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Control Systems Security Metrics and Risk Management

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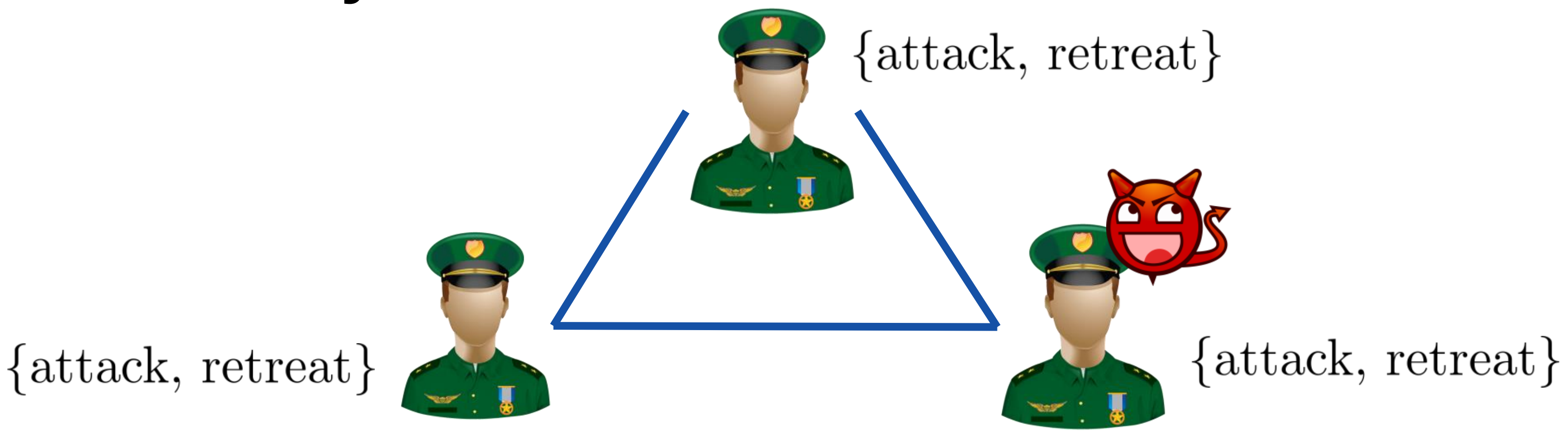
Example of Classic Cyber Security: The Byzantine Generals Problem



- Consider n generals and q unknown traitors among them. Can the $n - q$ loyal generals always reach an agreement?
- Traitors ("Byzantine faults") can do anything: different message to different generals, send no message, change forwarded message,…
- Agreement protocol exists iff $n \geq 3q + 1$
- If loyal generals use unforgeable signed messages ("authentication") then agreement protocol exists for any q! [Lamport *et al*., ACM TOPLAS, 1982]

- Application to linear consensus computations: See [Pasqualetti *et al*., CDC, 2007], [Sundaram and Hadjicostis, ACC, 2008]

Observations and Goals of Lecture

(q used as general proxy for overall attacker strength in the following)

- Resourceful attacker (large q) is hard/impossible to stop
- Actual q probably not known – Use varying q as input to risk study
- Large-scale industrial control systems are relatively unprotected today - Even small q may lead to substantial damage
- Smart defense can (significantly) increase the attacker's required q

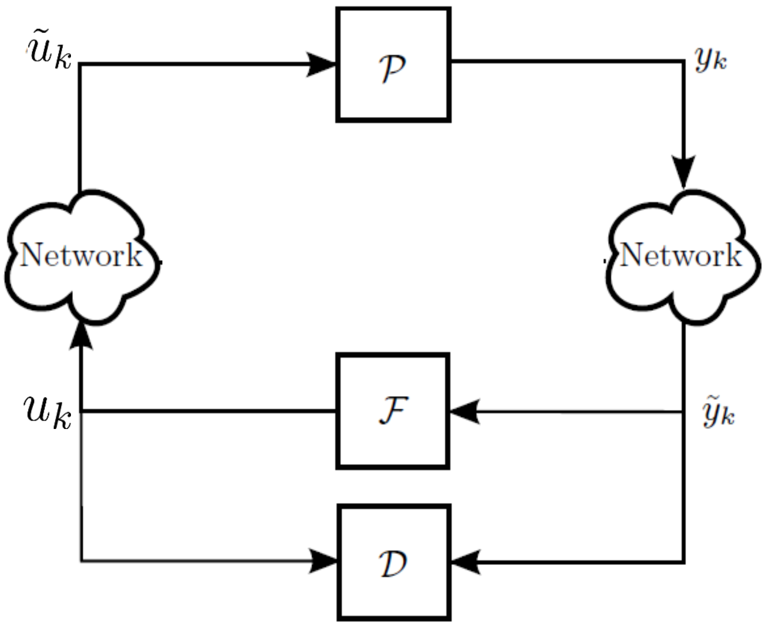
Goals of lecture

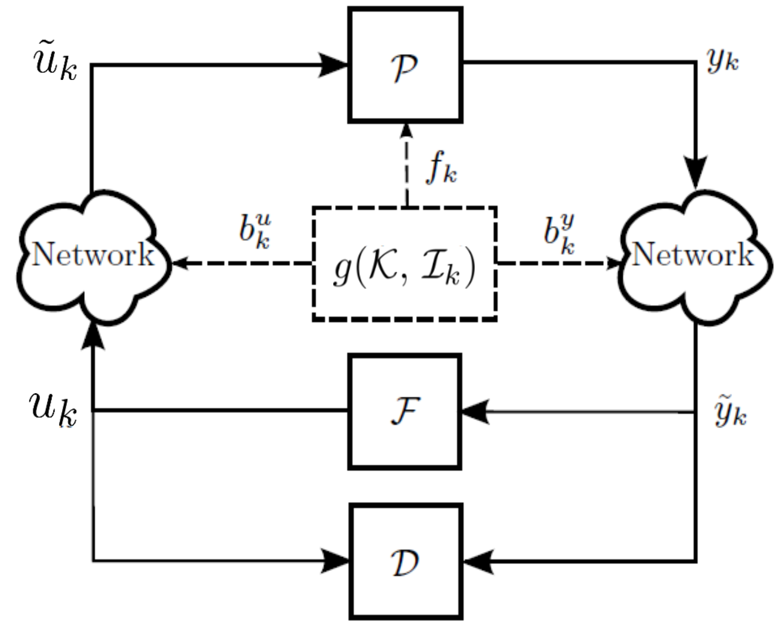
- Introduction to risk management and attack space
- Find signals susceptible to undetectable/unidentifiable attacks as fcn of q
- Introduce security metric (index) α and its computation
- Method to allocate defense to increase attacker's required q

Outline

- **Risk management**
- Attack detectability and security metric
- Attack identification and secure state estimation
- Security metric computation

Networked Control System under Attack





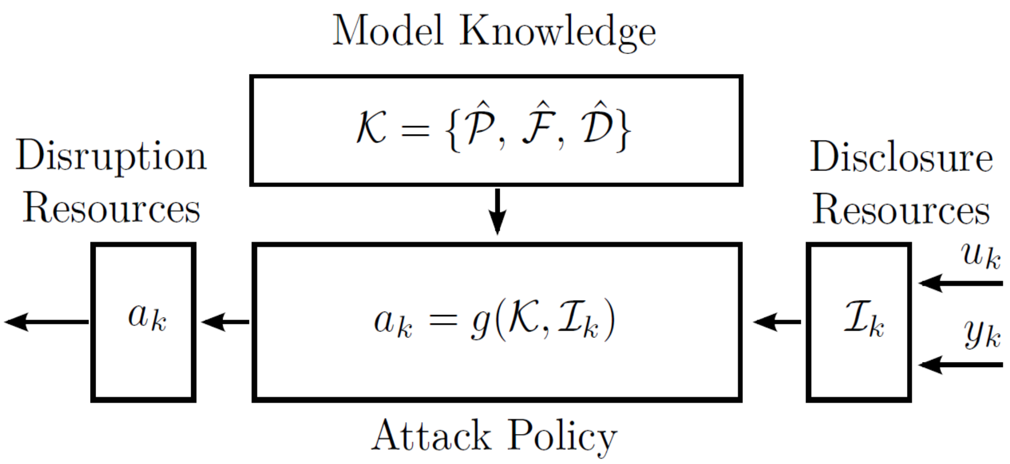
- Physical plant ($\mathcal{P}$)
- Feedback controller ($\mathcal{F}$)
- Anomaly detector ($\mathcal{D}$)
- Disclosure Attacks

- Physical Attacks f_k
- Deception Attacks

$$\tilde{u}_k = u_k + \Gamma^u b_k^u$$

$$\tilde{y}_k = y_k + \Gamma^y b_k^y$$

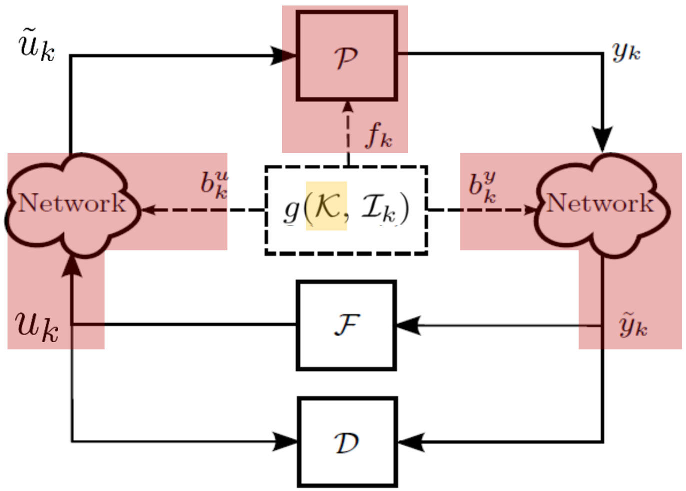
Adversary Model

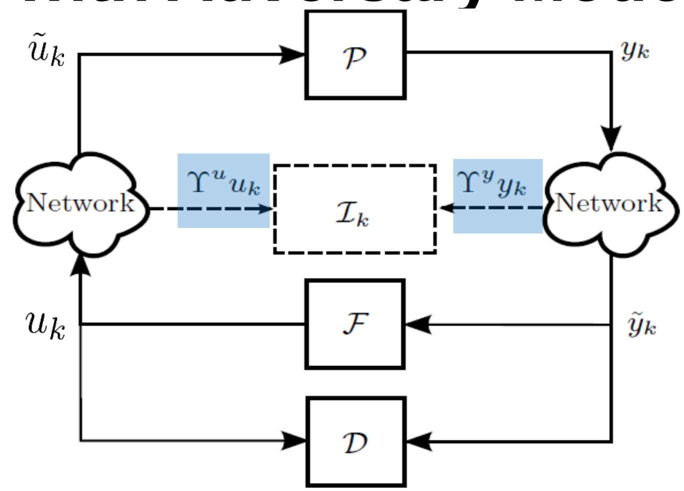


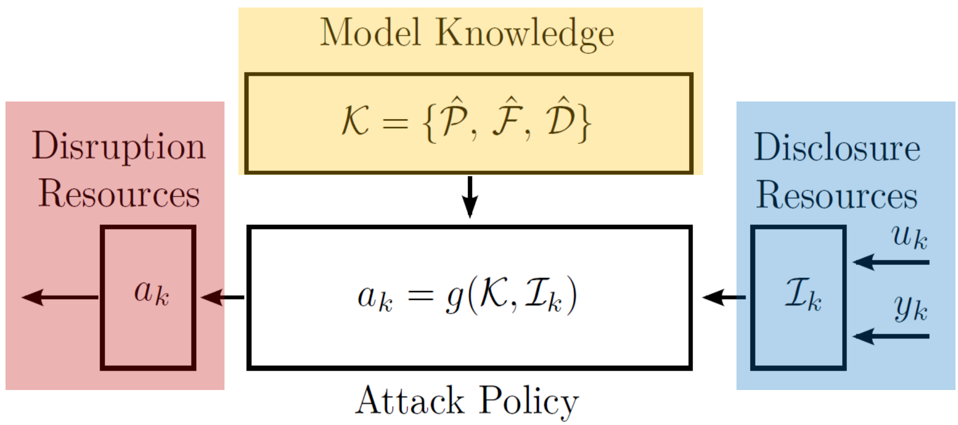
- **Attack policy:** Goal of the attack? Destroy equipment, increase costs,…
- **Model knowledge:** Adversary knows models of plant and controller? Possibility for stealthy attacks…
- **Disruption/disclosure resources:** Which channels can the adversary access?

[Teixeira *et al*., HiCoNS, 2012]

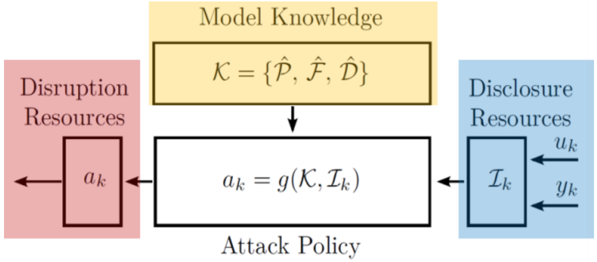
Networked Control System with Adversary Model

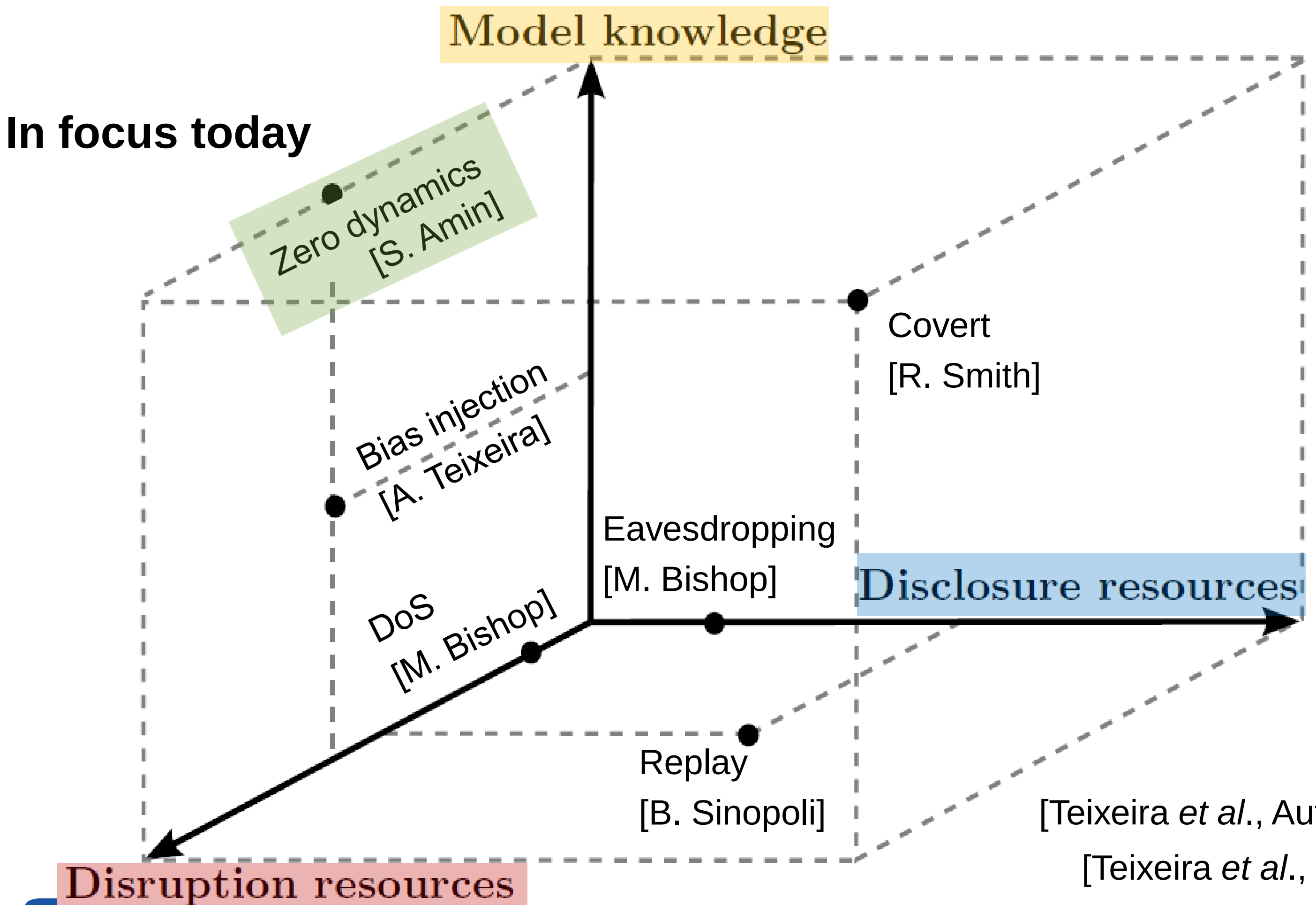






Attack Space





[Teixeira *et al*., Automatica, 2015]

[Teixeira *et al*., HiCoNS, 2012]

Why Risk Management?

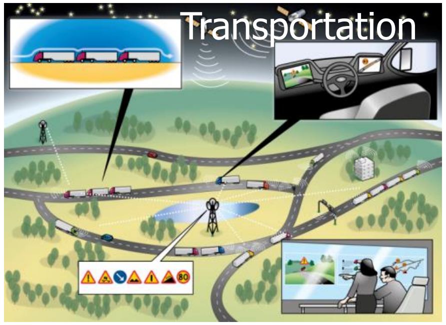
Complex control systems with numerous attack scenarios

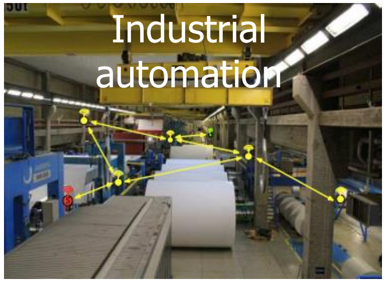
Examples: Critical infrastructures (power, transport, water, gas, oil) often with weak security guarantees

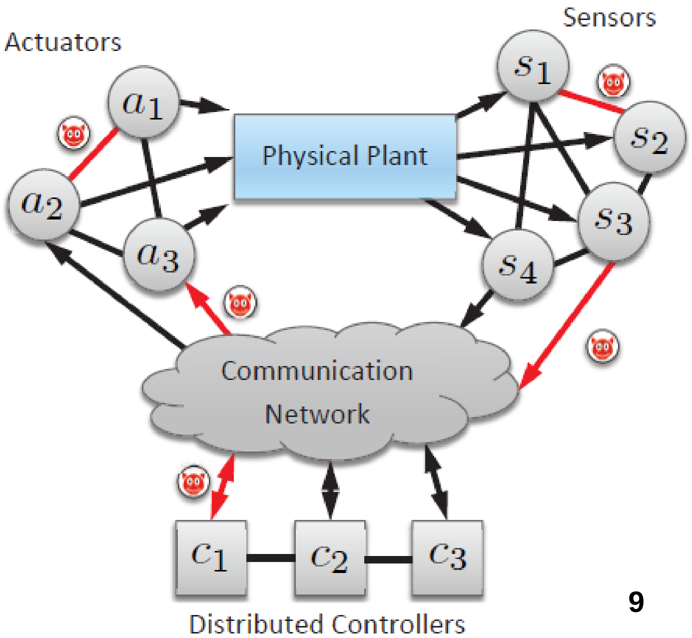
Too costly to secure the entire system against all possible attack scenarios

What scenarios to prioritize?

What components to protect/defend first?







Defining Risk

Risk = (Scenario, Likelihood, Impact)

Scenario

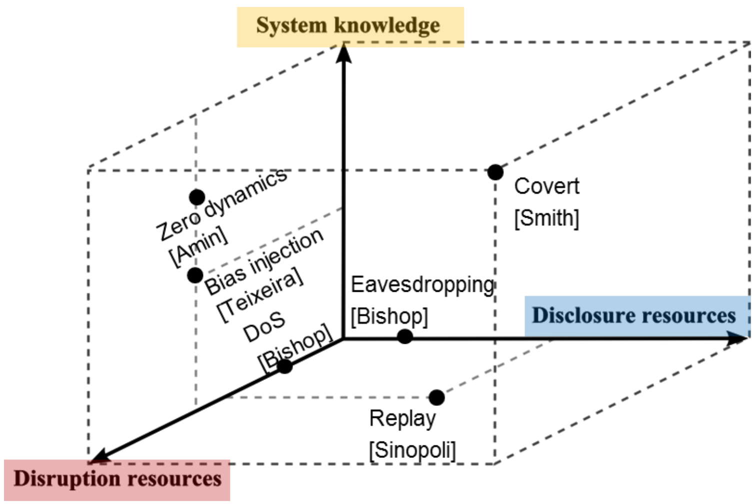
- How to describe the system under attack?

Likelihood

- How much effort does a given attack require?

Impact

- What are the consequences of an attack?

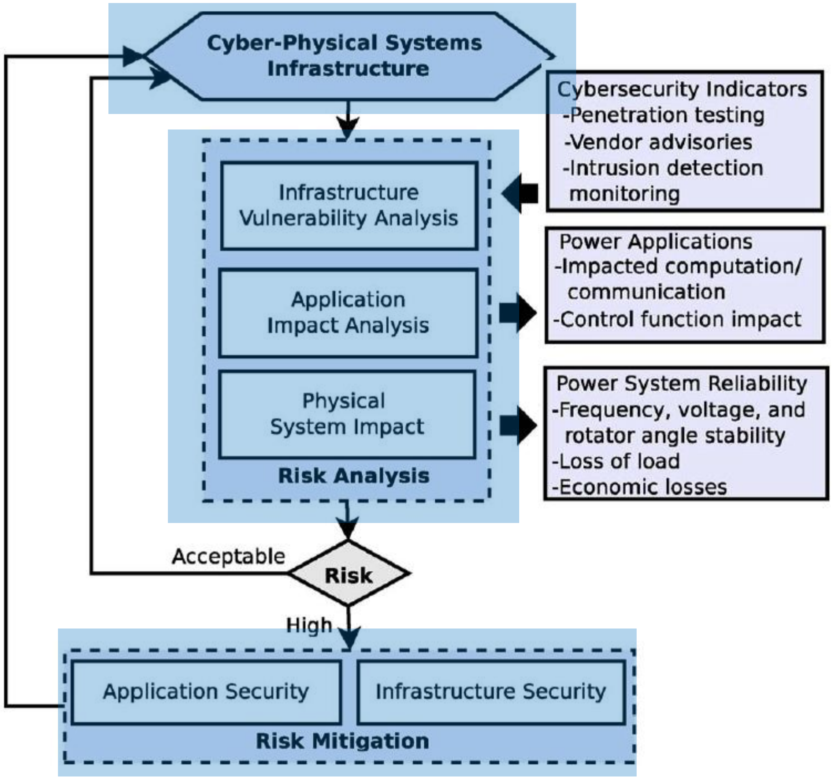


[Kaplan & Garrick, 1981], [Bishop, 2002]
([Teixeira *et al*., IEEE CSM, 2015])

Risk Management Cycle

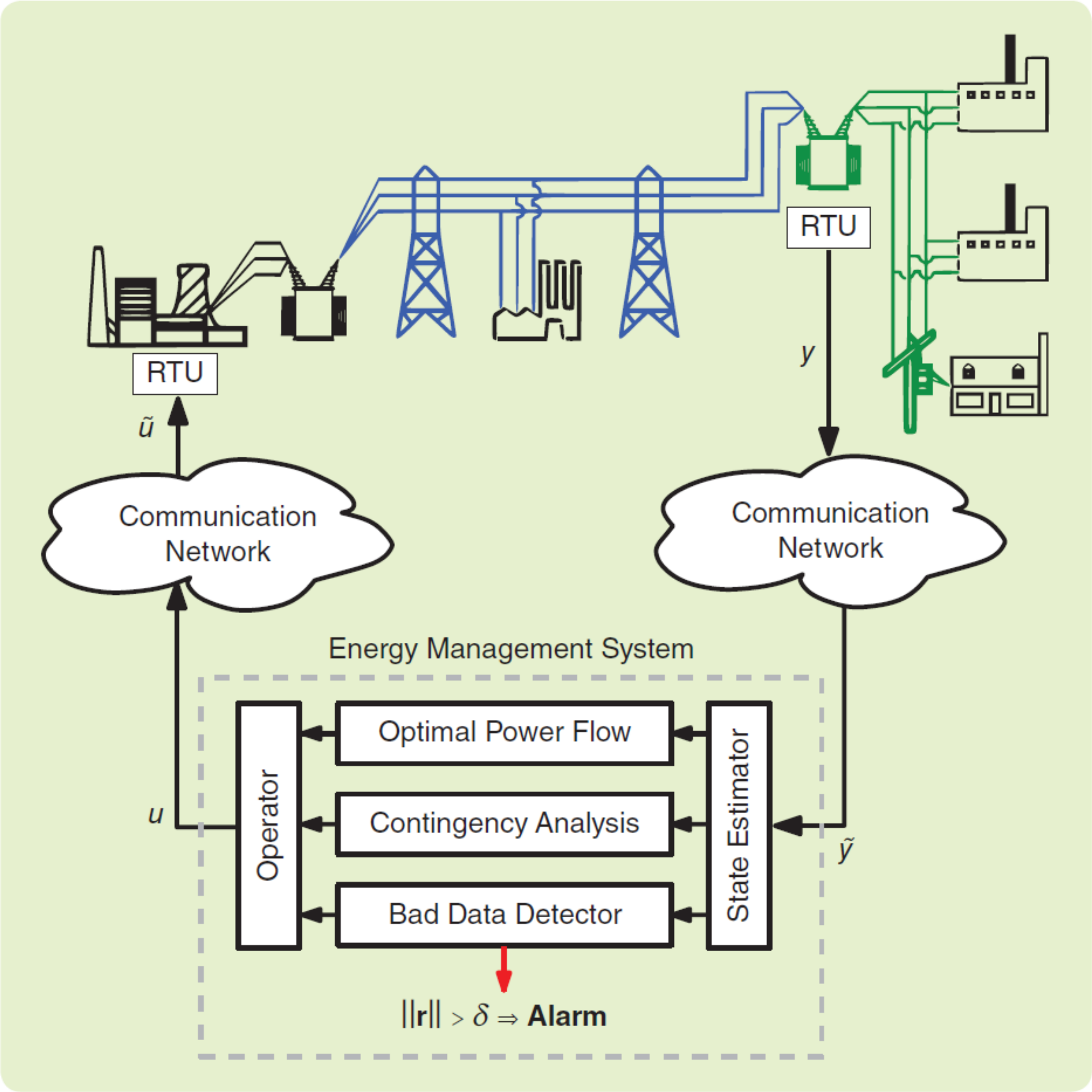
Main steps in risk management

- Scope definition
  - Models, Scenarios, Objectives
- Risk Analysis
  - **Threat Identification**
  - **Likelihood Assessment**
  - Impact Assessment
- Risk Treatment
  - **Prevention**, Detection, Mitigation

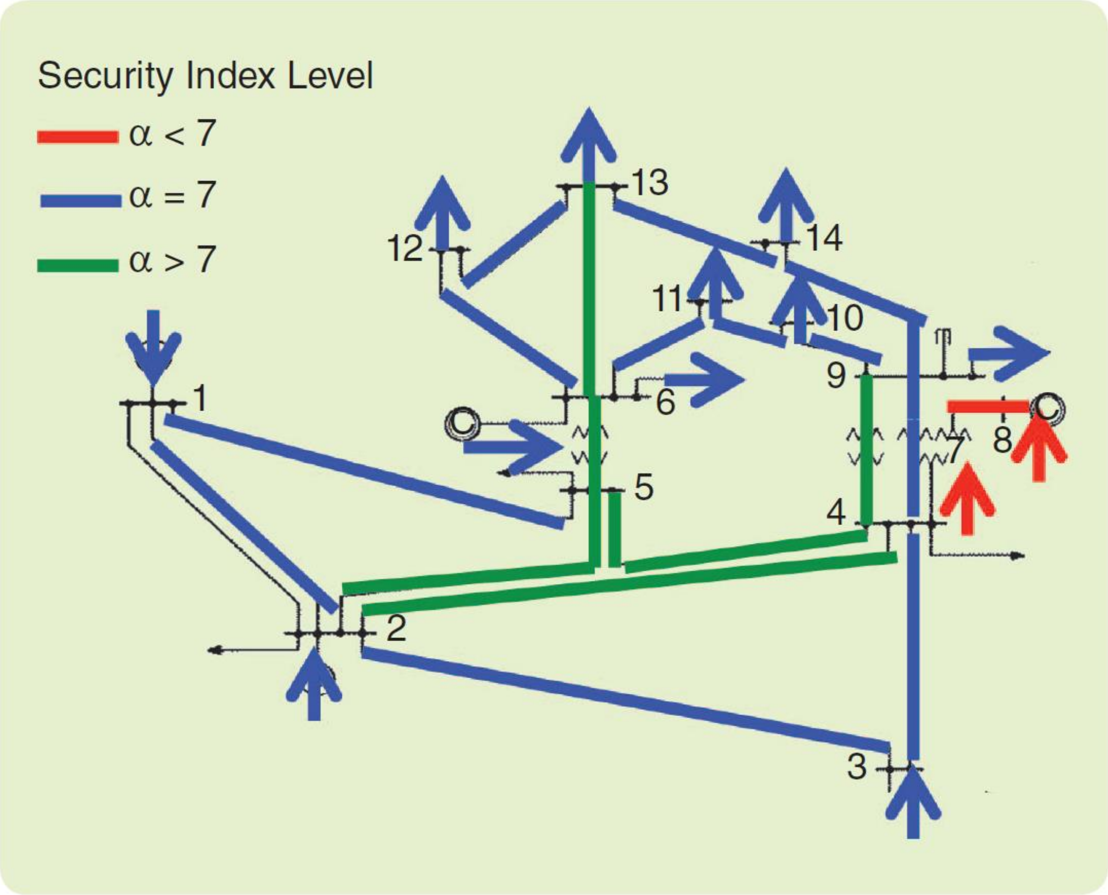


[Sridhar *et al*., Proc. IEEE, 2012]

Example 1: Power System State Estimator



Example 1: Power System State Estimator



Small security index α (to be defined) indicates sensors with inherent weak redundancy (~security). These should be defended first!

[Teixeira *et al*., IEEE CSM, 2015], [Vukovic *et al.*, IEEE JSAC, 2012]

Outline

- Risk management
- **Attack detectability and security metric**
- Attack identification and secure state estimation
- Security metric computation

Basic Notions: Input Observability and Detectability

$$x(k+1) = Ax(k) + Bu(k), \quad x(k) \in \mathbb{R}^n, \, u(k) \in \mathbb{R}^m$$
$$y(k) = Cx(k) + Du(k), \quad y(k) \in \mathbb{R}^p$$

Definitions:

1. The input u is *observable with knowledge of* $x(0)$ if $y(k) = 0$ for $k \geq 0$ implies $u(k) = 0$ for $k \geq 0$, provided $x(0) = 0$

2. The input u is *observable* if $y(k) = 0$ for $k \geq 0$ implies $u(k) = 0$ for $k \geq 0$ ($x(0)$ unknown)

3. The input u is *detectable* if $y(k) = 0$ for $k \geq 0$ implies $u(k) \to 0$ for $k \to \infty$ ($x(0)$ unknown)

[Hou and Patton, Automatica, 1998]

Basic Notions: Input Observability and Detectability

The Rosenbrock system matrix:

$$P(z) = \begin{bmatrix} A - zI & B \\ C & D \end{bmatrix} \in \mathbb{C}^{(n+p)\times(n+m)}$$

First observations:

- Necessary condition for Definitions 1-3
  $$\max_z \operatorname{rank} P(z) = m + n \Leftrightarrow \operatorname{normalrank} P(z) = m + n$$
- Fails if number of inputs larger than number of outputs ($m > p$)
- Necessary and sufficient conditions involve the *invariant zeros*:
  $$\sigma(P(z)) := \{z : \operatorname{rank} P(z) < \operatorname{normalrank} P(z)\}$$
  (Transmission zeros + uncontrollable/unobservable modes, Matlab command: `tzero`)

Basic Notions: Input Observability and Detectability

$$P(z) = \begin{bmatrix} A - zI & B \\ C & D \end{bmatrix} \in \mathbb{C}^{(n+p)\times(n+m)}$$

Theorems. Suppose (A, B, C, D) is minimal realization.

1. The input u is *observable with knowledge of* $x(0) \Leftrightarrow$
   $$\max_z \operatorname{rank} P(z) = m + n \Leftrightarrow \operatorname{normalrank} P(z) = m + n$$

2. The input u is *observable* $\Leftrightarrow$
   $$\forall z : \operatorname{rank} P(z) = m + n$$
   (no invariant zeros)

3. The input u is *detectable* $\Leftrightarrow$ (1) and
   $$\sigma(P(z)) \subseteq \{z : |z| < 1\}$$
   (invariant zeros are all stable = system is minimum phase)

[Hou and Patton, Automatica, 1998]

Basic Notions: Input Observability and Detectability

$$P(z) = \begin{bmatrix} A - zI & B \\ C & D \end{bmatrix}, \quad O(z) = \begin{bmatrix} A - zI \\ C \end{bmatrix}$$

Theorems. (A, B, C, D) possibly non-minimal realization

1. The input u is *observable with knowledge of* $x(0)$ $\Leftrightarrow$

   $$\max_z \operatorname{rank} P(z) = m + n \Leftrightarrow \operatorname{normalrank} P(z) = m + n$$

2'. The input u is *observable* $\Leftrightarrow$ (1) and

   $$\sigma(P(z)) = \sigma(O(z))$$

   (invariant zeros are all unobservable modes)

3'. The input u is *detectable* $\Leftrightarrow$ (1) and

   $$\sigma(P(z)) \setminus \sigma(O(z)) \subseteq \{z : |z| < 1\}$$

   (invariant zeros that are not unobservable modes are all stable)

[Hou and Patton, Automatica, 1998]

Fault Detection vs. Secure Control

Typical condition used in fault detection/fault tolerant control:

[Ding, Patton]

1. The input u is *observable with knowledge of* $x(0) \Leftrightarrow$

$$\max_z \operatorname{rank} P(z) = m + n \Leftrightarrow \operatorname{normalrank} P(z) = m + n$$

Typical conditions used in secure control/estimation:

2. The input u is *observable* $\Leftrightarrow$ [Sundaram, Tabuada]

$$\forall z : \operatorname{rank} P(z) = m + n$$

(no invariant zeros)

3/3'. The input u is *detectable* $\Leftrightarrow$ (1) and [Pasqualetti, Sandberg]

$$\sigma(P(z)) \subseteq \{z : |z| < 1\}$$

(invariant zeros are all stable = system is minimum phase)

Example 2

$$A = \begin{pmatrix} 0.9 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{pmatrix},\ B = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0.25 \end{pmatrix},\ C = \begin{pmatrix} 0.4 & 0.6 & 0 \\ 0.2 & 0 & 0.4 \end{pmatrix}$$

$$G(z) = C(zI - A)^{-1}B + D = \begin{pmatrix} \frac{0.2}{z-0.9} & \frac{0.3}{z-0.8} \\ \frac{0.1}{z-0.9} & \frac{0.1}{z-0.9} \end{pmatrix}$$

Invariant zeros = $\sigma\big(P(z)\big) = \{1.1\}$

[Note: $\text{normalrank}\, P(z) = n + \text{normalrank}\, G(z)$]

1. The input u is *observable with knowledge of* $x(0)$: Yes!
2. The input u is *observable:* No!
3. The input u is *detectable:* No!

With $x(0) = \begin{pmatrix} -0.705 \\ 0.470 \\ 0.352 \end{pmatrix}$ and $u(k) = 1.1^k \begin{pmatrix} -0.282 \\ 0.282 \end{pmatrix}$ then $y(k) = 0, k \geq 0$

OK for fault detection but perhaps not for security!

Attack and Disturbance Model

Consider the linear system $y = G_d d + G_a a$ (the controlled infrastructure):

$$x(k+1) = Ax(k) + B_d d(k) + B_a a(k)$$
$$y(k) = Cx(k) + D_d d(k) + D_a a(k)$$

- Unknown state $x(k) \in \mathbb{R}^n$ ($x(0)$ in particular)
- Unknown (natural) disturbance $d(k) \in \mathbb{R}^o$
- Unknown (malicious) attack $a(k) \in \mathbb{R}^m$
- Known measurement $y(k) \in \mathbb{R}^p$
- Known model A, B_d, B_a, C, D_d, D_a

- **Definition:** Attack signal a is *persistent* if $a(k) \nrightarrow 0$ as $k \to \infty$

- **Definition:** A (persistent) attack signal a is *undetectable* if there exists a simultaneous (masking) disturbance signal d and initial state $x(0)$ such that $y(k) = 0$, $k \geq 0$ (Cf. Theorem 3')

Undetectable Attacks and Masking

The Rosenbrock system matrix:

$$P(z) = \begin{bmatrix} A - zI & B_d & B_a \\ C & D_d & D_a \end{bmatrix}$$

- Attack signal $a(k) = {z_0}^k a_0$, $0 \neq a_0 \in \mathbb{C}^m$, $z_0 \in \mathbb{C}$, is *undetectable* iff there exists $x_0 \in \mathbb{C}^n$ and $d_0 \in \mathbb{C}^o$ such that

$$P(z_0) \begin{bmatrix} x_0 \\ d_0 \\ a_0 \end{bmatrix} = 0$$

- Attack signal is undetectable if indistinguishable from measurable (y) effects of natural noise (d) or uncertain initial states (x_0) [**masking]**

Example 2 (cont'd)

$$G(z) = C(zI - A)^{-1}B + D = \begin{pmatrix} G_d(z) & G_a(z) \end{pmatrix}$$

$$G_d(z) = (), \quad G_a = \begin{pmatrix} \frac{0.2}{z-0.9} & \frac{0.3}{z-0.8} \\ \frac{0.1}{z-0.9} & \frac{0.1}{z-0.9} \end{pmatrix}$$

Poles = $\{0.9, 0.9, 0.8\}$

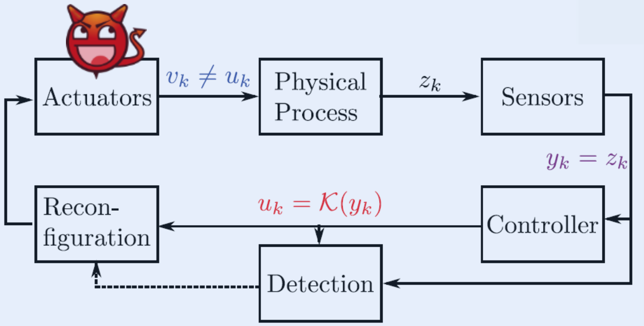
Invariant zeros = $\sigma\big(P(z)\big) = \{1.1\}$

Undetectable attack: $a(k) = 1.1^k \begin{pmatrix} -0.282 \\ 0.282 \end{pmatrix}$

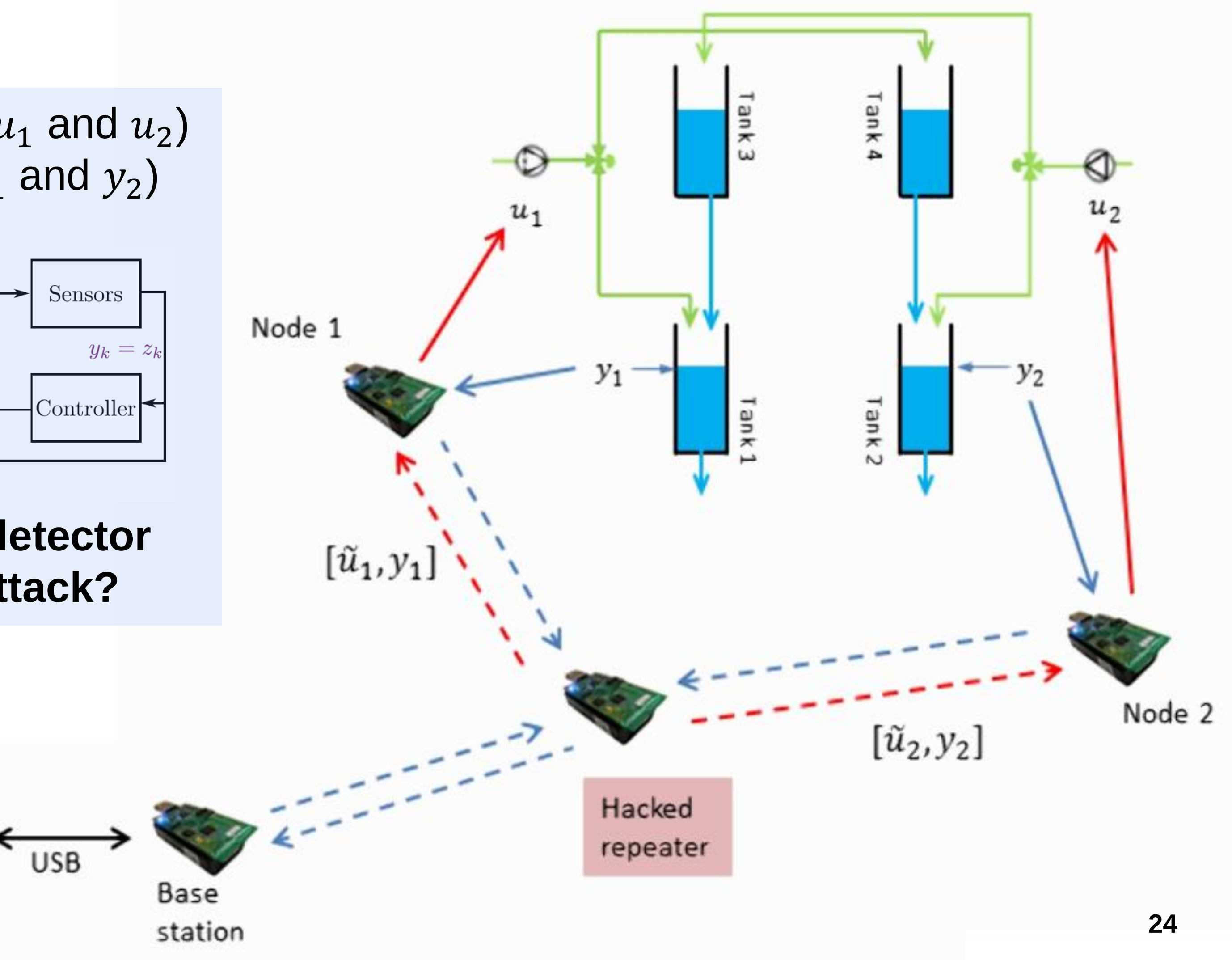
Masking initial state: $x_0 = \begin{pmatrix} -0.705 \\ 0.470 \\ 0.352 \end{pmatrix}$

Example 3: Stealthy Water Tank Attack

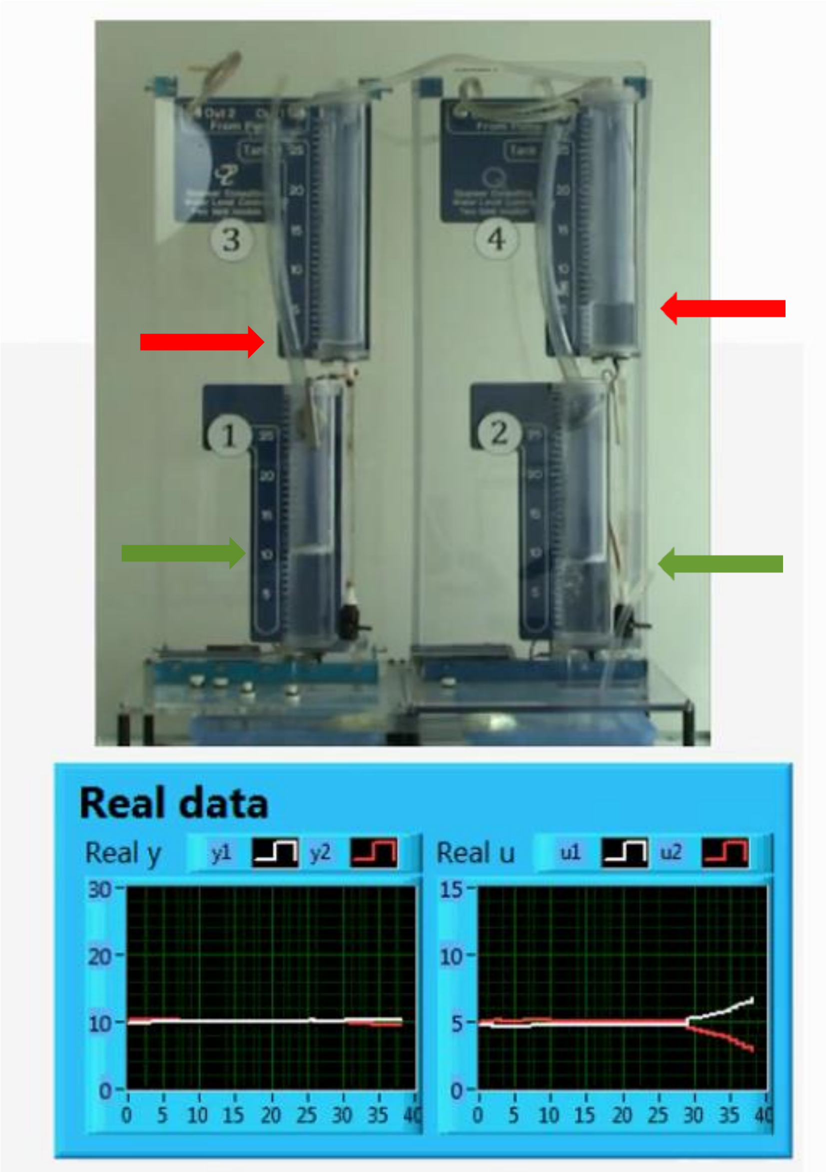
2 hacked actuators (u_1 and u_2)
2 healthy sensors (y_1 and y_2)



Can the controller/detector always detect the attack?



Example 3: Stealthy Water Tank Attack [Movie]

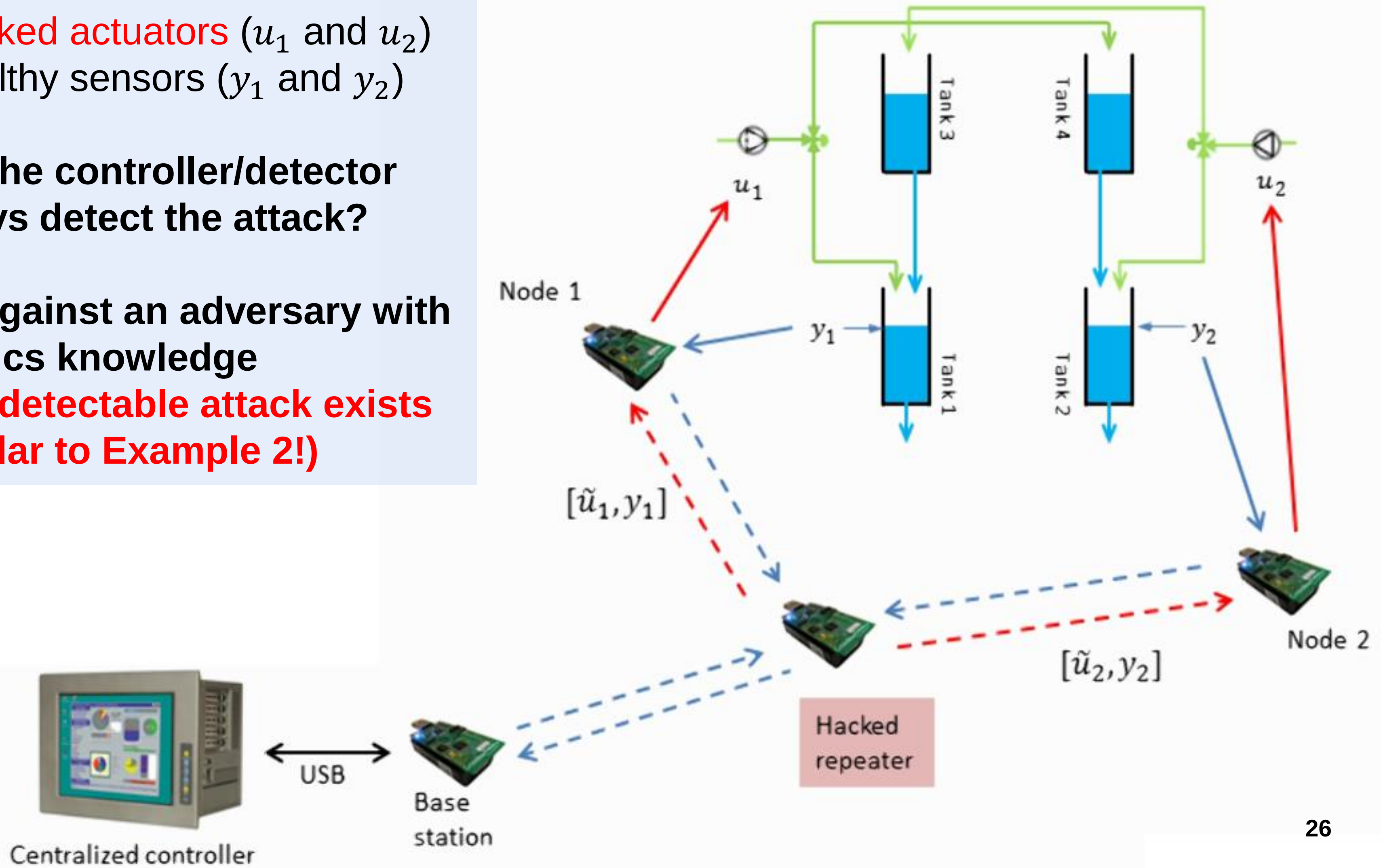


Example 3: Stealthy Water Tank Attack

2 hacked actuators (u_1 and u_2)
2 healthy sensors (y_1 and y_2)

Can the controller/detector always detect the attack?

Not against an adversary with physics knowledge
⇒ Undetectable attack exists
(Similar to Example 2!)



Undetectable Attacks and Masking (cont'd)

- Suppose operator observes the output $y(k)$, and does *not know* the true initial state $x(0)$ and true disturbance $d(k)$
- Let (x_0, d_0, a_0) be an undetectable attack, $0 = G_d d_0 + G_a a_0$ with initial state x_0

Consider the cases:

1. Un-attacked system $y = G_d(-d_0)$, with initial state $x(0) = 0$
2. Attacked system $y = G_a a_0$, with initial state $x(0) = x_0$

If initial states $x(0) = 0$ and $x(0) = x_0$ and disturbances $d = -d_0$ and $d = 0$ are equally likely, then impossible for operator to decide which case is true ⇒ **Attack is undetectable!**

Undetectable Attacks and Masking (cont'd)

- Suppose operator observes the output $y(k)$, and *knows* the true initial state $x(0) = 0$ and the disturbance $d(k) = 0, k \geq 0$
- Suppose system is asymptotically stable, $\rho(A) < 1$
- Let (x_0, a_0) be an undetectable attack, $0 = G_a a_0$ with initial state x_0

Consider the cases:

1. Un-attacked system $y_1(k) = 0, k \geq 0$, with initial state $x(0) = 0$
2. Attacked system $y_2(k) = (G_a a_0)(k) = -CA^k x_0 \to 0$ as $k \to \infty$, with initial state $x(0) = 0$

The attacked output y_2 is vanishing, and can be made arbitrarily close to y_1 by scaling $(x_0, a_0) \Rightarrow$ **Attack is asymptotically undetectable!**

The Security Index α_i

$$\begin{aligned} \alpha_i := \min_{|z_0|\geq 1, x_0, d_0, a_0^i} \quad & \|a_0^i\|_0 \\ \text{subject to} \quad & P(z_0) \begin{bmatrix} x_0 \\ d_0 \\ a_0^i \end{bmatrix} = 0 \end{aligned}$$

Notation: $\|a\|_0 \coloneqq |\mathrm{supp}(a)|$, a^i vector a with i-th element non-zero

Interpretation:

- Attacker persistently targets signal component a_i (condition $|z_0| \geq 1$)
- α_i is smallest number of attack signals that need to be simultaneously accessed to stage undetectable attack against signal a_i

Problem NP-hard in general (combinatorial optimization, cf. matrix *spark*). Generalization of static index in [Sandberg *et al.*, SCS, 2010]

$$x(k+1) = Ax(k) + B_d d(k) + B_a a(k)$$
$$y(k) = Cx(k) + D_d d(k) + D_a a(k)$$

Example 4: Simple Security Index

$$P(z) = \begin{bmatrix} A - zI & B_d & B_a \\ 0 & 0 & D_a \end{bmatrix} \qquad D_a = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Measurements not affected by physical states and disturbances
- 3 measurements
- 4 attacks with security indices:
  - $\alpha_1 = 3$
  - $\alpha_2 = 3$
  - $\alpha_3 = 3$
  - $\alpha_4 = \infty$ (By definition. Even access to all attack signals not enough to hide attack)

Special Case 1: Critical Attack Signals

Signal with $\alpha_i = 1$ can be undetectably attacked without access to other elements ⇒ **Critical Attack Signal**

$$P_i(z) = \begin{bmatrix} A - zI & B_d & B_{a,i} \\ C & D_d & D_{a,i} \end{bmatrix} \in \mathbb{C}^{(n+p)\times(n+o+1)}, \; P_d(z) = \begin{bmatrix} A - zI & B_d \\ C & D_d \end{bmatrix} \in \mathbb{C}^{(n+p)\times(n+o)}$$

Simple test, $\forall i$: If there is $z_0 \in \mathbb{C}$, $|z_0| \geq 1$, such that $\text{rank}\,[P_d(z_0)] = \text{rank}\,[P_i(z_0)]$, then $\alpha_i = 1$

Even more critical case: If $\text{normalrank}\,[P_d(z_0)] = \text{normalrank}\,[P_i(z_0)]$ then there is undetectable critical attack for all frequencies z_o

Holds generically when more disturbances than measurements ($o \geq p$)!

Special Case 2: Transmission Zeros

$$P(z) = \begin{bmatrix} A - zI & B_d & B_a \\ C & D_d & D_a \end{bmatrix}$$

[Amin *et al.*, ACM HSCC, 2010]
[Pasqualetti *et al.*, IEEE TAC, 2013]

Suppose $P(z)$ has full column normal rank. Then undetected attacks only at finite set of transmission zeros $\{z_0\}$

Solve

$$\alpha_i := \min_{|z_0|\geq 1, x_0, d_0, a_0^i} \quad \|a_0^i\|_0$$

$$\text{subject to} \quad P(z_0) \begin{bmatrix} x_0 \\ d_0 \\ a_0^i \end{bmatrix} = 0$$

by inspection of corresponding zero directions ⇒ **Easy in typical case of 1-dimensional zero directions**

Example 2 (cont'd)

$$G(z) = C(zI - A)^{-1}B + D = \begin{pmatrix} G_d(z) & G_a(z) \end{pmatrix}$$

$$G_d(z) = (), \quad G_a = \begin{pmatrix} \frac{0.2}{z-0.9} & \frac{0.3}{z-0.8} \\ \frac{0.1}{z-0.9} & \frac{0.1}{z-0.9} \end{pmatrix}$$

Invariant zeros = $\sigma\big(P(z)\big) = \{1.1\}$

Undetectable attack: $a(k) = 1.1^k \begin{pmatrix} -0.282 \\ 0.282 \end{pmatrix} \Rightarrow a_0 = \begin{pmatrix} -0.282 \\ 0.282 \end{pmatrix}$

Masking initial state: $x_0 = \begin{pmatrix} -0.705 \\ 0.470 \\ 0.352 \end{pmatrix}$

Only one signal satisfies α_i constraint! Since $||a_0||_0 = 2 \Rightarrow \alpha_{1,2} = 2$

Special Case 3: Sensor Attacks

$$P(z) = \begin{bmatrix} A - zI & 0 & 0 \\ C & D_d & D_a \end{bmatrix}$$

[Fawzi *et al*., IEEE TAC, 2014]
[Chen *et al.*, IEEE ICASSP, 2015]
[Lee *et al.*, ECC, 2015]

$P(z)$ only loses rank in eigenvalues $z_0 \in \{\lambda_1(A), \ldots, \lambda_n(A)\}$

Simple eigenvalues give one-dimensional spaces of eigenvectors x_0 ⇒ **Simplifies computation of $\boldsymbol{\alpha_i}$**

Example: Suppose $D_a = I_p$ (sensor attacks), $D_d = 0$, and system observable from each y_i, $i = 1, \ldots, p$:

- By the PBH-test: $\alpha_i = p$ or $\alpha_i = +\infty$ (if all eigenvalues stable, no persistent undetectable sensor attack exists)
- Redundant measurements increase α_i!

Special Case 4: Sensor Attacks for Static Systems

$$P(z) = \begin{bmatrix} I - zI & 0 & 0 \\ C & 0 & D_a \end{bmatrix}$$

[Liu *et al.*, ACM CCS, 2009]
[Sandberg *et al.*, SCS, 2010]

Since $A = I_n$ and $B_d = B_a = 0$, this is the steady-state case

Space of eigenvectors x_0 is n-dimensional ⇒ **Typically makes computation of α_i harder than in the dynamical case!**

Practically relevant case in power systems where $p > n \gg 0$

- Problem NP-hard, but power system imposes special structures in C (unimodularity etc.)
- Several works on efficient and exact computation of α_i using min-cut/max-flow and ℓ_1-relaxation ([Hendrickx *et al*., 2014], [Kosut, 2014], [Yamaguchi *et al*., 2015])

Example 4: Simple Security Index

$$P(z) = \begin{bmatrix} A - zI & B_d & B_a \\ 0 & 0 & D_a \end{bmatrix} \qquad D_a = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Special Case 4: Solution by MILP

Big M reformulation:

$$\alpha_i := \min_{x_0, a_0} \|a_0\|_0$$
$$\text{subject to}$$
$$0 = Cx_0 + D_a a_0$$
$$a_{0,i} = 1$$

$$\Rightarrow$$

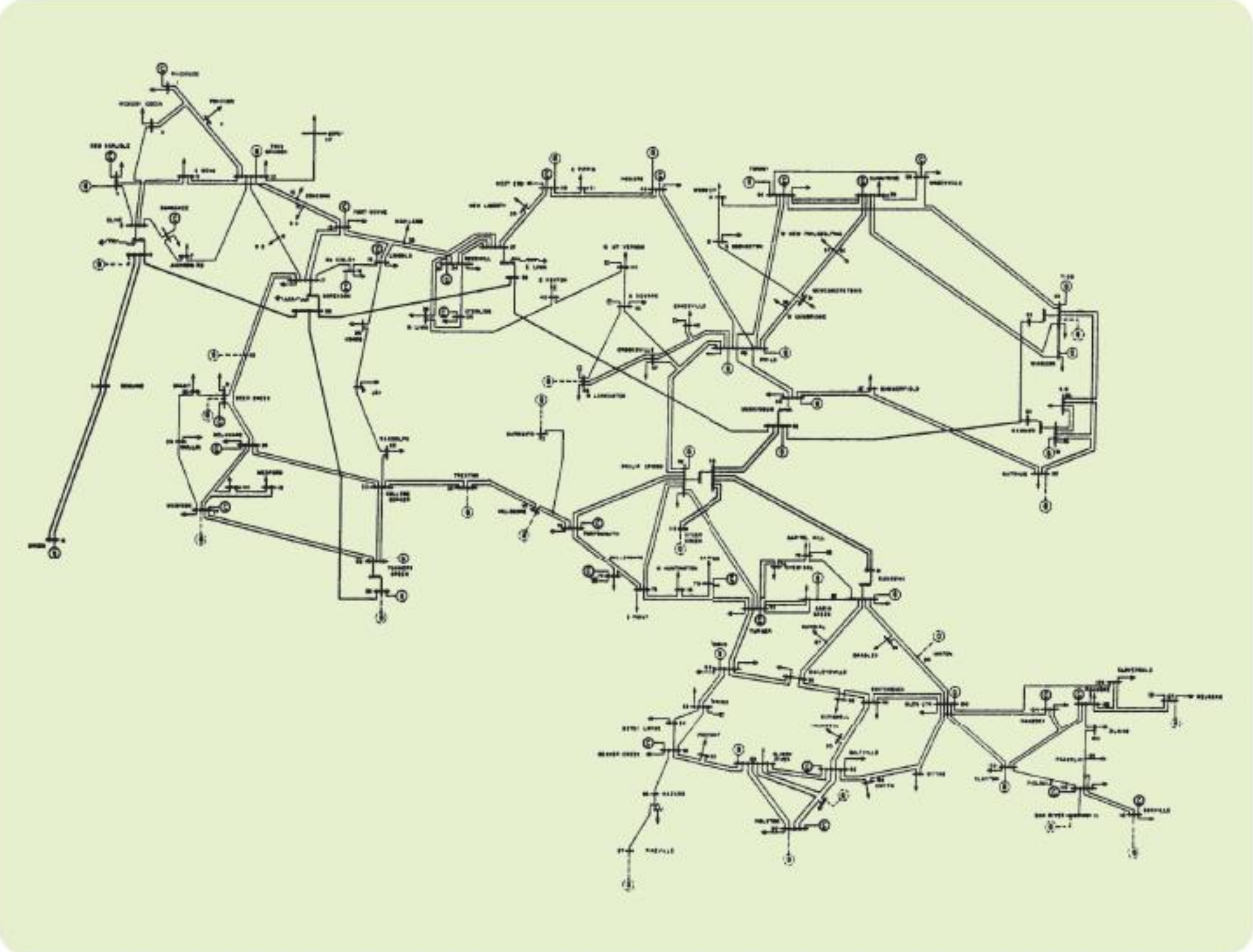
$$\alpha_i := \min_{z_0, x_0, a_0,} \sum_k z_k$$
$$\text{subject to}$$
$$0 = Cx_0 + D_a a_0$$
$$a_{0,i} = 1$$
$$-Mz \le a_0 \le Mz$$
$$z_k \in \{0, 1\}$$

Elementwise

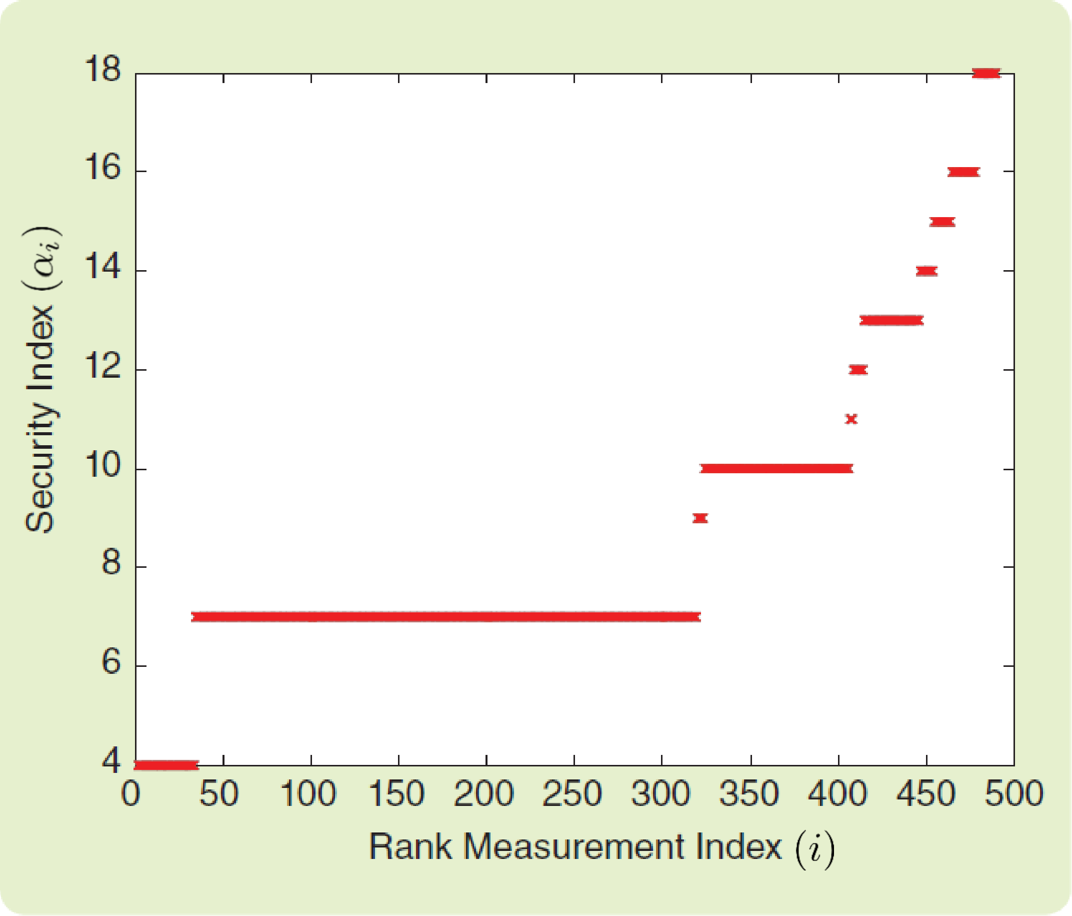
$$"\infty"$$

Example 1: Power System State Estimator for IEEE 118-bus System

- State dimension $n = 118$
- Number sensors $p \approx 490$



Example 1: Power System State Estimator for IEEE 118-bus System



- Computation time on laptop using min-cut method [Hendrickx *et al.*, IEEE TAC, 2014]: 0.17 sec
- Used for defense allocation in [Vukovic *et al.,* IEEE JSAC, 2012]

Summary So Far

- Basic risk management and control system attack space

- Dynamical security index α_i defined
  - Computation is NP-hard in general, but often “simple” in practically relevant cases:
    - One-dimensional zero-dynamics [Cases 2-3]
    - Static systems with special matrix structures (potential flow problems) [Case 4]
    - Dynamical models generally simplifies computation(!)
    - Redundant sensors increase α_i

- Fast computation enables greedy security allocation

Outline

- Risk management
- Attack detectability and security metric
- **Attack identification and secure state estimation**
- Security metric computation

Attack Identification

$$x(k+1) = Ax(k) + B_d d(k) + B_a a(k)$$
$$y(k) = Cx(k) + D_d d(k) + D_a a(k)$$

- Unknown state $x(k) \in \mathbb{R}^n$
- Unknown (natural) disturbance $d(k) \in \mathbb{R}^o$
- Unknown (malicious) attack $a(k) \in \mathbb{R}^m$
- Known measurement $y(k) \in \mathbb{R}^p$
- Known model A, B_d, B_a, C, D_d, D_a

- When can we decide there is an attack signal $a_i \neq 0$?
- Which elements a_i can we track (“identify”)?

- Not equivalent to designing an unknown input observer/secure state estimator (state not requested here). See end of presentation

Attack Identification

Definition: A (persistent) attack signal a is

- *identifiable* if for all attack signals $\tilde{a} \neq a$, and all corresponding disturbances d and $\tilde{d}$, and initial states $x(0)$ and $\tilde{x}(0)$, we have $\tilde{y} \neq y$;
- i*-identifiable* if for all attack signals a and $\tilde{a}$ with $\widetilde{a_i} \neq a_i$, and all corresponding disturbances d and $\tilde{d}$, and initial states $x(0)$ and $\tilde{x}(0)$, we have $\tilde{y} \neq y$

Interpretations:

- Identifiability ⇔ (different attack a ⇒ different measurement y) ⇔ attack signal is injectively mapped to y ⇒ attack signal is detectable
- i*-identifiable* weaker than *identifiable*
- $\forall i$: a *is* i*-identifiable* ⇔ a is *identifiable*
- a is i*-identifiable:* Possible to track element a_i, but not necessarily a_j, $j \neq i$

Theorem

Suppose that the attacker can manipulate at most q attack elements simultaneously ($\|a\|_0 \leq q$).

i. There exists persistent undetectable attacks $a^i \Leftrightarrow q \geq \alpha_i$;
ii. All persistent attacks are i-identifiable $\Leftrightarrow q < \alpha_i/2$;
iii. All persistent attacks are identifiable $\Leftrightarrow q < \min_i \alpha_i/2$.

Proof. Compressed sensing type argument. See [Sandberg and Teixeira, SoSCYPS, 2016] for details

Example 4: Simple Security Index (cont'd)

$$P(z) = \begin{bmatrix} A - zI & B_d & B_a \\ 0 & 0 & D_a \end{bmatrix} \qquad D_a = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Security indices: $\alpha_1 = 3$, $\alpha_2 = 3$, $\alpha_3 = 3$, $\alpha_4 = \infty$

Attacker with $q = 1$: Defender can identify (and thus detect) all attacks

$q = 2$: Defender can detect (not identify) all attacks against a_1, a_2, a_3 and identify all attacks against a_4

$q = 3 - 4$: Defender can identify all attacks against a_4. Exist undetectable attacks against a_1, a_2, a_3

Back to Risk Management

$$P(z) = \begin{bmatrix} A - zI & B_d & B_a \\ 0 & 0 & D_a \end{bmatrix} \qquad D_a = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Security indices: $\alpha_1 = 3,\ \alpha_2 = 3, \alpha_3 = 3, \alpha_4 = \infty$

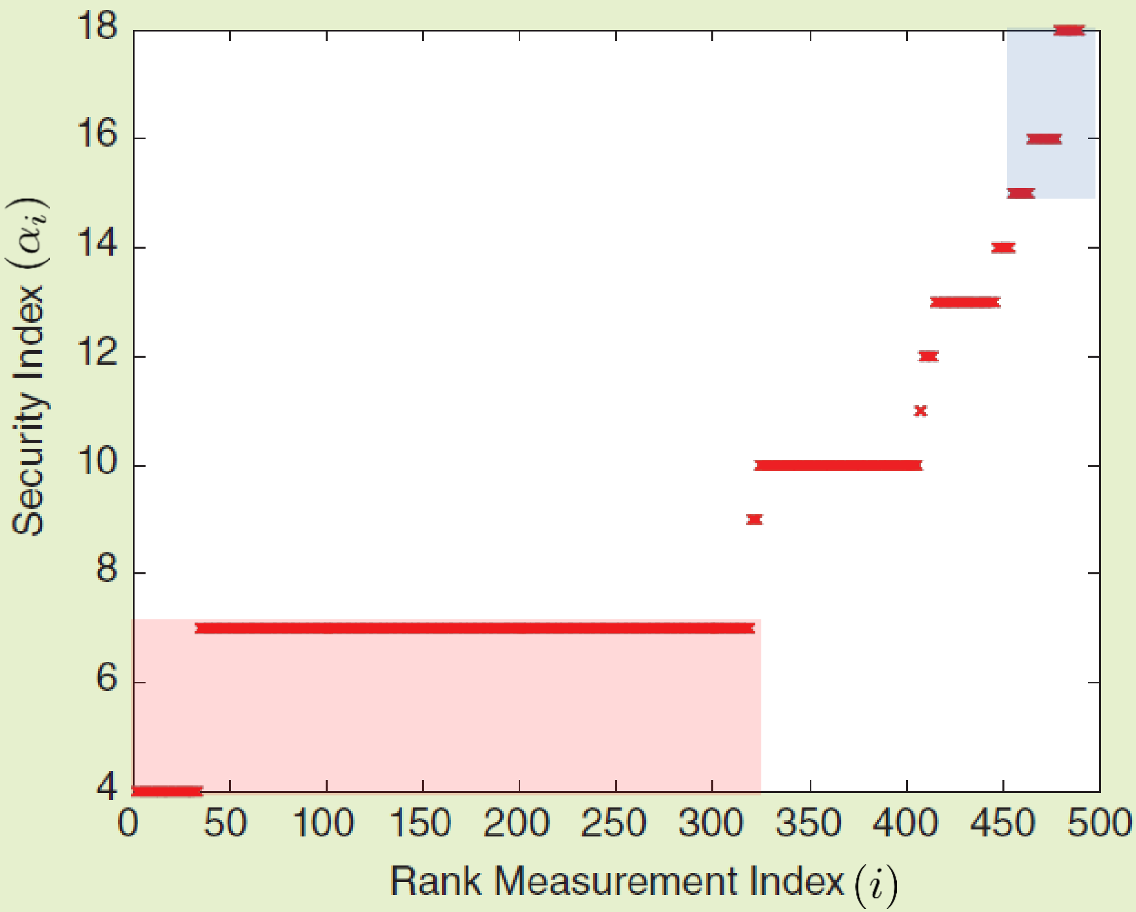
- Suppose the operator can choose to block *one* attack signal (through installing physical protection, authentication, etc.).
- Which signal a_1, a_2, a_3, or a_4 should she/he choose?
- Among the one(s) with lowest security index! Choose a_1.
- New attack model and security indices: $\alpha_2 = \alpha_3 = \alpha_4 = \infty$

$$D_a = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- By explicitly blocking one attack signal, all other attacks are implicitly blocked (they are identifiable)

Example 1: Power System State Estimator for IEEE 118-bus System

- Suppose number of attacked elements is $q \leq 7$



- Signals susceptible to undetectable attacks
- Signals were all attacks are identifiable
- Other signals will, if attacked, always result in non-zero output y

Outline

- Risk management
- Attack detectability and security metric
- Attack identification and secure state estimation
- **Security metric computation**

DC-Power Flow Measurement Matrix

$$C = \begin{bmatrix} P_1 D B^T \\ -P_2 D B^T \\ P_3 B D B^T \end{bmatrix}$$

(positive flow measurements)
(negative flow measurements)
(injection measurements)

B- directed incidence matrix of graph corresponding to power network topology

D - nonsingular diagonal matrix containing reciprocals of reactance of transmission lines

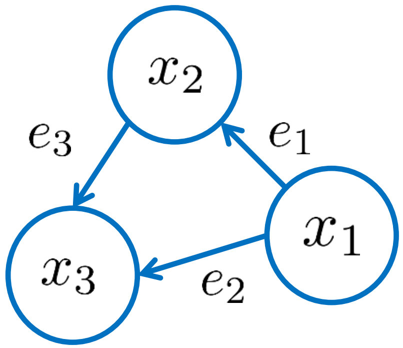
$P_{\cdot}$ - measurement selection matrices (rows of identity matrices)

More measurements than states, $m > n$. Redundancy!

Structure applies to all potential flow problems (water, gas,…)

[Hendrickx *et al*., IEEE TAC, 2014]

Example 5: DC-Power Flow Measurement Matrix

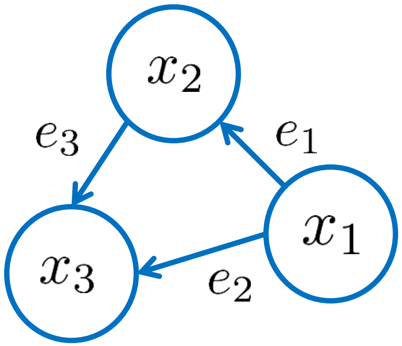


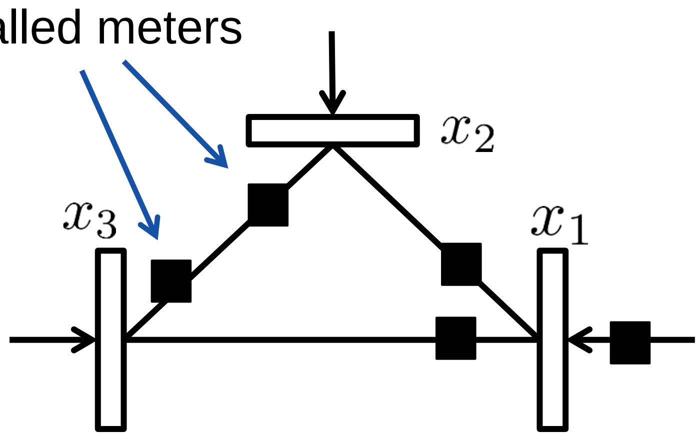
$$B^T = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}, \quad D = I$$

$$\text{Edge flows: } \begin{pmatrix} p_{12} \\ p_{13} \\ p_{23} \end{pmatrix} = DB^T x = \begin{pmatrix} x_1 - x_2 \\ x_1 - x_3 \\ x_2 - x_3 \end{pmatrix}$$

$$\text{Node injections: } BDB^T x = B \begin{pmatrix} p_{12} \\ p_{13} \\ p_{23} \end{pmatrix} = \begin{pmatrix} p_{12} + p_{13} \\ -p_{12} + p_{23} \\ -p_{13} - p_{23} \end{pmatrix}$$

Example 5: DC-Power Flow Measurement Matrix (cont'd)





$$C = \begin{bmatrix} P_1 D B^T \\ -P_2 D B^T \\ P_3 B D B^T \end{bmatrix}$$

$$P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}, \quad D = I$$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

Efficient Security Index Computation

Rewrite security index problem into equivalent form:

$$J_c := \min_{x \in \mathbb{R}^n} c^T g(DB^T x) + p^T g(BDB^T x) \equiv \|Cx\|_0$$

$$\text{subject to } B(:,k)^T x \neq 0 \quad \text{(enforce non-zero flow on edge } k\text{)}$$

- $g(\cdot)$ - Vector-valued indicator function (ex. $g\begin{pmatrix} -3 \\ 0 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$)
- $c \in \mathbb{R}_+^r$ - Encodes #edge flow meters (r edges)
- $p \in \mathbb{R}_+^n$ - Encodes #node injection meters (n nodes/states)

- Choose index k to activate sensor i so that $J_c = \alpha_i$

Restricted Binary Problem

$$J_c := \min_{x \in \mathbb{R}^n} c^T g(DB^T x) + p^T g(BDB^T x) \equiv \|Cx\|_0$$

$$\text{subject to } B(:,k)^T x \neq 0 \quad \text{(enforce non-zero flow on edge } k\text{)}$$

$$\Downarrow$$

$$J_b := \min_{x \in \{0,1\}^n} c^T g(DB^T x) + p^T g(BDB^T x) \equiv \|Cx\|_0$$

$$\text{subject to } B(:,k)^T x \neq 0 \quad \text{(enforce non-zero flow on edge } k\text{)}$$

Obviously $J_b \geq J_c$, but in fact we have…

Theorem.

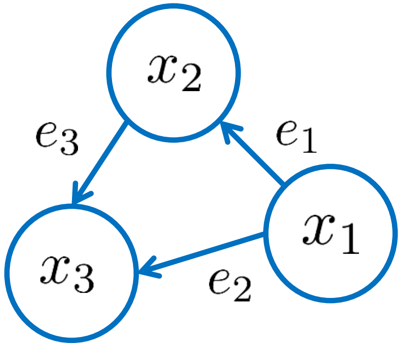
$$0 \leq J_b - J_c \leq \sum_{i=1}^{n} \max\{0, \max_{e \to v_i}\{p_i - c_e\}\}$$

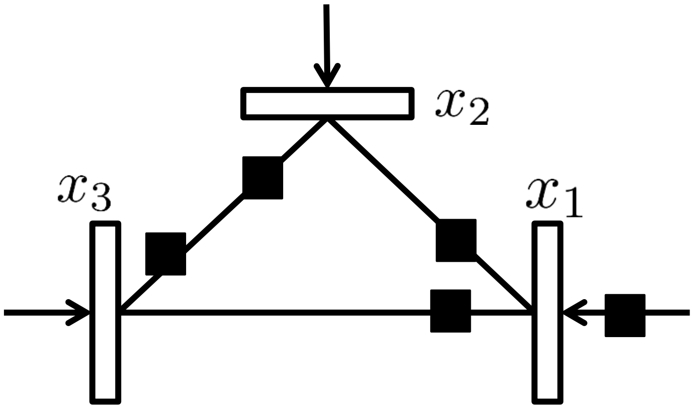
($e \to v_i$ ≔ set of edges connected to node with state x_i.)

[Hendrickx *et al*., IEEE TAC, 2014]

Example 5: DC-Power Flow Measurement Matrix (cont'd)

$$c = \begin{pmatrix} 1 & 1 & 2 \end{pmatrix}$$
$$p = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$



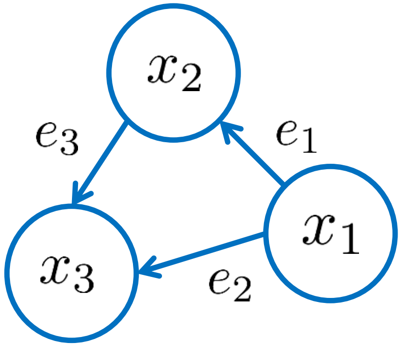


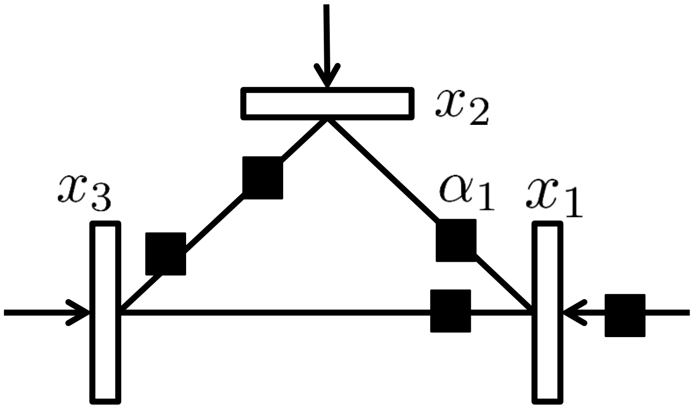
$$\begin{aligned} 0 \leq J_b - J_c &\leq \sum_{i=1}^{n} \max\{0, \max_{e \to v_i}\{p_i - c_e\}\} \\ &= \max\{0, \max\{0, 0\}\} + \max\{0, \max\{-1, -2\}\} + \max\{0, \max\{-1, -2\}\} \\ &= 0 \end{aligned}$$

Solve J_b to find exact security indices α_i!

Example 5: DC-Power Flow Measurement Matrix (cont'd)

$$c = \begin{pmatrix} 1 & 1 & 2 \end{pmatrix}$$
$$p = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

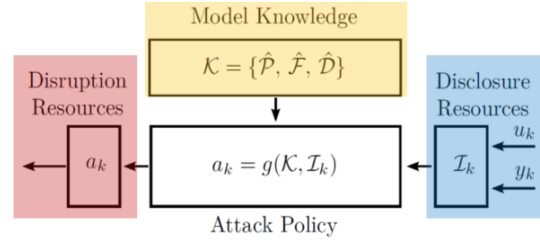


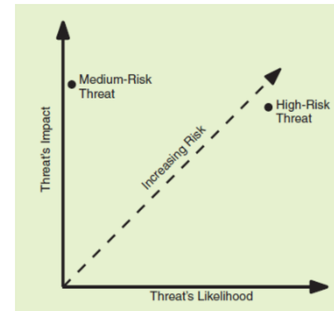


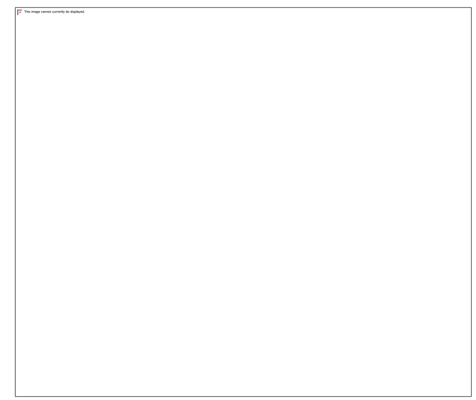
Compute α_1 in 4 steps:

1. Enforce flow across sensor 1 by choosing $x_1 = 1$ and $x_2 = 0$ [J_b constraint satisfied]
2. Test $x_3 = 0$: $\left||Cx|\right|_0 = 3$
3. Test $x_3 = 1$: $\left||Cx|\right|_0 = 4$
4. $J_b = \min\{3,4\} = 3 = \alpha_1$

Summary







- There is a need for CPS security
- Briefly introduced CPS attack models and concept of risk management

- Input observability and detectability
  ⇒ Undetectable attacks and masking initial states and disturbances

- A security metric α_i for risk management
  - Suppose attacker has access to q resources:
    - Undetectable attacks against a_i iff $q \geq \alpha_i$
    - Attack against a_i identifiable iff $q < \alpha_i/2$

- Many useful results in the fault diagnosis literature, especially for identifiable attacks: Unknown input observers, decoupling filters, etc.
- Future research direction: More realistic attacker models, estimate attack likelihoods and impacts, corporation with IT security,…

Further Reading

Introduction to CPS/NCS security

- Cardenas, S. Amin, and S. Sastry: “Research challenges for the security of control systems". Proceedings of the 3rd Conference on Hot topics in security, 2008, p. 6.
- Special Issue on CPS Security, IEEE Control Systems Magazine, February 2015
- D. Urbina *et al*.: ”Survey and New Directions for Physics-Based Attack Detection in Control Systems”, NIST Report 16-010, November, 2016

CPS attack models, impact, and risk management

- A. Teixeira, I. Shames, H. Sandberg, K. H. Johansson: ”A Secure Control Framework for Resource-Limited Adversaries”. Automatica, 51, pp. 135-148, January 2015.
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- D. Urbina *et al*.: ”Limiting The Impact of Stealthy Attacks on Industrial Control Systems”, 23rd ACM Conference on Computer and Communications Security, October, 2016

Further Reading

Detectability and identifiability of attacks

- S. Sundaram and C.N. Hadjicostis: “Distributed Function Calculation via Linear Iterative Strategies in the Presence of Malicious Agents”. IEEE Transactions on Automatic Control, vol. 56, no. 7, pp. 1495–1508, July 2011.
- F. Pasqualetti, F. Dörfler, F. Bullo: “Attack Detection and Identification in Cyber-Physical Systems”. IEEE Transactions on Automatic Control, 58(11):2715-2729, 2013.
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- R. Smith: “Covert Misappropriation of Networked Control Systems”. IEEE Control Systems Magazine, vol. 35, no. 1, pp. 82-92, February 2015.
- H. Sandberg and A. Teixeira: “From Control System Security Indices to Attack Identifiability”. Science of Security for Cyber-Physical Systems Workshop, CPS Week 2016

Further Reading

Security metrics (security index)

- O. Vukovic, K. C. Sou, G. Dan, H. Sandberg: "Network-aware Mitigation of Data Integrity Attacks on Power System State Estimation". IEEE Journal on Selected Areas in Communications (JSAC), 30:6, pp. 1108--1118, 2012.
- J. M. Hendrickx, K. H. Johansson, R. M. Jungers, H. Sandberg, K. C. Sou: "Efficient Computations of a Security Index for False Data Attacks in Power Networks". IEEE Transactions on Automatic Control: Special Issue on Control of CPS, 59:12, pp. 3194-3208, December 2014.
- H. Sandberg and A. Teixeira: “From Control System Security Indices to Attack Identifiability”. Science of Security for Cyber-Physical Systems Workshop, CPS Week 2016

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(KTH)

Secure State Estimation/Unknown Input Observer (UIO)

Secure state estimate $\hat{\boldsymbol{x}}$: Regardless of disturbance d and attack a, the estimate satisfies $\hat{x} \to x$ as $k \to \infty$

1. Rename and transform attacks and disturbances:

$$\begin{bmatrix} B_d \\ D_d \end{bmatrix} d + \begin{bmatrix} B_a \\ D_a \end{bmatrix} a = \begin{bmatrix} B_f \\ D_f \end{bmatrix} f, \quad \text{such that } \begin{bmatrix} B_f \\ D_f \end{bmatrix} \text{full column rank}$$

2. Compute security indices α_i with respect to f

Theorem: A secure state estimator exists iff

1. (C, A) is detectable; and

2. $q < \min_i \frac{\alpha_i}{2}$, where q is max number of non-zero elements in f.

Proof. Existence of UIO by [Sundaram *et al.*, 2007] plus previous theorem

How to Identify an Attack Signal?

Use decoupling theory from fault diagnosis literature [Ding, 2008]

Suppose that $y = G_d d + G_a a$ and

$$\text{normalrank}\,[G_d(z)] = m',$$
$$\text{normalrank}\,[G_d(z)\ G_a(z)] = m' + m''$$

Then there exists linear decoupling filter R such that

$$\begin{bmatrix} r \\ y' \end{bmatrix} = R(G_d d + G_a a) = \begin{bmatrix} 0 & \Delta \\ G'_d & G'_a \end{bmatrix} \begin{bmatrix} d \\ a \end{bmatrix},$$
$$\text{normalrank}\,[G'_d(z)] = \text{normalrank}\,[G'_d(z)\, G'_a(z)] = m'$$
$$\text{normalrank}\,[\Delta(z)] = m''$$

How to Identify an Attack Signal?

Suppose a is identifiable ($q < \min_i \alpha_i/2$)

1. Decouple the disturbances to obtain system $r = \Delta a$

2. Filter out uncertain initial state component in r to obtain $r' = \Delta a$

3. Compute left inverses of $\Delta_I := [\Delta_i]_{i \in I}$ formed out of the columns Δ_i of Δ, for all subsets $|I| = q$, $I \subseteq \{1, \dots, m\}$ (**Bottleneck! Compare with compressed sensing**)

4. By identifiability, if estimate $\hat{a}_I$ satisfies $r' = \Delta \hat{a}_I$, then $\hat{a}_I \equiv a$

(Similar scheme applies if a is only i-identifiable)