IT for Statistics and Learning
2023

Assignment 6
Assigned: Friday, Dec 15, 2023
Due: Thursdsy, Dec 21, 2023 M. Skoglund

Problem 6.1: Prove that if $P_{S|Z^n}$ is ε stable, then it is ε information stable for any Q.

Problem 6.2: Prove that if $\Pr(X \in [a,b]) = 1$ for $-\infty < a \leq b < \infty$ then X is σ^2-sub-Gaussian with $\sigma^2 = (b-a)^2/4$

Problem 6.3: Let $g(S, Z^n) = L_Q(S) - L_{Z^n}(S)$. Assume that $\ell(s, Z)$ is σ^2-sub-Gaussian for each s and let

$$i(S, Z^n) = \ln \frac{dP_{S,Z^n}}{d(P_S \otimes P_{Z^n})}$$

(assuming $P_{S,Z^n} \ll P_S \otimes P_{Z^n}$). Prove that

$$E_{P_{S,Z^n}}\left[\exp\left(\lambda g(S, Z^n) - \frac{\lambda^2\sigma^2}{2n} - i(S, Z^n)\right)\right] \leq 1$$

using the change-of-measure formula

$$E_{P_{X,Y}}\left[f(X,Y)\right] = E_{P_X \otimes P_Y}\left[e^{i(X,Y)} f(X,Y)\right]$$

Problem 6.4: Use the bound from Problem 6.3 to prove that

$$|E[g(S, Z^N)]| \leq \sqrt{\frac{2\sigma^2}{n} I(S; Z^n)}$$

if $\ell(s, Z)$ is σ^2-sub-Gaussian for each s

Problem 6.5: Consider the binary classifier for samples in $\mathbb{R}^2$ that separates points by a straight line, partitioning $\mathbb{R}^2$ into two half-spaces and assigning one label to one half-space and another label to points in the other half-space. Prove that the VC dimension of this set of classifiers is $d = 3$.