Control Over a Hybrid MAC Wireless Network

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***Abstract*—We consider the problem of performing control over large complex networked systems with packet drops. More specifically, we are interested in improving the performance of the regulation of control loops when the communication is made over low-cost wireless networks. In control over wireless networks it is common to use Contention-Free (CF) schemes where no losses occur with the price of low scalability and complicated scheduling policies. In this work we propose a hybrid MAC and control architecture, where a small number of control loops with high demand of attention are scheduled in a CF scheme and well regulated loops are scheduled in a lossy, asynchronous and highly scalable, Contention-Access (CA) scheme. We model and analyze the performance of such system with Markov Jump Linear System (MJLS) tools and compare it with other architecture types. Performance is evaluated using a quadratic cost function of the state.**

I. Introduction

There are major advantages in terms of increased productivity and reduced installation and maintenance costs in the use of wireless communication technology to build the Smart-grid infrastructure [1], [2]. The IEEE 802.15.4 standard [3] has received considerable attention as a low data rate and low power protocol for WSN applications in Smart-grids, industry, control, home automation and health care see [1], [2]. A recent interesting application of WSN is on integrating the automatic metering infrastructure and the Smart-grid power generation plants [5], [2]. In the Smart-grid context, the IEEE 802.15 smart utility networks task group 4g works for the physical layer amendments of IEEE 802.15.4 standard to support large and geographically diverse networks, such as process and factory automation. [5]

In terms of Medium Channel Access (MAC) control schemes, a contention-free scheme, such as Time Division Multiple Access (TDMA) [6], is the dominant MAC scheme for industrial control applications. This choice is motivated by the high determinism and high reliability provided by a Contention-Free (CF) scheme. The drawback of this access scheme is that it requires time synchronization and pre-scheduled fixed length time-slots by a centralized coordinator. Hence, for a large network such as the Smart-grid, a single CF MAC scheme might not be implementable and thus compromising the operation of the grid.

On the other hand, Contention-Access (CA) MAC schemes are largely used for monitoring applications because of its scalability, flexibility and easy configuration, clearly fitting the Smart-grid network demands, but its probabilistic data transmission due to packet drops, make it less suitable for process control [3], [2]. However, the CA MAC scheme guarantees low latency and high reliability if the sampling period is not fixed and large, such as the system with event-based controllers which are finding an increasingly research attention nowadays [9]. Therefore it is natural to incorporate different types of control strategies (e.g. typical time-driven and event-based control) in order to have the benefits of both types of controllers.

This work considers a hybrid MAC communication layer combining both CA and CF MAC schemes. The underlying idea is to provide two different control laws according to the protocol being used. In this setup, these control strategies should be chosen in conjunction with the properties of the associated protocol. Roughly speaking, the CA MAC is convenient to run control regulations with small set-point deviation and allows to multiplex several loops in the same network. Whereas communication based on dedicated time slots may be limited to isolated emergency (e.g. disturbance occurrence) when a loop needs a special attention.

From this original control scheme emerges a hybrid control structure. We will see that discrete-time Markov Jump Linear Systems (MJLS) provide a suitable framework to model the aforementioned control scheme. Conditions for analysis and control of such systems have been established in [7], [8]. In this paper, we exploit some of these tools to assess the stability and performance of the overall control law over a Networked Control System (NCS).

The outline of the paper is as follows. Section II defines the considered problem. In Section III, we introduce the details on a hybrid MAC communication and the design characteristics. Section IV presents the MJLS tools for stability and performance analysis. An example illustrates the hybrid architecture performance in Section V. Section VI concludes the paper.

II. Problem formulation

We consider several processes and controllers that communicate over a wireless network, as shown in Fig. 1. Let us assume that each of these processes is described by a linear stochastic differential equation

$$\begin{aligned} dx(t) &= Ax(t)dt + B(u(t)dt + dv(t)) \\ dv(t) &= w_1(t)dt + F(t)w_2(t)dt \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the plant state and $u(t) \in \mathbb{R}^m$ is the control signal. The disturbance $v(t)$ is composed by two noises, $w_1(t) \in \mathbb{R}^n$ and $w_2(t) \in \mathbb{R}^n$, with zero mean and uncorrelated increments with incremental covariance $R_{w_1}dt$ and $R_{w_2}dt$, respectively, with $R_{w_2} \gg R_{w_1}$. By considering

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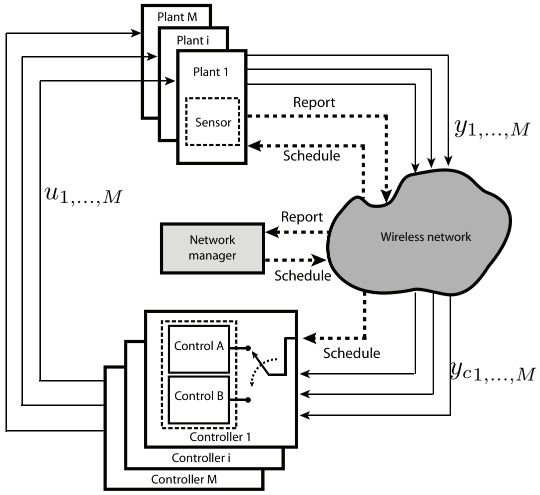


Fig. 1. General Wireless NCS framework with packet drops. A Network Manager is responsible for coordinating the sampling times of sensors and select the control law used by each controller.

zero-order-hold, a time-varying discrete-time sampled system is

$$\begin{aligned} x(kh+h) &= A_h x(kh) + B_h(u(kh)+v(kh)), \\ y(kh) &= x(kh), \\ v(kh) &= w_1(kh) + F(kh)w_2(kh), \end{aligned} \tag{2}$$

where $A_h = e^{Ah}$, $B_h = \int_0^h e^{As} ds B$ and the matrix $F(kh) = \theta_1(kh)$, where θ_1 is a Bernoulli process with a probability of occurrence ($\theta_1(kh) = 1$) equal to β and $1-\beta$, otherwise. This characterizes a sporadic occurrence of the disturbance w_2. The measured signal is sent across a wireless network and the resulting signal, received on the controller side, is represented by $y_c(kh)$. Besides, controllers are co-located with their respective processes and control signals use a reliable communication channel.

The use of wireless networks requires to take into account their inherent unreliability, namely data loss. A simple but very convenient way to model data loss relies on a Bernoulli process ([11], [12]). In that case, $y(kh)$ and $y_c(kh)$ are related by:

$$y_c(kh) = \theta_2(kh)y(kh) + (1-\theta_2(kh))y_c(kh-h) \tag{3}$$

where $\theta_2(kh)$ is a Bernoulli process with a probability of dropout ($\theta_2(kh) = 0$) equal to γ and a probability of successful delivery ($\theta_2(kh) = 1$) equal to $1-\gamma$.

Regarding the control law applied by $u(kh)$, we propose to employ a hybrid control scheme of the form

$$u(kh) = -L_{\theta_3(kh)} y_c(kh). \tag{4}$$

The controller is triggered on a Markovian jumping parameter $\theta_3(kh) \in \{0,1\}$, enabled through some network features that will be defined later. In the rest of the paper we will drop the notation kh for k in the sake of simplicity.

In the following section we introduce the system architecture of the propose scheme.

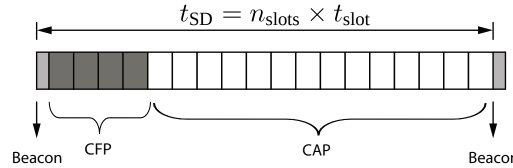


Fig. 2. Customized IEEE 802.15.4 Std superframe. A coordinator periodically transmits beacon messages used for network synchronization and configuration. The superframe is divided in Contention-Free (CFP) and Contention-Access Periods (CAP).

III. System Architecture

In this section we discuss the system architecture considering the hybrid aspect of process control and medium access control.

A. Networked Control System with Hybrid MAC

We consider the NCS of Fig. 1 with several control loops where information is exchanged between the process and controller, subject to channel access constraints. This control application requires a communication network which supports low latency, reliability, flexibility and scalability.

Now, we give the overview of a hybrid MAC which supports two channel access schemes: Contention-Access (CA) and Contention-Free (CF). Fig. 2 shows a superframe structure of a hybrid MAC which has a very similar structure with the IEEE 802.15.4 standard. Let us define the number of slots per superframe n_{slots}, the slot time t_{slot}, the superframe time t_{SD} and the number of Contention-Free Period (CFP) slots, n_{CFP}. A coordinator node in the network (Network Manager (NM) in Fig. 1), periodically sends the beacon frames in every superframe interval to identify its personal area network and to synchronize devices that communicate with it. The coordinator and devices communicate during active period t_{SD}. Each superframe duration is divided in a CFP composed of CFP slots and a Contention-Access Period (CAP). In the CFP, the dedicated bandwidth is used for time-critical data packets. In the CAP, a CA MAC is used to access the channel of non-time-critical data packets. Here we inverted the order of the appearance of the CAP and CFP with respect to the IEEE 802.15.4 standard. We do this in order to reduce the waiting time between a beacon from the coordinator and the sampling of the process. This is not in the current standard but could be changed in future revisions.

We now explain the design of the proposed NCS with hybrid MAC architecture. The state diagram of the hybrid architecture is depicted in Fig. 3.

The NM in Fig. 1 is responsible for, at each beacon time, configure the time for each process sensor to report measurements when in a disturbed condition ($\theta_1(k) = 1$), and assigning the type of control used by each controller. Each sensor is configured to send a warning message in the case its measurement exceeds a detection threshold d_{th}. In a *best-effort* (BE) mode, a given loop is running in a CA MAC scheme. In

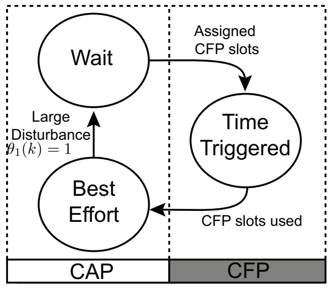


Fig. 3. State diagram of the control over a hybrid MAC architecture. A sensor and controller are in *best-effort* mode with CA MAC, when no disturbances occur. In case of a large perturbation ($\theta_1 = 1$) the NM assigns n_{CFP} slots to be used for *Time-Triggered* measurement in a CF MAC. The *Wait* mode models the system while it waits for the assigned CFP slots.

this case the sampling is given by a Bernoulli process, where the sensor sends a measurement with a $\Pr(\theta_4(k) = 1) = \alpha$ and does not try to transmit with $\Pr(\theta_4(k) = 0) = 1 - \alpha$. This models an event-based policy with average sampling as a function of the sampling probability α and the packet loss probability γ. We denote the sampling period as h_s. A fixed BE mode control law will be used in this mode. Additionally each sensor is configured to send a warning message if its measurement exceeds a detection threshold d_{th}.

When a large disturbance occurs, i.e. $\theta_1 = 1,\ F = 1$, and $y(k) \geq d_{th}$, the sensor reports to the NM, and enters a *wait* (WA) mode. In this mode the sensor and controller maintain their previous BE mode configuration and wait for a new configuration/schedule from the NM. The NM at the following beacon time, assigns n_{CFP} consecutive CFP slots to the loop and configures the controller to switch to a *time-triggered* (TT) mode. Notice that now the sampling period of the disturbed loop is $h_f = t_{slot} \ll h_s$. In this case, the loop is using CFP in the next superframe. It is worth to note that since in CFP there are no packet dropouts, i.e. $\theta_2(k) = 1,\ \forall k$. Once n_{CFP} slots are used, the corresponding controller re-sets the control law to the BE mode. When a disturbance does not occur, no CFP slots are used.

We now introduce the control laws used in the presented architecture.

B. Hybrid control law

In our framework, we use two types of control laws with different sampling rates. On one hand, when a loop is working in a BE mode, a Linear Quadratic (LQ) control is applied to perform an efficient and optimal regulation of the process according to an average sampling rate $h = h_s$ and $\theta_3(k) = 0$. On the other hand, if a large deviation of the set-point appears (i.e. large disturbance, $y(k) \geq d_{th}$), in which the loop requires a high attention, a suitable fast control should be set. In this latter case, a LQ control is applied but designed with a much smaller sampling period, $h = h_f = t_{slot} \ll h_s$ and $\theta_3(k) = 1$.

The optimal control problem is now defined to be finding the admissible control signal $u(kh)$ that minimizes the average cost function,

$$J = E\big[\sum_{k=0}^{\infty} x^T(k) Q_h x(k) + u^T(k) R_h u(k)\big], \tag{5}$$

where Q_h and R_h are positive semidefinite (details on their calculation in [10]).

The state-feedback gain of the LQ control is given by

$$L_{\theta_3(k)} = (R_h + B_h^T \bar{S}_h B_h)^{-1} (B_h^T \bar{S}_h A_h), \tag{6}$$

where $\bar{S}_h$ is the solution to the algebraic Riccati Equation

$$\begin{aligned}\bar{S}_h = A_h^T \bar{S}_h A_h + Q_h - (A_h^T \bar{S}_h B_h) \\ \times (B_h^T \bar{S}_h B_h + R_h)^{-1} (B_h^T \bar{S}_h A_h).\end{aligned} \tag{7}$$

The control law (4) is summarized as;

$$u(kh) = \begin{cases} -L_0 x(kh), & \text{with } \theta_3(k) = 0, \\ -L_1 x(kh), & \text{with } \theta_3(k) = 1. \end{cases} \tag{8}$$

Next we will present the closed-loop system details taking into account the different modes of the hybrid architecture.

C. Closed-loop System

Let us pick a single system (2) from the general multiple NCS scheme in Fig. 1. Given that the measurement communications are performed via the hybrid MAC scheme, the information collected on the controller side is

$$y_c(k) = \begin{cases} \theta_4(k)\theta_2(k)x(k) & \\ +(1-\theta_2(k))\theta_4(k)y_c(k) & \text{via CA MAC,} \\ x(k) & \text{via CF MAC.} \end{cases} \tag{9}$$

The control law switches according to the mode at which the plant is at a given step k, (8), with sampling period $h = t_{\mathrm{slot}} = h_s$. Gathering equations (8) and (9), the system (2) closed over the hybrid MAC can be rearranged as,

$$\bar{x}(k+1) = \mathcal{A}_{\theta(k)} \bar{x}(k) + \mathcal{B}_{\theta(k)} w(k), \tag{10}$$

where $\bar{x}(k)$ represents the extended state vector $[x^T(k) \quad y_c^T(k-1)]^T$ and $w(k) = [w_1^T(k) \quad w_2^T(k)]^T$. The state and input matrices depend on a set of jumping parameters $\theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ and are driven by the Markov model defined in Section III-D:

$$\mathcal{A}_{\theta(k)} = \begin{bmatrix} A_{h_s} - \theta_3 B_{h_s} L_1 - (1-\theta_3)\theta_2\theta_4 B_{h_s} L_0 & -(1-\theta_3)(1-\theta_2)\theta_4 B_{h_s} L_0 \\ (\theta_3 + \theta_2\theta_4)\mathbb{1} & (1-\theta_3)(1-\theta_2)\theta_4 \mathbb{1} \end{bmatrix}, \mathcal{B}_{\theta(k)} = \begin{bmatrix} B_{h_s} & B_{h_s}\theta_1 \\ 0 & 0 \end{bmatrix}.$$

We will now present the Markov model which captures the behavior of the system described above.

D. Markov Model

Taking into account the system architecture described above we propose a Markov model to characterize the state dynamics and events occurring in the system.

The Markov model is depicted in Fig. 4. This model is divided in three different interconnect sub-models following the architecture presented in Sec. III-A, Fig. 3. They model the BE, TT and WA modes. For simplicity reasons the model

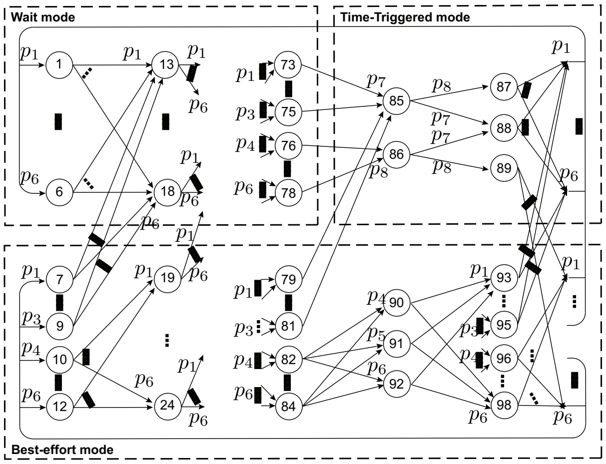


Fig. 4. Markov chain model of the Hybrid control and MAC architecture. This model is divided in best-effort, time-triggered and wait sub-models and incorporates probabilistic sampling, packet losses and occurrence of disturbances. The jump probabilities are defined as, $[p_1, ..., p_8] = [\alpha\gamma\beta, \alpha(1-\gamma)\beta, (1-\alpha)\beta, \alpha\gamma(1-\beta), \alpha(1-\gamma)(1-\beta), (1-\alpha)(1-\beta), \beta, 1-\beta]$

shows to the case where the number of CFP slots used is 2, i.e. $n_{\text{CFP}} = 2$, and total number of superframe slots $n_{\text{slots}} = 10$.

For each state in the best-effort mode there are six jumping possibilities ($p_1, ..., p_6$) in Fig. 4, which denote the combination of the following cases: sampling ($\theta_4(k)$), losing a packet ($\theta_2(k)$) and the occurrence of a disturbance ($\theta_1(k)$). This arises from the fact that the MAC scheme for this mode is a CA MAC. These probabilities can be grouped together in a vector $T_C = [\alpha\gamma\beta,\ \alpha(1-\gamma)\beta,\ (1-\alpha)\beta,\ \alpha\gamma(1-\beta),\ \alpha(1-\gamma)(1-\beta),\ (1-\alpha)(1-\beta)]$. The WA mode has the same state characteristics as the BE mode but different state transitions. The TT mode follows the WA mode, but a BE mode will follow a BE mode in the absence of disturbances. The states in the TT mode are characterized by no packet drops, Pr($\theta_2(k) = 1) = \gamma = 0$ and a deterministic sampling probability in the assigned CFP slot, Pr($\theta_4(k) = 1) = \alpha = 1$. Here we denote $p_7 = \beta$ and $p_8 = 1 - \beta$ in Fig. 4.

The corresponding transition matrix is defined by $\mathcal{P}$ and follows Fig. 4.

IV. Control over a hybrid MAC wireless network

In this section we present the existent MJLS tools to analyze the hybrid control and MAC architecture and propose a method to perform this analysis.

A. MJLS Analysis tools

Let us consider a general discrete-time Markov Jump Linear System (MJLS) $\mathcal{G}$ for a single plant as (2) [7],

$$\begin{aligned} x(k+1) &= A_{r(k)}x(k) + B_{r(k)}(u(k) + v(k)) \\ y(k) &= x(k) \\ v(k) &= w_1(k) + F_{r(k)}w_2(k) \end{aligned} \quad (11)$$

Moreover, $r(k)$ is a Markov chain with values in a finite set $\mathcal{N} = \{1, ..., N\}$ and transition probabilities

$$Pr(r(k+1) = j \mid r(k) = i) = p_{ij}(k),\ i, j \in \mathcal{N}. \quad (12)$$

The Markov chain has a transition probability matrix $\mathcal{P} = [p_{ij}]$ with distribution $\pi(i),\ i \in \mathcal{N}$. The transition probabilities are subject to the restrictions $p_{ij} \geq 0$ and $\sum_{j=1}^{N} p_{ij} = 1$ for any $i \in \mathcal{N}$. Furthermore, assume independence of the noises w_1 and w_2 from x and u for any given Markovian state. Let us define the operator $\mathcal{E}$ as follows: for $V = (V_1, ..., V_N)$, we set $\mathcal{E}(V, k) = (\mathcal{E}_1(V, k), ..., \mathcal{E}_N(V, k))$ as,

$$\mathcal{E}_i(V, k) = \sum_{j=1}^{N} p_{ij}(k)V_j \quad (13)$$

For now on we assume that when $r(k) = i$, the plant is in mode $i \in \mathcal{N}$ and: $A_{r(k)} = A_i,\ B_{r(k)} = B_i$ and $F_{r(k)} = F_i$.

In order to prove stability of the system in (11) the following must hold.

Lemma 1 (**[7], [12]**). *System (11) (without any input) is mean-square stable (MSS) if and only if there exist matrices $G_i > 0$ for $i \in \mathcal{N}$ that satisfy the following LMIs:*

$$G_i - A_i^T \mathcal{E}_i(G(k+1), k) A_i^T - S_i > 0 \text{ for } i \in \mathcal{N}.$$

The quadratic cost associated with system $\mathcal{G}$ with an admissible control law $u = (u(0), ..., u(T-1))$ and initial conditions (x_0, r_0) is denoted by $J(r_0, x_0, u)$ and is given by,

$$\mathcal{J} = E \sum_{i=1}^{N} \Big[\frac{1}{T} \big(\sum_{k=0}^{T-1} (x^T(k)\mathcal{Q}_i x(k) + u^T(k)\mathcal{R}_i u(k)) \big] + x^T(T)\mathcal{V}_i x(T) \big) \Big], \quad (14)$$

where $\mathcal{V}_i$, $\mathcal{Q}_i$ and $\mathcal{R}_i$ are positive semidefinite and mode i dependent, and T is the time horizon for cost evaluation. Solving the cost function $\mathcal{J}$ in (14) for a given control law $u(k) = -L_i x(k)$ gives raise to the following Lemma (inspired from [7] (Thm 4.2)).

Lemma 2. *Define a coupled recursive Lyapunov equation (CRLE) for $i \in \mathcal{N}$ and $k = T-1, ..., 0$, as*

$$X_i(k) = A_i^{C^T} \mathcal{E}_i(X(k+1), k) A_i^C + \begin{bmatrix} I & -L_{i(k)}^T \end{bmatrix} \Theta_i \begin{bmatrix} I \\ -L_{i(k)} \end{bmatrix} \quad (15)$$

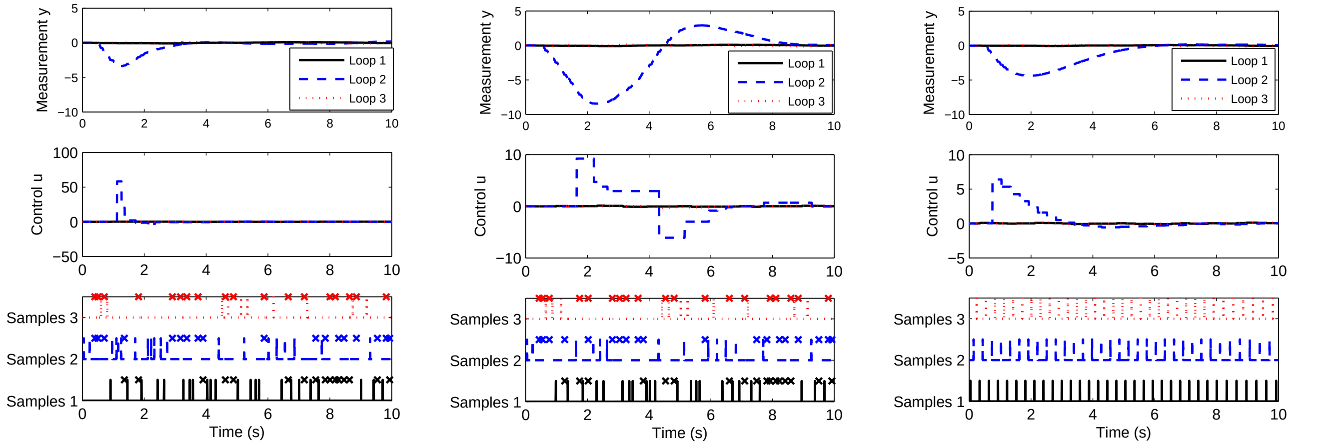
where $X_i(T) = \mathcal{V}_i$, $\Theta_i = \begin{bmatrix} \mathcal{Q}_i & 0 \\ 0 & \mathcal{R}_i \end{bmatrix}$ and $A_i^C = (A_i - B_i L_i)$.

The value of the cost (14) is given by,

$$\mathcal{J} = \sum_{i=1}^{N} \frac{1}{T} \big(x(0)^T X_i(0) x(0) + \sum_{k=0}^{T-1} \pi_i(k) tr((R_{w_1} + F_{i(k)}^T R_{w_2} F_{i(k)}) \mathcal{E}_i(X(k+1), k)) \big) \quad (16)$$

The CRLE values $X_i(k)$, are stored in a table for each given step k and are used to obtain the cost value $\mathcal{J}$.

Next, we propose a method to analyze stability and performance of the MJLS model for control over a hybrid MAC.



(a) Hybrid MAC scheme with hybrid control law $\{L_1, L_0\}$ and $\alpha = 0.33$.

(b) Pure CA MAC scheme with control law L_0 and $\alpha = 0.33$.

(c) Pure CF MAC scheme with control law L_0 and sampling period 0.3

Fig. 5. Closed loop time response analysis of three plants sharing a network under different MAC schemes. A disturbance occurs in plant 2 at time $t = 0.5$.

B. Methodology

We are now able to propose a method to analyze stability and performance of the MJLS model depicted in Sec. III and the tools presented above.

According to the closed-loop characteristics from Sec. III-C of each mode and the Markovian jump parameters previously defined in Sec. II and III-A, it is required to ensure that the overall system remains stable. The origin of a possible unstable behavior is twofold: i) Packet dropouts in the BE mode where loops share the network resources in a CA MAC and packets sent by sensors are subjected to loss and ii) Control law switching where we have to guaranty that jumping from a mode to another, does not result in any unstable behavior.

The effects of network communication over a hybrid MAC and the hybrid control scheme have been modelled through a set of jumping parameters $\theta_i(k)$, $i = \{1, ..., 4\}$, resulting in a MJLS of the form of (10). A previous result, from [7], on performance assessment in terms of a cost function (14) has been extended in Lemma 2 to deal with general control $u(k)$. To this end, the CRLE has been introduced and it can be shown that it is closely related to stability as defined in Lemma 1.

Let us summarize and describe the methodology as a procedure.

Procedure:

(a) Design separately the two LQ control, L_0 and L_1, with equations (6)-(7).

(b) Build the closed-loop state matrices structured as in (10) and assigned per each mode in the Markov model defined in Section III-D (see Fig. 4).

(c) Apply Lemma 1 to assess the stability of the overall scheme.

(d) Apply Lemma 2 to calculate the cost associated to the hybrid control over the hybrid MAC.

It is worth to note that if a stationary solution X_i is derived from (15) with a positive definite matrix Θ then it implies stability of the system and the step (d) can be skipped (proof is omitted because of the space limitation).

Next, we illustrate the hybrid MAC architecture with an example and analyze its performance when compared to other MAC schemes.

V. Example

As an example we consider the double integrator in the form of (1) with,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; R_{w_1} = \sqrt{0.1}I, \; R_{w_2} = \sqrt{22000}I.$$

The control laws L_0 and L_1 are set with, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $R = 0.01$, and sampling periods $h_s = 0.65s$, $h_f = 0.1s$, respectively. The cost function $\mathcal{J}$ (14) has parameters $\mathcal{Q}_i = \mathcal{V}_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathcal{R}_i = 0, \forall\ i \in \mathcal{N}$. Furthermore $x_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$.

The wireless network is composed of three control loops, with one sensor and one controller each as depicted in Fig. 1 and configure according Sec. III-A. The detection threshold is set to $d_{th} = 2$ for all sensor nodes. The network is simulated using the Truetime ([13]) wireless network block of the IEEE 802.15.4 Std., with no retransmissions due to collisions and channel busyness. With this setup we simulate a slotted α-persistent channel access. The sampling probability is set $\alpha \simeq 0.33$. The slot time $t_{\text{slot}} = 0.1s$, the number of slots on each superframe is set to $n_{\text{slots}} = 10$, and $n_{\text{CFP}} = 2$. Moreover, we define a pure CF MAC scheme as based on a traditional round-robin scheduling scheme with sequence $1, 2, 3, 1, 2, 3, ...,$ since it allows the highest fairness in the network.

A. Time Response Analysis

Fig. 5(a), 5(b), 5(c) show the time response analysis of the simulated system under Hybrid MAC, pure CA MAC and pure CF MAC schemes. The regulation of all the loops is successfully performed and the systems are stable under all MAC schemes.

A disturbance w_2 is active in plant 2, at $t = 0.5$. In the Hybrid MAC case the detection is reported from plant 2 at $t = 0.6s$. It is clear that the disturbance rejection is more efficient under the Hybrid MAC scheme when compared to the other two. The contribution of the fast control L_1 at $t = 1s$ during the TT mode clearly improves the regulation, even though it is observed a large communication loss (55%). The communication losses contribute for an average sampling period $h \simeq 0.65s$ in the Hybrid and CA MAC against $h = 0.3s$ for the CF MAC. Interestingly, the effect of the consecutive control action overcomes the worsen of performance given by a much larger sampling period.

Next we analyze the analytical and numerical quadratic cost performance of the hybrid architecture.

B. Performance Analysis

Here we investigate how the performance cost $\mathcal{J}$ in (14) using the Hybrid architecture proposed when compared to a pure CA and CF MAC schemes. At the same time we validate the Markov model presented in Sec. III-D, according to the TrueTime simulation model of the Hybrid architecture.

Considering the same system and parameters setup as in the example, we achieve the results presented in Table I for the of disturbance probability of 1%. The numerical value of the cost is calculated using (14) and the analytical value using (16). The Markov models of a pure CA and CF MAC are defined in [14]. The numerical analysis run-time was 2 minutes on a Intel Centrino 2 @ 2.0GHz. The analytical results were achieved through Monte Carlo simulations with duration of 10^6 simulation seconds.

The case of no disturbance is not presented here but the results shown that the CF MAC scheme outperforms the other two schemes. In this case the TT mode is not used in the hybrid architecture and it then a pure CA MAC. As shown in time response analysis the CA MAC decreases performance due to a much larger sampling period.

We observe in Table I that the Hybrid MAC has a lower quadratic control cost against the other two schemes as was expected from the time response analysis. Moreover, we observe that the performance of the undisturbed loops under the Hybrid MAC is worst since two CFP slots are used in loop 2 during the TT mode. Even though this occurs, the benefit of using the TT mode in loop 2, gives a lower overall control cost for all the loops. From Table I we confirm that the numerical and analytical values of the cost are fairly consistent and so the Markov models approximate the Truetime simulation.

TABLE I

$J_i = \{\text{numerical}, \text{analytical}\}$ COST (10^{-3}) OF A SINGLE PLANT SHARING A WIRELESS NETWORK UNDER THE VARIOUS MEDIUM ACCESS SCHEMES, WITH DISTURBANCE w_2 PROBABILITY $\Pr(\theta_1) = 1) = 0.01$.

Scheme	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	$\sum \mathcal{J}_i$
Hybrid	$\{5.5, 8.1\}$	$\{490, 640\}$	$\{5.3, 8.1\}$	$\{501, 656\}$
CF	$\{0.8, 0.8\}$	$\{690, 730\}$	$\{0.8, 0.8\}$	$\{692, 732\}$
CA	$\{4.5, 7.4\}$	$\{1190, 1250\}$	$\{4.3, 7.4\}$	$\{1197, 1398\}$

VI. Conclusion and Future Work

In this work we proposed a novel architecture for control over wireless networked systems in order to improve control performance. The design is based on allowing a hybrid control policy to run on a hybrid communication medium access scheme. We introduced a Markov model to characterize the proposed architecture and analysis tools from MJLS theory. Through analytical and numerical calculations we were able to show the effective improvement on performance of the control loops sharing the wireless network.

The future work will consist of replacing the α-persistent MAC to CSMA/CA with retransmissions, allowing more flexibility and less packet losses, but introducing delays. In accordance with this MAC we intend to design event-based controllers that guaranty stability and performance while in a BE mode.

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