Probability and Random Processes
2024

Assignment 1
Assigned: Thursday, Jan 18, 2024
Due: Thursday, Jan 25, 2024 M. Skoglund

Problem 1.1: For sequences $\{x_n\}$ and $\{y_n\}$,

1. define $\limsup x_n$ and $\liminf x_n$
2. prove that
$$\limsup(x_n + y_n) \leq \limsup x_n + \limsup y_n$$
3. if $\lim y_n < \infty$ exists, prove that
$$\limsup(x_n + y_n) = \limsup x_n + \lim y_n$$
4. let
$$a_n = \frac{1}{n} \sum_{k=1}^{n} x_k$$
prove that
$$\liminf x_n \leq \liminf a_n \leq \limsup a_n \leq \limsup x_n,$$
and conclude that if $\lim x_n$ exists, so does $\lim a_n$; does the converse hold?

Problem 1.2: Prove that a function $f(x)$ is continuous iff $f^{-1}(O)$ is open for every open $O \subset \mathbb{R}$

Problem 1.3: Define Lebesgue outer measure λ^*, and explain what goes wrong when trying to use λ^* as a universal measure for "length" on the real line. Then define Lebesgue measure λ and motivate the definition.

Problem 1.4: For a function $f(x)$, define what it means for f to be Lebesgue measurable. Based on the definition, argue that all continuous functions are Lebesgue measurable but there are Lebesgue measurable functions that are not continuous.