FEP3301

Computational Game Theory

Lecture 1

P2/2023

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Division of Network and Systems Engineering

Course objectives

- Upon completion you should be able to
  - Differentiate between GT models of multi-agent decision making
  - Formulate game theoretical models of problems
  - Solve decision making problems
  - Perform a critical evaluation of the literature

Course format

- Contact hours
  - 9 lectures of 2 hours each
  - 2-4 student presentation sessions

  - Starting today

- Non-contact hours
  - 3 homework assignments
  - 1 take home exam

Student presentations

- Goal
  - Peruse a research paper
    - Explain main results
    - Appropriateness of the model
    - Validity of the results
- List of papers
  - https://people.kth.se/~gyuri/teaching/FEP3301/paperlist.shtml
- Expression of interest via e-mail
  - Favorite topic
  - Ordered list of 3 papers you are interested in
    - pick from the list
    - or propose a paper you like (complexity welcome)
- Pairing process
  - FCFS – Greedy algorithm

Needed to pass…

- Active participation
  - at the lectures and during the presentations
- Homework and take home exam
  - To be handed in approx. every two weeks
    - Peer-reviewed
  - Worth 66 pts in total
- Good presentation
  - Worth 10 pts

- You need **55** points to pass (~72%)

8 ECTS

Course schedule - Lectures

https://people.kth.se/~gyuri/teaching/FEP3301/schedule.shtml

Course schedule

Student presentations

Occasion	Date	Time	Location
1	Wed. 22 Nov 2023	13.15-15.00	Ivar Herlitz
2	Wed. 6 Dec 2023	13.15-15.00	Gustaf Dahlander
3?			

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The door-opening game

Other Examples

- Resource management
  - Allocation:
    - Communication/computing system (Internet) – bandwidth, computing power – fairness?
    - Radio spectrum: allocation of spectrum so as to maximize some notion of welfare
  - Placement: Storage and caching - peering between ASs in the Internet: establishment of links
  - Transportation/routing: Given a capacitated network and traffic demands, how to choose routes
  - Scheduling: loads in smart distribution grids
- Security
  - Wireless communication: Jamming
  - Intrusion detection:
    - Passive: Investment in mitigation/detection schemes
    - Active: How to perturb system state so that an attack can be detected at a low cost
- Economics
  - Online advertising: design mechanism for pricing ad locations and maximize click-through rate
  - Electricity markets

What is a game?

- A set of players
- A set of actions
- Likes – preferences over outcomes
- Many assumptions
  - Around the players
    - Rationality
    - Strategic reasoning
    - Available information - uncertainty
  - Around the actions
    - Timing

What is game theory about?

- Model decision making behavior of individuals
  - Outcome depends on the behavior of other individuals
  - Individuals seek their self interests
- Questions to be answered
  - What is the solution?
    - How many are there (existence)?
  - How to reach a solution (learning)?
    - What solution will emerge?
  - Computational complexity of finding a solution?
  - Efficiency of the solution?

A Bit of History

- Origins
  - Decision theory
- Some notable works
  - A. Cournot “Recherches sur les principes mathématiques de la théorie des richesses”, 1838
  - E. Borel “La théorie du jeu et les equations intégrales a noyau symétrique ‟ 1921, (two player games)
  - J. von Neumann, "Zur Theorie der Gesellschaftsspiele", *Mathematische Annalen*, 100, pp. 295–300 (1928).
  - J. von Neumann, O. Morgenstern, “Theory of Games and Economic Behavior”, 1944
    - Two person zero-sum games
  - J. Nash, “Non-cooperative Games”, The Annals of Mathematics, Second Series, Vol. 54, No. 2, (Sep., 1951), pp. 286-295

- Nobel Prizes
  - 1995 - John Harsányi, John Nash, Reinhard Selten (Non-cooperative games)
  - 2005 - Robert Aumann, Thomas Schelling (Cooperative and NC games)
  - 2007 - Leonid Hurwicz, Eric Maskin, Roger Myerson (Mechanism design)
  - 2012 – Alvin E. Roth, Lloyd S. Shapley (Stable allocations and market design)
  - 2014 – Jean Tirole (Market power and regulation, Mech.design)

Types of games

- Possibility of binding agreements
  - Non-cooperative vs. cooperative/coalitional
- Timing and type of feedback
  - Static - Strategic
  - Dynamic - Extensive, repeated, stochastic, differential, evolutionary, …
- Information available for decision making
  - Perfect vs. imperfect vs. incomplete information
- Cardinality of the set of actions and players
  - Finite vs. infinite
  - Discrete vs. continuous

Strategic games

Strategic games

- Players

- Players know each others' possible decisions
  - And the effects of those decisions on themselves

- Players prefer some outcomes over others
  - Goal: obtain best outcome
    maximize own utility

- Each player makes a decision
  - Once
  - Simultaneously

Formal definition

- A strategic game <N,(A_i),($\succcurlyeq_i$)> consists of
  - The set of players
    - A finite set *N*
  - The set of actions available to player *i*
    - For each player a non-empty set A_i
  - The preference relation of player *i*
    - $\forall i \in N$ a preference relation $\succcurlyeq_i$ on $A = \times_{j \in N} A_j$

Preference relation: complete, reflexive, transitive binary relation

?

Actions, consequences, payoff

- Consequences often more important than the actions

- Extend the definition with consequences
  - Define function $A \to C$
  - Preference relation over C

- The consequence can be non-deterministic
  - Probability space Ω
  - A and Ω induce a lottery on C
    - $A x \Omega \to C$
  - Preference relation interpreted over the lottery

- Introduce payoff function
  - $u_i : A \to R$, such that $u_i(a) \geq u_i(b) \Leftrightarrow a \succcurlyeq_i b$

Example: wireless uplink power allocation

Games in Normal Form

- Representation of a game $G=<N,(A_i),(u_i)>$
  - $N=\{1,2\}$
  - $A_1=\{a_{11},a_{12},a_{13}\}$, $A_2=\{a_{21},a_{22}\}$
  - $u_1(.,.)$, $u_2(.,.)$

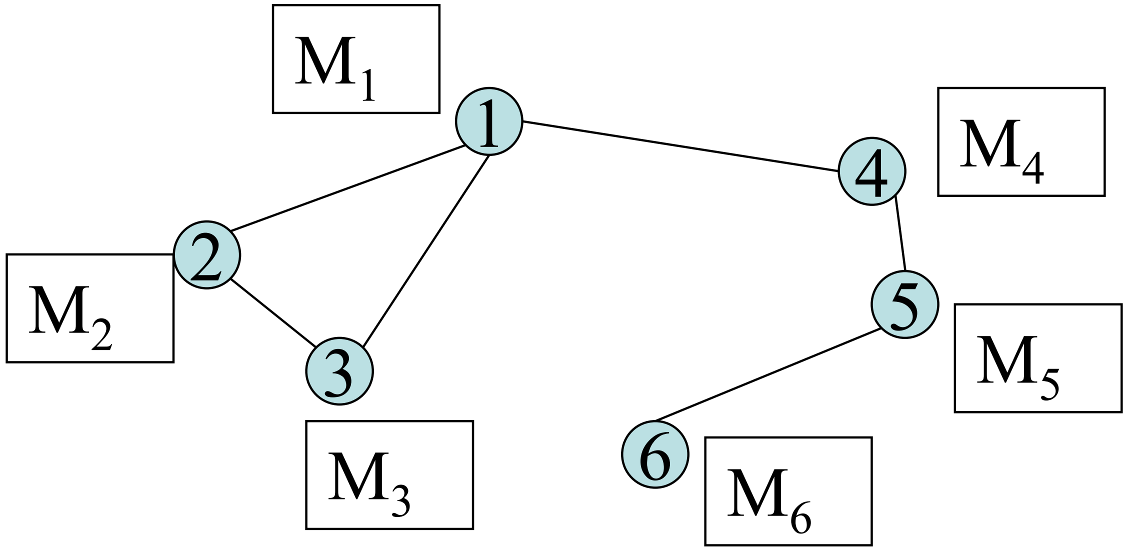
		Player 2's actions	
		a_{21}	a_{22}
Player 1's actions	a_{11}	$u_1(a_{11},a_{21})$, $u_2(a_{11},a_{21})$	$u_1(a_{11},a_{22})$, $u_2(a_{11},a_{22})$
	a_{12}	$u_1(a_{12},a_{21})$, $u_2(a_{12},a_{21})$	$u_1(a_{12},a_{22})$, $u_2(a_{11},a_{22})$
	a_{13}	$u_1(a_{13},a_{21})$, $u_2(a_{13},a_{21})$	$u_1(a_{13},a_{22})$, $u_2(a_{11},a_{22})$

- Requires $O(|N|(max|A_i|)^{|N|})$ entries

Graphical games

- Not all players influence each others' payoff directly
- Represent players as vertices of a graph *G*
- Payoffs in local game matrices (normal form)
  - contains only actions of neighboring players

- Requires $O(|N|(max|A_i|)^d)$ entries
  - *d* is the maximum local neighborhood



An example

- Prisoner's dilemma

	Do not confess	Confess
Do not confess	3,3	0,4
Confess	4,0	1,1

Payoff = 4 – (#years in prison)

- What should they do?

Another example

- Stag hunt game by R.J. Aumann

	L	R
U	9,9	0,8
D	8,0	7,7

- What should they do?

Aumann, R.J. (1990), "Nash Equilibria are not Self-Enforcing," in J-J Gabsewicz, J-F Richard, and L. Wolsey (eds), *Economic Decision-Making: Games, Econometrics, and Optimisation*, Amsterdam: North-Holland, 201-206.

Strong Pareto Efficiency

- For someone to win others have to lose

- An action a^* is strongly Pareto efficient if there is no action a for which
  - $a \succcurlyeq_i a^*$ for $\forall\, i \in N$ and
  - $a \succ_i a^*$ for some $i \in N$

- Can we reach such a solution in a game?

Example revisited

- Prisoner's dilemma

	Do not confess	Confess
Do not confess	3,3	0,4
Confess	4,0	1,1

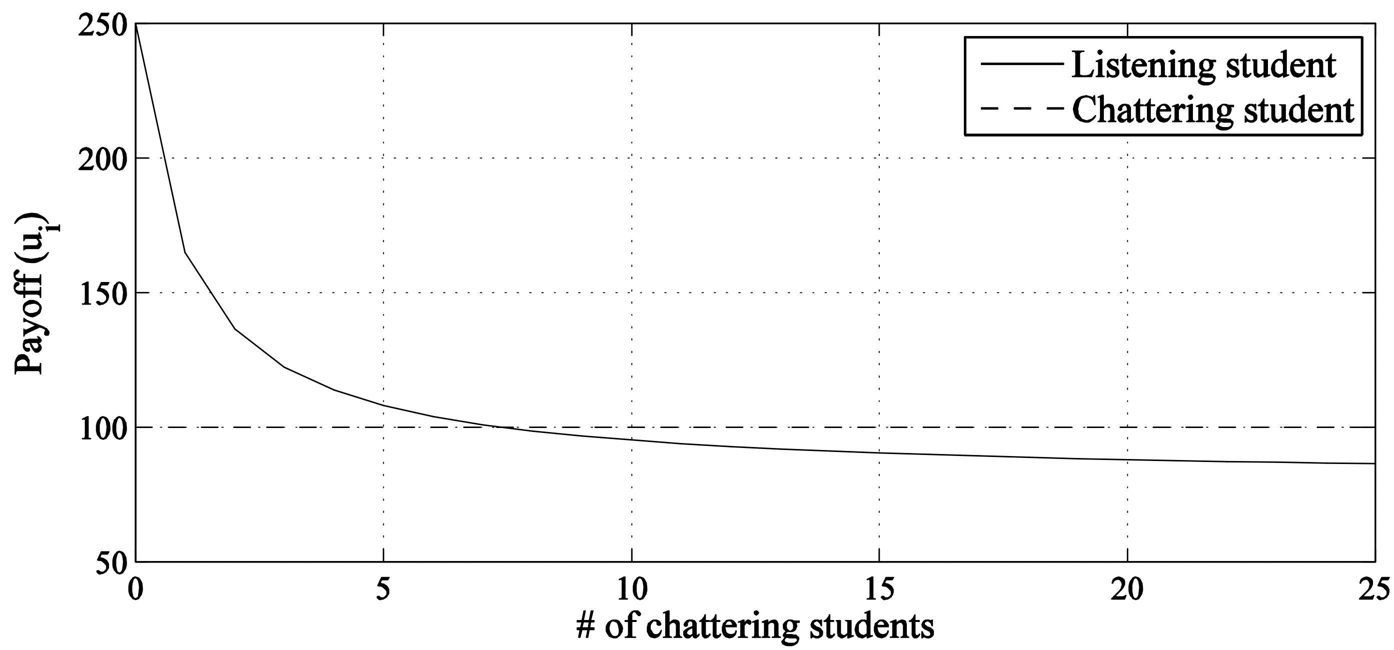
- Which outcomes are Pareto efficient?
- Would players choose those?

An experiment

- Class of N students (you ☺)
- Student *i* has two options during class
  - Chatter
    - $u_i=100$
  - Pay attention
    - $u_i=250-170\times N_C/(N-1)$
    - N_C = # of chattering students

- Would you chatter or rather pay attention?

The payoff of the experiment



2-person version

	Red (chatter)	Blue (listen)
Red (chatter)	100,100	100, 80
Blue (listen)	80,100	250,250

Solution concepts of games

- What is a reasonable solution for a game?

- Variety of solution concepts
  - Equilibria
    - Dominant strategy equilibrium
    - Nash equilibrium and its refinements
  - Iterated elimination of strictly dominated strategies
  - Rationalization

- Questions regarding the solutions
  - Existence

  - Uniqueness - cardinality

  - Complexity of the calculation
  - Feasibility/convergence/emergence
  - Efficiency

Dominant Strategy

- a_i^* is dominant strategy for player *i* in *G*=<*N*,(*A*$_i$),($\succcurlyeq_i$)>

$$(a_i^*, a_{-i}) \succcurlyeq_i (a_i, a_{-i}) \quad \forall a \in A$$

	Do not confess	Confess
Do not confess	3,3	0,4
Confess	4,0	1,1

Dominant Strategy Equilibrium

- a_i^* is dominant strategy for player i in G=<N,(A_i),($\succcurlyeq_i$))>

$$(a_i^*, a_{-i}) \succcurlyeq_i (a_i, a_{-i}) \quad \forall a \in A$$

	Do not confess	Confess
Do not confess	3,3	0,4
Confess	4,0	1,1

- The profile $a^* \in A$ is a dominant strategy equilibrium if

$$(a_i^*, a_{-i}) \succcurlyeq_i (a_i, a_{-i}) \quad \forall a \in A, i \in N$$

  - Best response to every collection of actions of the other players

	L	R
U	9,9	0,8
D	8,0	7,7

Nash equilibrium

- A profile from which no player has an interest to deviate

	Do not confess	Confess
Do not confess	3,3	0,4
Confess	4,0	1,1

- If players reach a Nash equilibrium, they will stay there

Nash equilibrium (pure)

- Nash equilibrium of a strategic game $< N, (A_i), (\succcurlyeq_i) >$ is a profile $a^* \in A$ of actions such that

$$(a^*_{-i}, a^*_i) \succcurlyeq_i (a^*_{-i}, a_i) \quad \text{for } \forall a_i \in A_i$$

- No player can gain by deviating from a^*_i given that the others choose a^*_{-i}

Best response function

- Set valued function
  - $B_i(a_{-i})=\{a_i \in A_i: (a_{-i},a_i) \succcurlyeq_i (a_{-i},a'_i)$ for $\forall a'_i \in A_i$
- Nash equilibrium is a profile a^* such that
  - $a^*_i \in B_i(a^*_{-i})$ for all $i \in N$

Example revisited

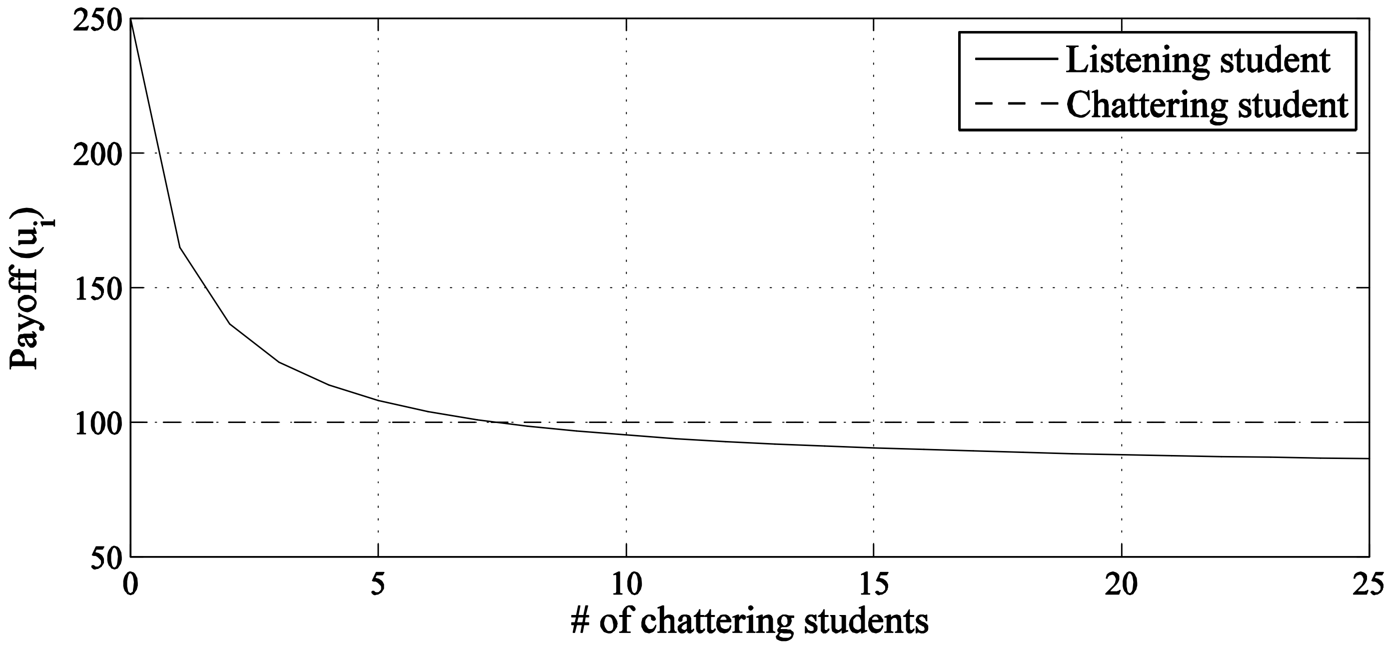
- Stag hunt game by R.J. Aumann

	L	R
U	9,9	0,8
D	8,0	7,7

- How many NE are there?
- Which NE is more likely to happen?
  - What if the players can communicate?

Aumann, R.J. (1990), “Nash Equilibria are not Self-Enforcing,” in J-J Gabsewicz, J-F Richard, and L. Wolsey (eds), *Economic Decision-Making: Games, Econometrics, and Optimisation*, Amsterdam: North-Holland, 201-206.

Experiment revisited



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N. J. Vriend,"Demonstrating the Possibility of Pareto Inferior Nash Equilibria", in Journal of Economic Education 31(4), pp. 358-362, 2000

Example

- Battle of the Sexes (Bach or Stravinsky)

	Sports	Theatre
Sports	3,2	0,0
Theatre	0,0	2,3

- How many NE are there?

Another example

- Hawk and Dove (aka, Game of chicken)

	Dove	Hawk
Dove	3,3	1,4
Hawk	4,1	0,0

- How many NE are there?

Yet another example?

- Matching pennies

	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

- How many NE are there?

Existence of Nash equilibria

- The strategic game $< N, (\mathrm{A_i}), (\succcurlyeq_i) >$ has a Nash equilibrium if for all $i \in N$
  - the set A_i of actions of player i is a nonempty compact convex subset of a Euclidean space

  and the preference relation $\succcurlyeq_i$ is
  - continuous
  - convex on A_i.

- Proof
  - based on Kakutani's fixed point theorem

(Debreu '52,Glicksberg '52, Fan '52)

Notes on the existence results

- The equilibrium is not necessarily unique
  - Which equilibrium is an appropriate solution?
- The existence is not guaranteed for finite games!
  - For none of the examples considered before…
- Best response functions can be used to find equilibria
  - Not very efficient

Summary

- Brief overview of game theoretic models
- Strategic games
  - Formal definition
  - Existence of Nash equilibria

- Next time
  - Strictly competitive games
    - Maxminimization vs. Nash equilibria
  - Mixed strategy equilibria
  - Rationalizability
  - IEDS, IEWS

Literature

- M. Osborne, A. Rubinstein, “A Course in Game Theory”, MIT press, 1994
- D. Fudenberg, J. Tirole, “Game Theory”, MIT press, 1991
- Nisan, Roughgarden, Tardos, Vazirani (eds.), “Algorithmic Game Theory”, Cambridge UP, 2007

- Kakutani, "A generalization of Brouwer’s fixed point theorem". Duke Mathematical Journal 8 (3) pp. 457–459, 1941