinct efficiency peak of the EGU leads to on-off or thermostat strategies.

DYNAMIC PROGRAMMING ANALYSIS

To solve the optimal control problem it is discretisized on a time and state grid over the time interval $[0,N]$ with time step Δt. It then becomes

$$\min \phi(E_N - E_0) + \sum_{k=0}^{N-1} P_{fuel,k}$$

$$\text{subj. to} \begin{cases} E_{k+1} = E_k + (P_{gen,k} - P_{pm,k} - d_{e,k})\Delta t \\ P_{fuel,k} = P_{gen,k} + d_{c,k} \\ E_k \in [E_{min}, E_{max}] \\ P_{gen,k} \in S_k \cap [P_{gen,min}, P_{gen,max}] \end{cases} \quad (15)$$

where k is the time index. We thus minimize P_{fuel} and the relation between control signal P_{gen} and P_{fuel} is given by (4). The function $\phi(E_N - E_0)$ ensures that the final state E_N is equal to the initial state E_0 by penalizing deviations. The set of admissible controls S_k is time varying and contains the controls taking the system from the current state to states reachable in the next time step. If the time and state grid is made too coarse, the solution will contain discretization noise. It is therefore important to choose the resolution of the grid carefully.

We assume that the energy storage supplies the system with the necessary power, and we thus put no constraint on the maximum or minimum power of this component.

A problem of this type is conveniently solved using dynamic programming since it handles nonlinearities and constraints in a straightforward way [15]. Dynamic programming is based on the Bellman equation and solves the problem via backwards recursion:
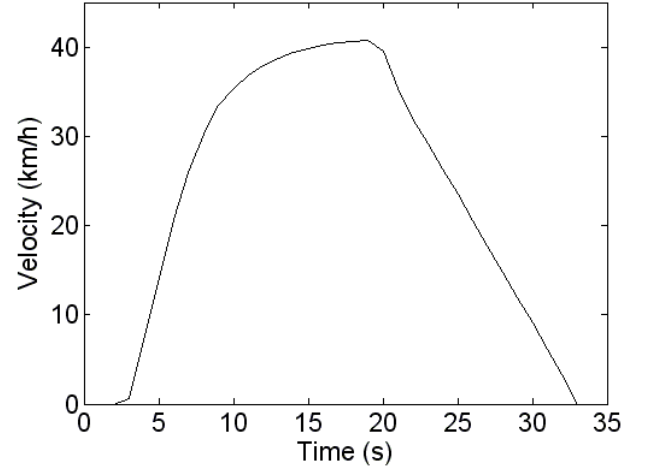

Figure 4: The velocity profile of the optimal control problem.

$$J^*(n,E_n) = \min_{P_{gen,n}} \{P_{fuel,n} + J(n+1,E_{n+1})\} \quad (16)$$

where $J^*(n,E_n)$ is the optimal cost-to-go at time instant n and state E_n.

We construct a driving cycle and obtain P_{pm} necessary for solving (15) using a comprehensive simulation model developed in-house at Scania. It is essentially based on standard models of vehicle dynamics. To facilitate evaluation of the optimal control, we choose a simple velocity profile shown in Fig. 4. This consists of acceleration, constant speed and deceleration phases, and should give good insight into how system properties affect the optimal control.

We solve (15) on this driving cycle for two vehicle setups. To investigate if the conclusions from the previous section hold, the setups chosen differ in RC and β. Their respective efficiency curves are shown in Fig. 5.

Setup 1 has a distinct efficiency peak and thus a high β-value, β_{high}. Setup 2 has a less distinct efficiency peak and $\beta = \beta_{low}$. To allow for EGU shut-off, P_{idling} is put to zero, and there is thus no fuel injection when P_{gen} is zero.

We use $R_1C_1 = 100R_2C_2$ to vary the energy storage characteristics. The vehicle setups are thus:

1. β_{high}, R_1C_1
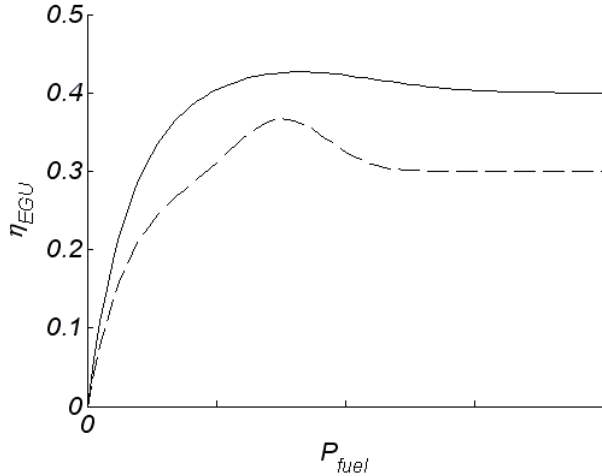
2. β_{low}, R_2C_2

Figure 5: The engine and generator efficiency for the two cases studied using dynamic programming. Dashed curve has a high β-value and solid curve has a low.

with $R_1C_1 > R_2C_2$.

Setup 1 can correspond to a vehicle using an otto engine and relatively large energy storage with low internal resistance. Setup 2 can be a diesel engine vehicle with small energy storage with high internal resistance.

Typically, $\beta_{\text{diesel engine}} < \beta_{\text{fuel cell}} < \beta_{\text{otto engine}}$. Battery based energy storages have larger storage capacity but also higher internal resistance compared to a storage based on supercapacitors.

The optimal control P_{gen}^* of the respective vehicle setups on the velocity profile in Fig. 4 is shown in Fig. 6. We see that Setup 1 has a characteristic on-off, or bang-bang, behavior where the EGU is either operated at maximum efficiency or shut off. Having knowledge about the future time history of the system, the dynamic programming solution method chooses the optimal time to switch. This would not be possible in real-time and some criterion for when to switch would have to be defined. The optimal control of Setup 2 is not bang-bang. Instead it follows the propulsion motor power in the acceleration up to a certain point, and then back to zero. This control ensures that the energy level is high, thus minimizing dissipation in the energy storage. These results are in accordance with the ones derived in the previous section. The trade-off when looking at the instantaneous efficiency thus also holds over a driving cycle.

CONCLUSIONS

A simple model of a series HEV was described. Its characteristic features are energy storage internal resistance and varying efficiency of the EGU. Using some simplifications, we were able to show analytically that the optimal operation point of the EGU depends on these characteristic features. The properties of the EGU
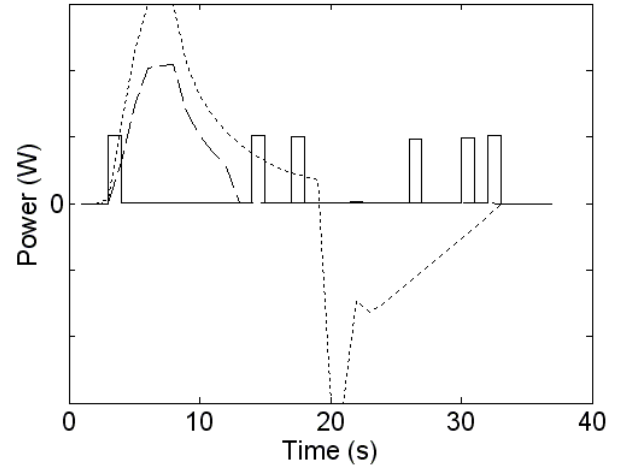
Figure 6: The optimal control P_{gen}^* of the two vehicle setups over the chosen velocity profile. The propulsion motor power corresponds to the dotted line. The optimal control of Setup 1, with β_{high} and R_1C_1, is shown as a solid line. The optimal control of Setup 2, with β_{low} and R_2C_2, is the dashed line. We see that the first vehicle setup has a distinct on-off behavior, while Setup 2 is not on-off.

maximum efficiency operation point are here described by the sharpness of the EGU efficiency peak β, with a higher β giving a more distinct efficiency peak. The optimal operation point of the EGU depends on β and on the value of $E(t)/RC$. $E(t)/RC$ is large if the energy level is large during operation and the internal resistance is low, and vice versa. For large values of β and $E(t)/RC$ the optimal operation point of the EGU is not altered by the inclusion of the energy storage, which should result in on-off strategies operating the EGU at maximum efficiency. If β and $E(t)/RC$ are small, the optimal operation point is significantly altered by the inclusion of the energy storage, and should most likely change during operation. The analysis indicates that we should measure $E(t)$ to determine the optimal operation point. Our conclusions based on instantaneous efficiency were tested on a complete driving cycle using deterministic dynamic programming. The optimal control turned out to correspond to what intuition told us based on the analytical results.

The main conclusion is thus that the optimal control of series HEVs can differ significantly depending on powertrain characteristics. An EGU with a distinct efficiency peak together with an energy storage with low internal resistance and high energy levels during operation results in on-off strategies. On the other hand, an EGU with a less distinct efficiency peak together with an energy storage with high internal resistance which is operated at lower energy levels results in a completely different control, which is non-trivial to determine. This conclusion is somewhat intuitive, but a contribution of the present work is to investigate this issue analytically. Intermediate cases should exhibit a mix of the strategies, and the strategy could also possibly vary during operation. This makes the optimal control of this kind of systems still more difficult. The trade-off between these powertrain characteristics is the mechanism behind all optimal control problems using a similar model setup.

Besides the powertrain characteristics investigated here there can be many other factors that affect the optimal control. If the cost function is modified to include for example noise, drivability, passenger comfort, engine transients or emissions, the optimal control can be different. This could possibly decrease the feasibility of an on-off strategy. In this work we focused on supercapacitors, but other types of energy storages are also interesting for optimal control problems. Though, modelling for example batteries is likely to become more complicated than supercapacitors.

ACKNOWLEDGMENTS

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