Supervisory Control Design for Networked Systems with Time-Varying Communication Delays

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Abstract: This paper proposes a supervisory control structure for networked systems with time-varying delays. The control structure, in which a supervisor triggers the most appropriate controller from a multi-controller unit, aims at improving the closed-loop performance relative to what can be obtained using a single robust controller. Our analysis considers average dwell-time switching and is based on a novel multiple Lyapunov-Krasovskii functional. We develop analysis conditions that can be verified by semi-definite programming, and show that associated state feedback synthesis problem also can be solved using convex optimization. Small and large scale networked control systems are used to illustrate the effectiveness of our approach.

1. INTRODUCTION

Networked control systems are distributed systems that use shared communication networks to exchange information between system components such as sensors, controllers and actuators (Zhang et al. [2001], Walsh et al. [2002]). The networked control system architecture promises advantages in terms of increased flexibility, reduced wiring, and lower maintenance costs, and is finding its way into a wide variety of applications, ranging from automobiles and automated highway systems, to process control and power distribution systems, see *e.g.*, Walsh et al. [2002] – Navet et al. [2005].

The use of a shared communication network introduces time-varying information delays and losses which may deteriorate system performance, even to the point where the closed-loop system becomes unstable. A conservative approach is to design a robust controller that considers the worst-case (maximal) delay. However, this might cause poor performance if the actual delay is only rarely close to its upper bound. Therefore, there is currently a renewed interest in adapting the control law to the delay evolution (*e.g.* Chen et al. [2006], Hirche et al. [2006], Jiang et al. [2009]). Inspired by the delay evolution that we have experienced in applications, see Figure 1, we design supervisory control scheme in the sense of Morse [1996]. This control architecture consists of a finite number of controllers, each designed for a bounded delay variation (corresponding, *e.g.*, to low, medium and high, network load) and a supervisor which orchestrates the switchings among them.

The analysis of switched systems with fixed time-delays is challenging and has attracted significant attention in the literature, *e.g.* Xie and Wang [2005], Hirche et al. [2006],

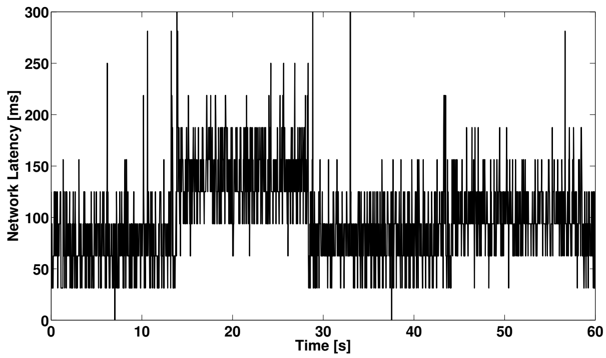

Fig. 1. The figure shows a recorded delay trace from the multi-hop wireless networking protocol used for networked control in Witrant et al. [2007]. The delay exhibits distinct mode changes (here corresponding to one, two or three-hop communication) and varies around its piecewise constant mode-dependent mean. Similar behavior was reported by Kuwata et al. [2008], who measured the delay of sensor data sent over a CAN bus. Their delay varied between 10-20 ms, but increased abruptly to around 150 ms under certain network conditions.

Sun et al. [2006], Yan and Özbay [2008]. Only recently, attempts to analyze switched systems with time-varying delays have begun to appear. Distinctively, Jiang et al. [2009] have constructed multiple Lyapunov-Krasovskii functionals to guarantee the stability under minimum dwell-time condition for interval time-varying delays.

In this work, we analyze our proposed supervisory control structure by combining a novel multiple Lyapunov-Krasovskii functional with the assumption of average dwell-time switching. The average dwell-time concept, in-

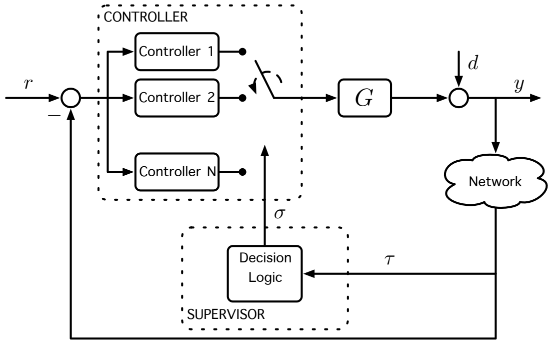

Fig. 2. The general block scheme of the proposed supervisory control structure.

troduced by Hespanha and Morse [1999], is a natural deterministic abstraction of load changes in communication networks, where minimal or maximal guarantees for the duration of a certain traffic condition is hard to guarantee. We demonstrate that the existence of a multiple Lyapunov-Krasovskii functional that ensures closed-loop stability under average dwell-time switching can be verified by solving a set of linear matrix inequalities. In addition, we show that the state feedback synthesis problem for the proposed supervisory control structure also can be solved via semi-definite programming.

The organization of the rest of the paper is as follows. Section 2 introduces the control structure used throughout the paper and formalizes the relevant analysis and synthesis problems. In Section 3, the multiple Lyapunov-Krasovskii functionals are constructed for analyzing the exponential stability of supervisory control system under average dwell-time switchings. Then LMI conditions that verify the existence of multiple Lyapunov functionals are derived. State-feedback synthesis conditions are given in Section 4. Numerical examples are used to demonstrate the effectiveness of the proposed techniques in Section 5.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes the n–dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices, and the notation $P > 0$ for $P \in \mathbb{R}^{n\times n}$, means that P is a symmetric and positive definite matrix. Additionally, '$\star$' represents symmetric terms in symmetric matrices and in quadratic forms, $\otimes$ denotes the Kronecker product, and $\mathbb{R}_{\geq 0}$ is the interval $[0, \infty)$.

2. SYSTEM MODELING

First consider the supervisory control system setup in Figure 2. The switched linear system with time-varying delays is

$$\begin{array}{ll} \dot{x}(t) = Ax(t) + A_{\sigma(t)}x\big(t - \tau_{\sigma(t)}(t)\big) & \forall t \in \mathbb{R}_{\geq 0} \\ x(t) = \phi(t) & \forall t \in [-h_{M+1}, 0] \end{array} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $A \in \mathbb{R}^{n\times n}$ and $A_{\sigma(t)} \in \mathbb{R}^{n\times n}$ are the known system matrices and $\sigma : \mathbb{R}_{\geq 0} \mapsto \mathcal{M}$ with $\mathcal{M} = \{1, \ldots, M\}$, for each $t \in \mathbb{R}_{\geq 0}$, is the switching control signal. $\tau_{\sigma(t)}(t)$ is the time-varying delay function satisfying

$$h_1 \leq \underline{h}_{\sigma(t)} \leq \tau_{\sigma(t)}(t) \leq \overline{h}_{\sigma(t)} \leq h_{M+1} \ .$$

Here, $\phi(t) \in C([-h_{M+1}, 0], \mathbb{R}^n)$ is the initial function where $C([-h_{M+1}, 0], \mathbb{R}^n)$ is the Banach space of continuous functions defined on $[-h_{M+1}, 0]$.

Definition 1. The system (1) is exponentially stable under the switching signal $\sigma(t)$ if there exist positive γ and α such that the solution of $x(t)$ of the system (1) satisfies

$$||x(t)|| \leq \gamma ||x(t_0)||_c \ e^{-\alpha(t-t_0)}, \quad t \geq t_0 \tag{2}$$

where $||x(t_0)||_c \triangleq \sup\limits_{-h_{M+1}\leq\theta\leq 0} \{||x(t_0 + \theta)||, ||\dot{x}(t_0 + \theta)||\}$.

In order to guarantee exponential stability, we will put restrictions on the switching signal $\sigma(t)$. Specifically, we will assume that the signal satisfies an average dwell-time condition in the following sense.

Definition 2. (Liberzon [2003]). We denote the number of jumps of a switching signal σ on the interval (t, T) by $N_\sigma(T, t)$. Then we say that σ has the *average dwell-time* τ_a if there exist two positive numbers N_0 and τ_a such that

$$N_\sigma(T, t) \leq N_0 + \frac{T - t}{\tau_a}, \quad \forall T > t \geq 0\,. \tag{3}$$

Let $\mathcal{S}[\tau_a]$ be the set of all switching signals satisfying the above condition.

According to Liberzon [2003], the constant N_0 affects the overshoot bound for Lyapunov-type stability but otherwise does not change stability properties of the switched system. For this reason, we also choose $N_0 = 0$ as commonly used in the literature.

We consider two specific problems in this paper. The first is to verify that a given switched linear system (1) is exponentially stable under average dwell-time switching. The second one is to design state feedback controllers for each mode i such that the supervisory control system is exponentially stable with guaranteed convergence rate under average dwell-time switching.

3. EXPONENTIAL STABILITY ANALYSIS USING MULTIPLE LYAPUNOV–KRASOVSKII FUNCTIONALS

The exponential stability of switched system (1) is equivalent to the existence of scalar $\alpha \in \mathbb{R}_{\geq 0}$ such that $e^{\alpha t}||x(t)||$ asymptotically converges to zero for each $\sigma \in \mathcal{S}[\tau_a]$. In order to characterize the rate of convergence of the system (1), let us consider the change of variables $\xi(t) \triangleq e^{\alpha t}x(t)$. Then we have:

$$\begin{aligned} \dot{\xi}(t) &= \alpha e^{\alpha t}x(t) + e^{\alpha t}\dot{x}(t) \\ &= \alpha\xi(t) + e^{\alpha t}\big(Ax(t) + A_i x(t - \tau_i(t))\big) \\ &= \alpha\xi(t) + A\xi(t) + A_i e^{\alpha t} x\big(t - \tau_i(t)\big) \\ &= (\alpha I_n + A)\xi(t) + e^{\alpha\tau_i(t)} A_i \xi\big(t - \tau_i(t)\big)\,, \end{aligned} \tag{4}$$

where $\tau_i(t) \in [h_i, h_{i+1})$, $\forall i \in \mathcal{M}$.

Because of the change of variables, the switched system model (4) has a time-varying coefficient. Similar to Seuret et al. [2004], we can rewrite the system model (4) in a polytopic form to get rid of time-varying term by using the bounded delay values for each mode. For this purpose, we express the term $e^{\alpha\tau_i(t)}$ as a convex combination of the bounds $e^{\alpha h_i}$ and $e^{\alpha h_{i+1}}$:

$$e^{\alpha\tau_i(t)} = \lambda_1(t)e^{\alpha h_i} + \lambda_2(t)e^{\alpha h_{i+1}}\,, \quad \forall i \in \mathcal{M} \tag{5}$$

where $\lambda_1(t), \lambda_2(t) \in \mathbb{R}_{\geq 0}$ and $\lambda_1(t) + \lambda_2(t) = 1$, $\forall t \in \mathbb{R}_{\geq 0}$. Hence, the delayed differential equation (1) is rewritten as

$$\dot{\xi}(t) = \bar{A}\xi(t) + \sum_{j=1}^{2} \lambda_j(t)\bar{A}_{ij}\xi\big(t-\tau_i(t)\big) \quad (6)$$

where $\bar{A} \triangleq (\alpha I_n + A)$ and $\bar{A}_{ij} \triangleq \varrho_{ij}A_i$ with $\varrho_{ij} \triangleq e^{\alpha h_{i+j-1}}$ when $\tau_i(t) \in [h_i, h_{i+1})$, $\forall i \in \mathcal{M}$.

We combine a novel multiple Lyapunov-Krasovskii functional with the dwell-time approach of Hespanha and Morse [1999] to establish exponential stability of the switched system (1). For ease of notation, we state the theorem for the case of two delay modes only, but the approach extends immediately to a system with M modes.

Theorem 3. There exists a finite constant τ_a such that the switched linear system (1) is exponentially stable over $\mathcal{S}[\tau_a]$ with a given decay rate α for time-varying delays $\tau_i(t) \in [h_i, h_{i+1})$, $\forall i \in \{1, 2\}$ if there exist real matrices $P_i > 0$, $Q_k^i > 0$, $R_k^i > 0$, $S_k^i > 0$ and $T_k^i > 0$, $\forall i, k \in \{1, 2\}$ and a constant scalar $\mu > 1$ satisfying $P_i \le \mu P_j$, $Q_k^i \le \mu Q_k^j$, $R_k^i \le \mu R_k^j$, $S_k^i \le \mu S_k^j$ and $T_k^i \le \mu T_k^j$, $\forall i, j, k \in \{1, 2\}$ such that the LMIs $\Gamma_j^i < 0$ given in (7) and (8) hold for all $i, j \in \{1, 2\}$.

Remark 1. Note that the decay rate α enters the LMIs (7) and (8) in $\bar{A}$ and $\bar{A}_{ij}$.

Proof of Theorem 3: Analogously to Hespanha and Morse [1999], our claim follows if we can find the Lyapunov Krasovskii functionals $V_i(t)$ that guarantee decay rate α while in mode i and a constant $\mu > 1$ such that $V_i(t) \le \mu V_j(t)$ $\forall i, j \in \{1, 2\}$. Then (1) is exponentially stable for every switching signal σ with average dwell-time $\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}$.

The following Lyapunov-Krasovskii functional inspired from Shao [2008] is considered

$$\begin{aligned} V_i(t) &= \xi(t)^T P_i \xi(t) + \sum_{k=1}^{2} \int_{t-h_k}^{t} \xi(s)^T Q_k^i \xi(s) ds \\ &+ \sum_{k=1}^{2} \int_{t-h_{k+1}}^{t} \xi(s)^T R_k^i \xi(s) ds \\ &+ \sum_{k=1}^{2} \int_{-h_k}^{0} \int_{t+s}^{t} h_k \dot{\xi}(\theta)^T S_k^i \dot{\xi}(\theta) d\theta ds \\ &+ \sum_{k=1}^{2} \int_{-h_{k+1}}^{-h_k} \int_{t+s}^{t} \delta_k \dot{\xi}(\theta)^T T_k^i \dot{\xi}(\theta) d\theta ds \ , \end{aligned} \quad (9)$$

(denoting $\delta_k \triangleq h_{k+1} - h_k$).

The derivative of $V_i(t)$ along the trajectory of the system is given by

$$\begin{aligned} \dot{V}_i(t) &= 2\xi(t)^T P_i \dot{\xi}(t) + \xi(t)^T \left[\sum_{k=1}^{2} \left(Q_k^i + R_k^i \right) \right] \xi(t) \\ &- \sum_{k=1}^{2} \xi(t-h_k)^T Q_k^i \xi(t-h_k) - \sum_{k=1}^{2} \xi(t-h_{k+1})^T R_k^i \xi(t-h_{k+1}) \\ &+ \dot{\xi}(t)^T \left[\sum_{k=1}^{2} \left(h_k^2 S_k^i + \delta_k^2 T_k^i \right) \right] \dot{\xi}(t) - \sum_{k=1}^{2} \int_{t-h_k}^{t} h_k \dot{\xi}(s)^T S_k^i \dot{\xi}(s) ds \\ &- \sum_{k=1}^{2} \int_{t-h_{k+1}}^{t-h_k} \delta_k \dot{\xi}(s)^T T_k^i \dot{\xi}(s) ds \ . \end{aligned} \quad (10)$$

Using the Jensen's inequality (Gu et al. [2003]), the integral terms in the preceding equality are bounded as

$$\begin{aligned} &- \int_{t-h_k}^{t} h_k \dot{\xi}(s)^T S_k^i \dot{\xi}(s) ds \\ &\qquad \le -\big(\xi(t) - \xi(t-h_k)\big)^T S_k^i \big(\xi(t) - \xi(t-h_k)\big) \end{aligned}$$

and

$$\begin{aligned} &- \int_{t-h_{k+1}}^{t-h_k} \delta_k \dot{\xi}(s)^T T_k^i \dot{\xi}(s) ds \\ &\quad \le - \int_{t-h_{k+1}}^{t-\tau_k(t)} (h_{k+1} - \tau_k(t)) \dot{\xi}(s)^T T_k^i \dot{\xi}(s) ds \\ &\quad - \int_{t-\tau_k(t)}^{t-h_k} (\tau_k(t) - h_k) \dot{\xi}(s)^T T_k^i \dot{\xi}(s) ds \\ &\le -\big(\xi(t-\tau_k(t)) - \xi(t-h_{k+1})\big)^T T_k^i \big(\xi(t-\tau_k(t)) - \xi(t-h_{k+1})\big) \\ &- \big(\xi(t-h_k) - \xi(t-\tau_k(t))\big)^T T_k^i \big(\xi(t-h_k) - \xi(t-\tau_k(t))\big) \ . \end{aligned}$$

Then this yields

$$\begin{aligned} \dot{V}_i(t) &\le \xi(t)^T \Bigg[\bar{A}^T P_i + P_i \bar{A} + \sum_{k=1}^{2} \left(Q_k^i + R_k^i - S_k^i \right) \\ &+ \bar{A}^T \sum_{k=1}^{2} \left(h_k^2 S_k^i + \delta_k^2 T_k^i \right) \bar{A} \Bigg] \xi(t) + 2\xi(t)^T \Bigg[P_i \sum_{j=1}^{2} \left(\lambda_j(t) \bar{A}_{ij} \right) \\ &+ \bar{A}^T \sum_{k=1}^{2} \left(h_k^2 S_k^i + \delta_k^2 T_k^i \right) \sum_{j=1}^{2} \left(\lambda_j(t) \bar{A}_{ij} \right) \Bigg] \xi(t-\tau_i(t)) \\ &- \xi(t-h_1)^T (Q_1^i + T_1^i + S_1^i) \xi(t-h_1) + 2 \sum_{k=1}^{2} \xi(t)^T S_k^i \xi(t-h_k) \\ &- \xi(t-h_2)^T \left(Q_2^i + S_2^i + \sum_{k=1}^{2} T_k^i + R_1^i \right) \xi(t-h_2) \\ &- \xi(t-h_3)^T (T_2^i + R_2^i) \xi(t-h_3) + 2 \sum_{k=1}^{2} \xi(t-\tau_k(t))^T T_k^i \\ &\times \xi(t-h_k) - 2 \sum_{k=1}^{2} \xi(t-\tau_k(t))^T T_k^i \xi(t-\tau_k(t)) + \xi(t-\tau_i(t))^T \\ &\times \sum_{j=1}^{2} \left(\lambda_j(t) \bar{A}_{ij} \right)^T \left[\sum_{k=1}^{2} \left(h_k^2 S_k^i + \delta_k^2 T_k^2 \right) \right] \sum_{j=1}^{2} \left(\lambda_j(t) \bar{A}_{ij} \right) \\ &\times \xi(t-\tau_i(t)) + 2 \sum_{k=1}^{2} \xi(t-\tau_k(t))^T T_k^i \xi(t-h_{k+1}) \\ &= \psi(t)^T \tilde{\Gamma}^i(t) \psi(t) \ , \end{aligned}$$

where $\psi(t) = \operatorname{col}\{\xi(t), \xi(t-\tau_1(t)), \xi(t-\tau_2(t)), \xi(t-h_1), \xi(t-h_2), \xi(t-h_3)\}$.

Note that the time derivative of $V_i(t)$ is bounded by a quadratic function in $\psi(t)$, i.e.

$$\dot{V}_i(t) \le \psi(t)^T \tilde{\Gamma}^i(t) \psi(t) \ ,$$

with

$$\tilde{\Gamma}^i(t) = \lambda_1(t) \tilde{\Gamma}_1^i + \lambda_2(t) \tilde{\Gamma}_2^i$$

for all $i \in \{1, 2\}$. Then, for two different modes, we write the following two matrices:

$$\Gamma_j^1 = \left[\begin{array}{c|cc|ccc|cccc} \Phi_1 & P_1\bar{A}_{1j} & 0 & S_1^1 & S_2^1 & 0 & h_1\bar{A}^T S_1^1 & \delta_1\bar{A}^T T_1^1 & h_2\bar{A}^T S_2^1 & \delta_2\bar{A}^T T_2^1 \\ \hline \star & -2T_1^1 & 0 & T_1^1 & T_1^1 & 0 & h_1\bar{A}_{1j}^T S_1^1 & \delta_1\bar{A}_{1j}^T T_1^1 & h_2\bar{A}_{1j}^T S_2^1 & \delta_2\bar{A}_{1j}^T T_2^1 \\ \star & \star & -2T_2^1 & 0 & T_2^1 & T_2^1 & 0 & 0 & 0 & 0 \\ \hline \star & \star & \star & -\Xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\Xi_2^1 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\Xi_3^1 & 0 & 0 & 0 & 0 \\ \hline \star & \star & \star & \star & \star & \star & -S_1^1 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -T_1^1 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -S_2^1 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -T_2^1 \end{array}\right] < 0 \ \forall j \in \{1,2\} \tag{7}$$

$$\Gamma_j^2 = \left[\begin{array}{c|cc|ccc|cccc} \Phi_2 & 0 & P_2\bar{A}_{2j} & S_1^2 & S_2^2 & 0 & h_1\bar{A}^T S_1^2 & \delta_1\bar{A}^T T_1^2 & h_2\bar{A}^T S_2^2 & \delta_2\bar{A}^T T_2^2 \\ \hline \star & -2T_1^2 & 0 & T_1^2 & T_1^2 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -2T_2^2 & 0 & T_2^2 & T_2^2 & h_1\bar{A}_{2j}^T S_1^2 & \delta_1\bar{A}_{2j}^T T_1^2 & h_2\bar{A}_{2j}^T S_2^2 & \delta_2\bar{A}_{2j}^T T_2^2 \\ \hline \star & \star & \star & -\Xi_1^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\Xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\Xi_3^2 & 0 & 0 & 0 & 0 \\ \hline \star & \star & \star & \star & \star & \star & -S_1^2 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -T_1^2 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -S_2^2 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -T_2^2 \end{array}\right] < 0 \ \forall j \in \{1,2\} \tag{8}$$

where $\Phi_i = P_i\bar{A} + \bar{A}^T P_i + \sum_{k=1}^2 \left(Q_k^i + R_k^i - S_k^i\right)$, $\Xi_1^i = Q_1^i + T_1^i + S_1^i$, $\Xi_2^i = Q_2^i + S_2^i + \sum_{k=1}^2 T_k^i + R_1^i$ and $\Xi_3^i = R_2^i + T_2^i \ \ \forall i \in \{1,2\}$.

$$\tilde{\Gamma}_j^1 = \left[\begin{array}{c|cc|ccc} \Phi_1 & P_1\bar{A}_{1j} & 0 & S_1^1 & S_2^1 & 0 \\ \hline \star & -2T_1^1 & 0 & T_1^1 & T_1^1 & 0 \\ \star & \star & -2T_2^1 & 0 & T_2^1 & T_2^1 \\ \hline \star & \star & \star & -\Xi_1^1 & 0 & 0 \\ \star & \star & \star & \star & -\Xi_2^1 & 0 \\ \star & \star & \star & \star & \star & -\Xi_3^1 \end{array}\right] + \phi_1^T \sum_{k=1}^{2} \left(h_k^2 S_k^1 + \delta_k^2 T_k^1\right)\phi_1 \,, \tag{11}$$

where $\phi_1 = \left[\ \bar{A}\ \bar{A}_{1j}\ 0_{n\times 4n}\ \right]$ for all $j \in \{1,2\}$, and

$$\tilde{\Gamma}_j^2 = \left[\begin{array}{c|cc|ccc} \Phi_2 & 0 & P_2\bar{A}_{2j} & S_1^2 & S_2^2 & 0 \\ \hline \star & -2T_1^2 & 0 & T_1^2 & T_1^2 & 0 \\ \star & \star & -2T_2^2 & 0 & T_2^2 & T_2^2 \\ \hline \star & \star & \star & -\Xi_1^2 & 0 & 0 \\ \star & \star & \star & \star & -\Xi_2^2 & 0 \\ \star & \star & \star & \star & \star & -\Xi_3^2 \end{array}\right] + \phi_2^T \sum_{k=1}^{2} \left(h_k^2 S_k^2 + \delta_k^2 T_k^2\right)\phi_2 \,, \tag{12}$$

where $\phi_2 = \left[\ \bar{A}\ 0_n\ \bar{A}_{2j}\ 0_{n\times 3n}\ \right]$ for all $j \in \{1,2\}$.

After applying the Schur complement twice to $\tilde{\Gamma}_j^i$ to form Γ_j^i, we arrive at an equivalent condition in terms of the new matrices Γ_j^i:

$$\Gamma^i(t) = \lambda_1(t)\Gamma_1^i + \lambda_2(t)\Gamma_2^i < 0 \quad \forall i \in \{1,2\}\,. \tag{13}$$

As argued above, the condition is satisfied if Γ_1^i and Γ_2^i are both negative definite.

By guaranteeing that $\Gamma^i(t) < 0$, we ensure that the dynamics in each fixed mode is exponentially stable with decay rate α. However, to guarantee stability for the switched system under the average dwell-time assumption, we also need to guarantee that

$$V_i(t) \le \mu V_j(t)\,, \quad \forall i,j \in \{1,2\} \tag{14}$$

for some constant $\mu > 1$. Noting that $V_i(t)$ is linear in P_i, Q_k^i, R_k^i, S_k^i and T_k^i, (14) is implied by the following conditions:

$$P_i \le \mu P_j\,,\ Q_k^i \le \mu Q_k^j\,,\ R_k^i \le \mu R_k^j\,,\ S_k^i \le \mu S_k^j\,,\ T_k^i \le \mu T_k^j$$

for all $i,j,k \in \{1,2\}$. This concludes the proof. □

Remark 2. The analysis procedure extends immediatelly to the system with M modes. However, the LMIs grow in both size and number. As distinct from the two-mode case, we need to check $2M$ LMIs (extensions of (7), (8)) whose dimensions are $2(2M+1)n \times 2(2M+1)n$, and $M(M-1)$ additional LMIs (*e.g.*, $P_i \le \mu P_j$). The LMIs use $M(4M+1)$ matrix variables, each with $n(n+1)/2$ decision variables.

Proposition 4. A lower bound on the average dwell-time ensuring the global stability of switched delay system (1) is determined by solving the following optimization problem

$$\alpha_\circ = \begin{cases} \underset{\substack{P_i>0, Q_k^i>0,\\ R_k^i>0, S_k^i>0, T_k^i>0}}{\text{maximize}} & \alpha \\ \text{subject to} & \begin{array}{l} \Gamma_j^i < 0, \\ P_i \le \mu P_j, Q_k^i \le \mu Q_k^j, \\ R_k^i \le \mu R_k^j, S_k^i \le \mu S_k^j, \\ T_k^i \le \mu T_k^j \end{array} \end{cases} \tag{15}$$

for all $i,j,k \in \{1,2\}$. Then the lower bound on average dwell-time is calculated as $\tau_a^\circ \triangleq \frac{\ln\mu}{\alpha_\circ}$.

4. STATE-FEEDBACK CONTROLLER DESIGN

In this section, we will extend our analysis conditions to state feedback design for exponential stability of the supervisory control structure introduced in Section 2. More precisely, we consider a linear time-invariant plant

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where the control input is a mode-dependent linear feedback of the delayed state vector, *i.e*,

$$u(t) = K_i x(t - \tau_i(t)) \tag{16}$$

when $\sigma(t) = i$ (and hence, $\tau_i(t) \in [h_i, h_i + 1)$), $i \in \mathcal{M}$. The design problem is to find feedback gain matrices K_i that ensure closed-loop stability for all switching signals in $\mathcal{S}[\tau_a]$. Clearly, this problem is closely related to the stability analysis problem considered in Section 3, since the supervisory control structure induces a switched linear system on the form (1) with $A_{\sigma(t)} = BK_{\sigma(t)}$. We have the following result:

Theorem 5. For a given decay rate α, there exists a state-feedback control of the form (16) which exponentially stablizes system (1) over $\mathcal{S}[\tau_a]$ for time-varying delays $\tau_i(t) \in [h_i, h_{i+1})$, $\forall i \in \{1, 2\}$ if there exist real constant matrices $\tilde{X}_i > 0$, $\tilde{P}_i > 0$, $\tilde{Q}^i_k > 0$, $\tilde{R}^i_k > 0$, $\tilde{S}^i_k > 0$ and $\tilde{T}^i_k > 0$, $\forall i, k \in \{1, 2\}$ and a constant scalar $\mu > 1$ such that the LMIs given in (19) and (20) for all $j \in \{1, 2\}$, and $\tilde{P}_i \le \mu \tilde{P}_j$, $\tilde{Q}^i_k \le \mu \tilde{Q}^j_k$, $\tilde{R}^i_k \le \mu \tilde{R}^j_k$, $\tilde{S}^i_k \le \mu \tilde{S}^j_k$ and $\tilde{T}^i_k \le \mu \tilde{T}^j_k$, $\forall i, j, k \in \{1, 2\}$ are feasible. A stabilizing control law is given by (16) with gain $K_i = \tilde{Y}_i \tilde{X}_i^{-1}$ for all $i \in \{1, 2\}$.

Proof: The structure of (7) and (8) is not suitable for the synthesis of a state-feedback controller due to the presence of multiple product terms $\bar{A}S^i_k$, $\bar{A}T^i_k$, $\bar{A}_{ij}S^i_k$ and $\bar{A}_{ij}T^i_k$. These product terms prevent finding a linearizing change of variable even after congruence transformation. Briat et al. [2010] have used a relaxation approach to solve the problem and decouple the multiple product at the expense of an increase of the conservatism. Let (19) and (20) be called as Θ^1_j and Θ^2_j, respectively, in the rest of this paper. Then we prove that $\Theta^i_j < 0$ $\forall i \in \{1, 2\}$ implies the feasibility of (7) and (8). Note that Θ^i_j can be decomposed as

$$\Theta^i_j = \Theta^i_j|_{X_i=0} + U_i^T X_i V_i + V_i^T X_i U_i < 0 \quad \forall i \in \{1, 2\}$$

where $U_1 = \begin{bmatrix} -I_n & \bar{A} & \bar{A}_{1j} & 0_{n\times 4n} & I_n & 0_{n\times 4n}\end{bmatrix}$, $V_1 = \begin{bmatrix} I_n & 0_{n\times 11n}\end{bmatrix}$, $U_2 = \begin{bmatrix} -I_n & \bar{A} & 0_n & \bar{A}_{1j} & 0_{n\times 3n} & I_n & 0_{n\times 4n}\end{bmatrix}$ and $V_2 = \begin{bmatrix} I_n & 0_{n\times 11n}\end{bmatrix}$. Then invoking the projection lemma (Gahinet and Apkarian [1994]), the feasibility of $\Theta^i_j < 0$ implies the feasibility of the LMIs

$$\mathcal{N}_{U_i}^T \Theta^i_j|_{X=0} \mathcal{N}_{U_i} < 0 \tag{21}$$

$$\mathcal{N}_{V_i}^T \Theta^i_j|_{X=0} \mathcal{N}_{V_i} < 0 \tag{22}$$

where $\mathcal{N}_{U_i}$ and $\mathcal{N}_{V_i}$ are basis of the null space of U_i and V_i, respectively. After some tedious calculations, we can show that LMIs (17) and (18) are equivalent to (7) and (8) showing that $\Theta^i_j < 0$ $\forall i \in \{1, 2\}$ implies the feasibility of (7) and (8). Moreover, LMI (22) characterizes the conservatism of the relaxation.

Since LMI (17) and (18) do not include any multiple product, it can easily be used for controller design. Hence, it is possible to use congruence transformations and change of variables so as to design the state-feedback controller. Performing a congruence transformation with respect to matrix $I_{12n} \otimes X^{-1}$ and applying the following linearizing change of variables $\tilde{X}_i \triangleq X_i^{-1}$, $\tilde{P}_i \triangleq X_i^{-T} P_i X_i^{-1}$, $\tilde{Q}^i_k \triangleq X_i^{-T} Q^i_k X_i^{-1}$, $\tilde{R}^i_k \triangleq X_i^{-T} R^i_k X_i^{-1}$, $\tilde{S}^i_k \triangleq X_i^{-T} S^i_k X_i^{-1}$, $\tilde{T}^i_k \triangleq X_i^{-T} T^i_k X_i^{-1}$, $\tilde{\Xi}^i_1 \triangleq X_i^{-T} \Xi^i_1 X_i^{-1}$, $\tilde{\Xi}^i_2 \triangleq X_i^{-T} \Xi^i_2 X_i^{-1}$, $\tilde{\Xi}^i_3 \triangleq X_i^{-T} \Xi^i_3 X_i^{-1}$ and $\tilde{Y}_i = K_i X_i^{-1}$, $\forall i, k \in \{1, 2\}$ yields LMI (19) and (20). □

5. NUMERICAL EXAMPLES

5.1 Small Scale Example

We consider the following linear system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.024 \end{bmatrix} u(t) \tag{23}$$

with a time-varying communication delay between sensor and controller that behaves as shown in Figure 3. The supervisor generates the switching signals shown in Figure 3 to trigger the suitable controller. The supervisory controller is

$$u(t) = \begin{cases} K_L x\big(t - \tau_L(t)\big) & \text{if } \tau_L(t) \in [0.05, 0.15) \\ K_H x\big(t - \tau_H(t)\big) & \text{if } \tau_H(t) \in [0.15, 0.40) \end{cases}. \tag{24}$$

The lower bound on the average dwell-time τ_a° is determined as 0.2340 s for given $\mu = 1.4$ by solving the optimization problem (15). Indeed, the maximum exponential decay rate $\alpha_\circ$ is computed as 1.44 in (15). As a result, the mode-dependent switching controllers are calculated as

$$K_L = [-744.6069 \ \ -74.5248] \ ,$$
$$K_H = [-578.0139 \ \ -57.6035] \ .$$

Additionally, using same Lyapunov Krasovskii functional, we design a classical state-feedback controller to compare the non-switching and switching control performance (in terms of rise time, settling time and maximum overshoot). For this reason, we apply the same Lyapunov Krasovskii functional (9) with $Q_k = 0$, $R_k = 0$, $S_k = 0$, $T_k = 0$ $\forall k > 1$ and $\tau(t) \in [0.05, 0.4)$. Then we define the maximum exponential decay α as 1.33 and the resulting controller

$$K = [-603.4151 \ \ -60.2007] \ .$$

The average dwell time of communication delays seen in Figure 3 satisfies the condition $\tau_a^\circ < \tau_a = 0.35$ s. Therefore, the supervisory control system is globally exponentially stable for given switching signal.

The switching control system is faster than the non-switching conrol system as shown in Figure 4. In addition to simulation results, we can compare the exponential decay rates of the switching and non-switching control systems. It is clearly seen that $\alpha_\circ > \alpha$ (1.44 > 1.33). This result shows that switching controller has slightly better performance than non-switching one.

5.2 Large Scale Example: Wide-Area Power Networks

To demonstrate the applicability of our methods to real system of higher dimension, we consider the IEEE nine-bus system (Anderson and Fouad [2003]) shown in Figure 5. We select the second order (swing) model with phase and

$$
\begin{bmatrix}
-X_1-X_1^T & X_1^T\bar{A}+P_1 & X_1^T\bar{A}_{1j} & 0 & 0 & 0 & 0 & X_1^T & h_1S_1^1 & \delta_1T_1^1 & h_2S_2^1 & \delta_2T_2^1\\
\star & \sum_{k=1}^{2}\left(Q_k^1+R_k^1-S_k^1\right)-P_1 & 0 & 0 & S_1^1 & S_2^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & -2T_1^1 & 0 & T_1^1 & T_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & -2T_2^1 & 0 & T_2^1 & T_2^1 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & -\Xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & -\Xi_2^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & -\Xi_3^1 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -P_1 & -h_1S_1^1 & -\delta_1T_1^1 & -h_2S_2^1 & -\delta_2T_2^1\\
\star & \star & \star & \star & \star & \star & \star & \star & -S_1^1 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -T_1^1 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -S_2^1 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -T_2^1
\end{bmatrix} < 0 \quad (17)
$$

$$
\begin{bmatrix}
-X_2-X_2^T & X_2^T\bar{A}+P_2 & 0 & X_2^T\bar{A}_{2j} & 0 & 0 & 0 & X_2^T & h_1S_1^2 & \delta_1T_1^2 & h_2S_2^2 & \delta_2T_2^2\\
\star & \sum_{k=1}^{2}\left(Q_k^2+R_k^2-S_k^2\right)-P_2 & 0 & 0 & S_1^2 & S_2^2 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & -2T_1^2 & 0 & T_1^2 & T_1^2 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & -2T_2^2 & 0 & T_2^2 & T_2^2 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & -\Xi_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & -\Xi_2^2 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & -\Xi_3^2 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -P_2 & -h_1S_1^2 & -\delta_1T_1^2 & -h_2S_2^2 & -\delta_2T_2^2\\
\star & \star & \star & \star & \star & \star & \star & \star & -S_1^2 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -T_1^2 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -S_2^2 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -T_2^2
\end{bmatrix} < 0 \quad (18)
$$

$$
\begin{bmatrix}
-\tilde{X}_1-\tilde{X}_1^T & \bar{A}\tilde{X}_1+\tilde{P}_1 & \varrho_{1j}B\tilde{Y}_1 & 0 & 0 & 0 & 0 & \tilde{X}_1^T & h_1\tilde{S}_1^1 & \delta_1\tilde{T}_1^1 & h_2\tilde{S}_2^1 & \delta_2\tilde{T}_2^1\\
\star & \sum_{k=1}^{2}\left(\tilde{Q}_k^1+\tilde{R}_k^1-\tilde{S}_k^1\right)-\tilde{P}_1 & 0 & 0 & \tilde{S}_1^1 & \tilde{S}_2^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & -2\tilde{T}_1^1 & 0 & \tilde{T}_1^1 & \tilde{T}_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & -2\tilde{T}_2^1 & 0 & \tilde{T}_2^1 & \tilde{T}_2^1 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & -\tilde{\Xi}_1^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & -\tilde{\Xi}_2^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & -\tilde{\Xi}_3^1 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -\tilde{P}_1 & -h_1\tilde{S}_1^1 & -\delta_1\tilde{T}_1^1 & -h_2\tilde{S}_2^1 & -\delta_2\tilde{T}_2^1\\
\star & \star & \star & \star & \star & \star & \star & \star & -\tilde{S}_1^1 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -\tilde{T}_1^1 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\tilde{S}_2^1 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\tilde{T}_2^1
\end{bmatrix} < 0 \quad (19)
$$

$$
\begin{bmatrix}
-\tilde{X}_2-\tilde{X}_2^T & \bar{A}\tilde{X}_2+\tilde{P}_2 & 0 & \varrho_{2j}B\tilde{Y}_2 & 0 & 0 & 0 & \tilde{X}_2^T & h_1\tilde{S}_1^2 & \delta_1\tilde{T}_1^2 & h_2\tilde{S}_2^2 & \delta_2\tilde{T}_2^2\\
\star & \sum_{k=1}^{2}\left(\tilde{Q}_k^2+\tilde{R}_k^2-\tilde{S}_k^2\right)-\tilde{P}_2 & 0 & 0 & \tilde{S}_1^2 & \tilde{S}_2^2 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & -2\tilde{T}_1^2 & 0 & \tilde{T}_1^2 & \tilde{T}_1^2 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & -2\tilde{T}_2^2 & 0 & \tilde{T}_2^2 & \tilde{T}_2^2 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & -\tilde{\Xi}_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & -\tilde{\Xi}_2^2 & 0 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & -\tilde{\Xi}_3^2 & 0 & 0 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & -\tilde{P}_2 & -h_1\tilde{S}_1^2 & -\delta_1\tilde{T}_1^2 & -h_2\tilde{S}_2^2 & -\delta_2\tilde{T}_2^2\\
\star & \star & \star & \star & \star & \star & \star & \star & -\tilde{S}_1^2 & 0 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -\tilde{T}_1^2 & 0 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\tilde{S}_2^2 & 0\\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\tilde{T}_2^2
\end{bmatrix} < 0 \quad (20)
$$

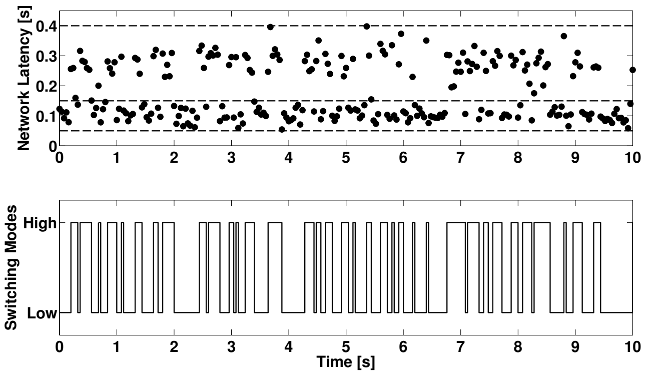

Fig. 3. The communication delays in the network with corresponding swithcing modes of supervisory controller.

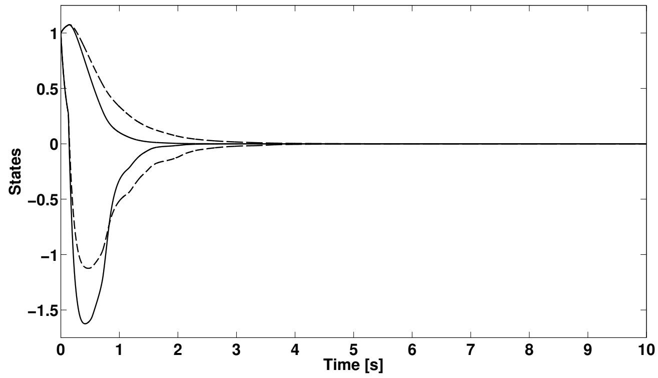

Fig. 4. State trajectory of the closed-loop system under supervisory control (solid line) and a single mode-independent state feedback (dashed). Both simulations are performed from the same initial values.

$$\frac{d}{dt}\begin{bmatrix}\delta_1\\ \omega_1\\ \delta_2\\ \omega_2\\ \delta_3\\ \omega_3\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ -0.0432 & -0.0702 & 0.0209 & 0 & 0.0223 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0.1248 & 0 & -0.2372 & -0.2594 & 0.1124 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0.3761 & 0 & 0.3554 & 0 & -0.7315 & -0.5515 \end{bmatrix}\begin{bmatrix}\delta_1\\ \omega_1\\ \delta_2\\ \omega_2\\ \delta_3\\ \omega_3\end{bmatrix} + \begin{bmatrix}0\\ 0.1471\\ 0\\ 0\\ 0\\ 0\end{bmatrix} u(t) \tag{25}$$

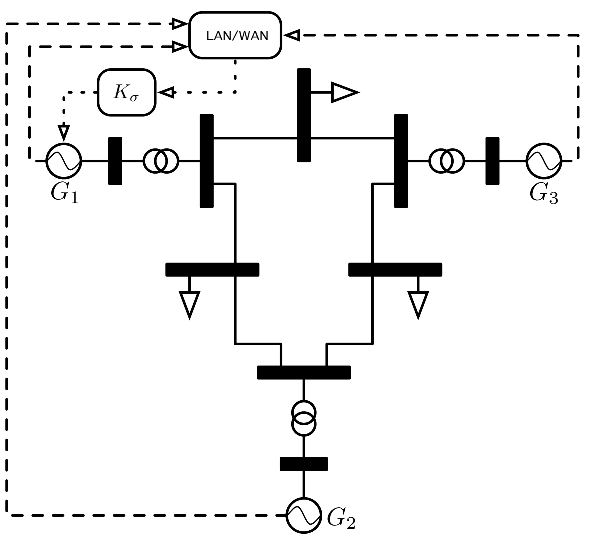

Fig. 5. IEEE nine-bus power system.

frequency (δ_i, ω_i) for the all generators $i \in \{1, 2, 3\}$. Then the numerical model (25) is obtained using the Power System Analysis Toolbox developed by Milano [2010].

We assume that the phase and frequency of each bus can be measured and is communicated to a central control location. In wide-area power systems, the communication delays vary depending on communication technologies, protocols and network load. In this example, we assume that the delay varies between 50 and 200 ms and that mode changes are such that the average dwell-time is guaranteed to be at least 0.9 seconds.

The supervisory controller is designed for the delay intervals [50, 100) and [100,200) ms. We design supervisory control gains targeting a decay rate of $\alpha = 0.375$, and use $\mu = 1.4$ to guarantee $\tau_a = 0.8972$.

Solving the LMIs for this larger system takes a few hours, but appears to be numerically stable. We find the feedback gains

$$K_L = [-35.14\ -27.00\ -1.72\ 23.82\ 23.06\ -28.72]\ ,$$
$$K_H = [-30.77\ -24.47\ -2.98\ 21.26\ 22.18\ -22.89]\ .$$

We note that for this example, the feedback gains are rather similar and conjecture that a single controller could have been used with satisfactory performance.

6. CONCLUSION

This paper has been dedicated to analysis and synthesis of a supervisory controller for networked control systems with multiple time-varying delays. The main contribution of this paper is to develop a stability analysis and a state feedback synthesis technique for a supervisory control system that switches among a multi-controller unit based on the current network state. Remarkably, both the analysis and state feedback synthesis problems can be solved via convex optimization over linear matrix inequalities. Finally, examples were given to show the effectiveness of the proposed analysis and synthesis techniques.

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