2015 IEEE 54th Annual Conference on Decision and Control (CDC)
December 15-18, 2015. Osaka, Japan

Demand response for aggregated residential consumers with energy storage sharing

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***Abstract*— A novel distributed algorithm is proposed in this paper for a network of consumers coupled by energy resource sharing constraints, which aims at minimizing the aggregated electricity costs. Each consumers is equipped with an energy management system that schedules the shiftable loads accounting for user preferences, while an aggregator entity coordinates the consumers demand and manages the interaction with the grid and the shared energy storage system (ESS) via a distributed strategy. The proposed distributed coordination algorithm requires the computation of Mixed Integer Linear Programs (MILPs) at each iteration. The proposed approach guarantees constraints satisfaction, cooperation among consumers, and fairness in the use of the shared resources among consumers. The strategy requires limited message exchange between each consumer and the aggregator, and no messaging among the consumers, which protects consumers privacy. Performance of the proposed distributed algorithm in comparison with a centralized one is illustrated using numerical experiments.**

***Index Terms*—Demand response, mixed integer linear programming, distributed scheduling algorithms.**

I. Introduction

Residential areas are responsible for nearly 40% of the energy consumption in developed countries, which are known to have significant potential for energy and cost savings, as well as for load shifting, compared to industry and transportation [1]. To take advantage of this potential, Demand Response (DR) has received increased attention in recent years since it can efficiently support load balancing and economical/environmental cost reduction [2]. DR is commonly defined as changes in electricity use by consumers in response to changes in the electricity price over time [2]. Effective DR policies naturally require smart appliances, which can be switched on or off in response to specific DR signals, e.g., price signals. Several works have proposed load management strategies and scheduling smart appliances, accounting for price information (e.g., see [2], [3], [4], [5], [6], [7], [8], and [9]).

The aforementioned works do not consider electrical storage systems (ESS), while it would be more flexible and efficient for consumers to manage their energy use in response to time-varying electricity prices and network congestion, taking advantage of the capability of these devices to store energy and release it when it is more convenient [10], [11], [12], [13], and [14]. Since storage devices are still expensive, a reasonable solution to afford the expense and benefit from the use of an ESS would be to share it among several consumers. Therefore, the households should be coordinated by an aggregator/coordinator. Aggregators are new entities in the electricity market that act as mediators between users and the utility operator, and possess the technology to perform DR signals and communicate with both users and utilities [15]. In [16], an algorithm is built on the alternating directions method of multipliers (ADMM), focusing on decentralized algorithms for Electric Vehicles charging. In addition, a coordination framework based on ADMM is proposed in [17] to negotiate among the households and a coordinator, with the main goal being to minimize the imbalance among communities, while including objectives and constraints for each community and taking into account each user's quality of life/activities.

The main contribution of this paper is to propose a novel distributed optimization algorithm for scheduling of smart appliances in residential areas sharing an ESS. Here, in addition to the detailed modeling of loads and ESS, state of health of the ESS is taken into account. The scheduling problem is formulated as a MILP problem with the aim of minimizing the aggregated electricity cost. In this study, an ESS is integrated with the aggregator to increase level of comfort and profit for the users. The role of the aggregator is thus to provide DR services to the grid operator and economical incentives to the users to reshape their demand profiles, guaranteeing that the benefits of using a shared resources are fairly allocated to all the users. The aggregator negotiates with the home users to shift their loads and increase the ESS benefits, providing incentives to the users to shift their consumption.

The rest of the paper is structured as follows. Section II describes the system and the connections between the two layers (aggregator and apartment) of the system. Section III proposes a distributed scheduling algorithm for smart appliances and EESs to cope with peak-demand shaving, and fairly devoting the monetary profits to the apartments (based on their flexibility in load shifting). Section IV presents a simple motivation example to show the coupling between ESS and apartment consumption. In addition, preliminary simulation results in this section, illustrate the performance of the proposed distributed algorithm in comparison with a centralized one. Finally, Section V provides conclusions and

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This work is supported by the European Institute of Technology (EIT) Information and Communication Technology (ICT) Labs, the EU 7th Framework Programme (FP7/2007-2013, grant agreement n° 608224), the Swedish Energy Agency, Swedish Foundation for Strategic Research through the ICT-Psi project, the Swedish Governmental Agency for Innovation Systems (VINNOVA), and the Knut and Alice Wallenberg Foundation.

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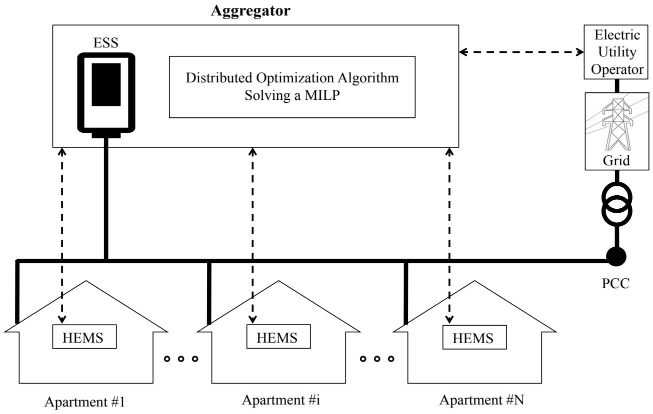

Fig. 1. Schematic of interconnected apartments and aggregator.

suggestions for future studies.

II. System description

In this paper, we consider *active* apartments, i.e., houses where effective DR policies are enabled through the integration of smart appliances, scheduling algorithms, energy management systems, and information exchange over wireless communication technologies. As depicted in Figure 1, we consider a small-scale community which can range from the apartments in one building to a small district of a city. The overall system forms a microgrid, with one single point of common coupling (PCC) with the distribution grid. Each apartment is equipped with a Home Energy Management System (HEMS), which is responsible for locally operating end-user smart appliances. Each HEMS is connected to an aggregator entity via a communication network, which aims at coordinating the apartments, scheduling the ESS and managing the interaction with the distribution grid. The apartments are independent from each other and coupled only through the shared ESS and the PCC power limits. In this structure, the aggregator coordinates the apartments through energy shift request signals. In this negotiation, the aggregator provides economical incentives to home users accepting to modify their energy pattern.

In the next section we describe the distributed approach to the energy management of the system under consideration.

III. Problem formulation

In this section we describe a distributed approach to solve the problem of coordinating a set of smart appliances located in N apartments sharing an ESS such that each apartment can profit from the use of the ESS while technical and operational constraints, as well as user preferences, are satisfied. In order to manage a large set of appliances we propose an iterative two-layer hierarchical approach. The approach is based on a distributed algorithm with problems formulated at the apartment and at the aggregator levels being MILP problems. The local HEMSs are coordinated by the aggregator in order to come up with an agreement throughout negotiation iterations and provide a feasible solution to the centralized problem. We remark that the proposed algorithm is suitable for model predictive control frameworks: it can be implemented in a hierarchical fashion where the problems at apartment levels are executed in parallel and the updates of aggregated demand profile is carried out by the high-level aggregator. In the following we first describe the appliance and the ESS modeling, based on the study [14].

A. Appliances and ESS constraints

This section is based on the mathematical formulation illustrated in [14]. The scheduling horizon is discretized into T uniform time slots. The number of appliances in the apartment a is denoted by N^{a}, and n^i denotes the number of un-interruptible energy phases for each appliance. The energy assigned to energy phase j of appliance i for apartment a during the whole period of time slot k is denoted by p_k^{ija}. Binary decision variables (x_k^{ija}) are required to indicate whether a particular energy phase is being processed or not. Moreover, two other sets of binary decision variables are needed to model the decision problem. One is denoted as s_k^{ija}, with a value of one indicating that, in appliance i, energy phase j is already finished by time slot k. The other set is denoted as t_k^{ija}. These decision variables are used to indicate whether at time slot k, appliance i is making a transition between running phase $j-1$ to j.

The constraint that is enforced to make sure that the energy phases fulfill their energy requirement is as

$$\sum_{k=1}^{T} p_k^{ija} = E^{ija}, \quad \forall i, j, \tag{1}$$

where E^{ija} is the energy requirements for energy phase j in appliance i. To determine that an energy phase is being processed during time slot k, while the limitation on lower and upper power assignment to the phase are satisfied, the constraint

$$\underline{p}_k^{ija} x_k^{ija} \le p_k^{ija} \le \overline{p}_k^{ija} x_k^{ija}, \quad \forall i, j, k, \tag{2}$$

is enforced, and the $\underline{p}_k^{ija}$ and $\overline{p}_k^{ija}$ are the lower and upper limits. Also, the power safety constraint is imposed as

$$\sum_{i=1}^{N} \sum_{j=1}^{n^i} p_k^{ija} \le \overline{P}_k, \quad \forall k, \tag{3}$$

where $\overline{P}_k$ is the upper limit of the total energy assigned at time slot k, for each apartment. The limits on energy phases process time are imposed as

$$\underline{T}^{ija} \le \sum_{k=1}^{m} x_k^{ija} \le \overline{T}^{ija}, \quad \forall i, j, \tag{4}$$

where the $\underline{T}^{ija}$ and $\overline{T}^{ija}$ are the lower and upper limits of the number of time slots for the related energy phase to be processed. To satisfy the sequential processing of the energy phases of an appliance and sequential operation of appliances, the following constraints are imposed respectively

$$\begin{array}{rcll} x_k^{ija} & \le & s_k^{i(j-1)a}, & \forall i, k, \ \forall j = 2, \ldots, n^i, \\ x_k^{ija} & \le & s_k^{\tilde{i} n^{\tilde{i}} a}, & \forall k, \end{array} \tag{5}$$

with the $\tilde{i}$ being the index of the appliance which must be finished before the appliance with i index can start running. To make sure that the energy phases are un-interruptible the following constraint is imposed.

$$\begin{array}{rcll} x_k^{ija} & \leq & 1-s_k^{ija} & \forall i,j,k \\ x_{k-1}^{ija}-x_k^{ija} & \leq & s_k^{ija} & \forall i,j,\, \forall k=2,\dots,m \\ s_{k-1}^{ija} & \leq & s_k^{ija} & \forall i,j,\, \forall k=2,\dots,m. \end{array} \quad (6)$$

To count the number of time slots spent between the energy phases in an appliance and impose lower and upper limits on this number, the constraints

$$t_k^{ija} = s_k^{i(j-1)a} - (x_k^{ija}+s_k^{ija}) \forall i,j,\, \forall k=2,\dots,n^i, \quad (7)$$

$$\underline{D}^{ija} \leq \sum_{k=1}^{T} t_k^{ija} \leq \overline{D}^{ija}, \forall i,\, \forall j=2,\dots,n^i, \quad (8)$$

are considered, where $\underline{D}^{ija}$ and $\overline{D}^{ija}$ are between-phase delay lower and upper bounds, respectively. Finally, to meet the household preferences and finishing a particular appliance within a specified time interval, the constraint

$$x_k^{ija} \leq TP_k^{ia} \quad \forall i,j,k, \quad (9)$$

is enforced, where TP_k^{ia} is the time preference interval.

To include an EES the following set of constraints is defined [14]. The ESS dynamics are modeled as follows:

$$b_k^s = \alpha b_{k-1}^s + \eta_c b_{k-1}^c - \eta_d b_{k-1}^d, \quad \forall k=2,\dots,T, \quad (10)$$

where α is a constant energy degradation in each sampling interval, b_k^s is the energy level at time slot k and η^c and η^d are efficiencies accounting for the losses during charging and discharging. The power exchanged with the EES during time slot k is denoted by b_k^c (or b_k^d) when the EES is charging (or discharging). The following limits on the energy level and the power exchanged with the ESS are enforced:

$$\underline{b}^s \leq b_k^s \leq \overline{b}^s, \quad \forall k, \quad (11)$$

where $\underline{b}^s$ and $\overline{b}^s$ denote the lower and the upper bounds respectively, and

$$0 \leq b_k^c \leq \overline{b}_k^c x_k^c, \quad 0 \leq b_k^d \leq \overline{b}_k^d x_k^d, \quad \forall k, \quad (12)$$

where the binary decision variables x_k^c and x_k^d indicate whether the EES is charging or discharging in time slot k, respectively. The bounds on the power exchanged with the ESS are $\overline{b}_k^c$ and $\overline{b}_k^d$ for charging and discharging respectively. Further, the constraint

$$x_k^c + x_k^d \leq 1, \quad \forall k, \quad (13)$$

has to be satisfied to rule out the simultaneous charging and discharging during the same time slot. To take the state of health of EES into account, the total number of charging and discharging cycles during a day must be limited to a determined number N^c, and the constraints

$$\begin{array}{rcll} x_k^c - x_{k-1}^c & \leq & c_k^t, & \forall k=2,\dots,m \\ x_k^d - x_{k-1}^d & \leq & d_k^t, & \forall k=2,\dots,m \\ \sum_{i=1}^{m} c_k^t + d_k^t & \leq & N^c\,, & \end{array} \quad (14)$$

are imposed, where the auxiliary binary decision variables c_k^t and d_k^t determine the transition time slots to start charging and discharging, respectively. Finally, it is reasonable to assume that the initial and the final energy levels (b_0^s and b_T^s respectively) in the EES are the same, since the final energy level is also the initial condition for the next day scheduling. Hence, the following equality constraint on the initial and final SOC is enforced

$$b_0^s = b_T^s. \quad (15)$$

B. Centralized problem

The centralized scheduling problem for a network of apartments with a shared ESS is formulated as the following MILP:

$$\begin{array}{ll} \min & \sum_{k=1}^{T} c_k p_k^{\text{grid}} \\ \text{s.t.} & \text{constraints } (1)-(15) \\ & \sum_{a=1}^{N}\sum_{i=1}^{N^a}\sum_{j=1}^{n^i} p_k^{ija} + b_k^c - b_k^d = p_k^{\text{grid}} \\ & \underline{p}_k^{\text{grid}} \leq p_k^{\text{grid}} \leq \overline{p}_k^{\text{grid}}. \end{array} \quad (16)$$

When the number of smart appliances increases, solving the centralized problem can be computationally prohibitive. Thus, we proposed the following distributed algorithm.

C. Description of the distributed algorithm

Here we describe the proposed distributed algorithm. The algorithm comprises an initialization step, and the definition of MILP problems at the apartment and aggregator levels.

Parameters and variables involved in the algorithm: Table I reports all the other parameters and variables defined in the algorithm.

TABLE I: Parameters and variables involved in the algorithm

l	iteration number within current time step
N	number of apartments/single-family houses
N^a	number of appliances of apartment a
$p^{\text{grid},l}$	total exchanged power with the grid at iteration l
$p^{ija,l}$	power feeding into the apartment a (per appliance and per energy phase) at iteration l
β	penalty on the unsatisfied share of energy shift required by the Aggregator
γ	reward on the redistributed part of unsatisfied energy shift
$\underline{p}^{\text{grid}}$, $\overline{p}^{\text{grid}}$	lower and upper bounds on the power exchanged with the grid
$p^{\text{app},l}$	aggregated demand at iteration l
E^{app}	total energy requirements
$G^{\text{TOT},l}$	total profit due to the ESS at iteration l
	at the end of the horizon
$G^{\text{a},l}$	profit per apartment at iteration l
$\Delta p^{\text{AGG},l}$	energy shift required by the Aggregator at iteration l
$\delta p^{\text{a},l}$	accepted energy shift by apartment a at iteration l
$\tilde{p}^{\text{a},l}$	unsatisfied share of energy shift by apartment a at iteration l
$\delta p^{+\text{a},l}$, $\delta p^{-\text{a},l}$	redistributed energy shift by apartment a at iteration l
	("+" for energy increase and "-" for energy decrease)

Initialization: The following problem is solved for each apartment a, $\forall a = 1, \dots, N$:

$$\begin{array}{ll} \min & \sum\limits_{k=1}^{T} c_k \left(\sum\limits_{i=1}^{N^a} \sum\limits_{j=1}^{n^i} p_k^{ija,0} \right) \\ \text{s.t.} & \text{constraints } (1) - (9). \end{array} \tag{17}$$

The aggregated demand profile, resulting from solving Problem (17) for each apartment, represents the solution of the centralized problem (16) without considering any shared ESS, which is fully separable in that case. The sum of the optimal values for each apartment is an upper bound on the optimal solution of the problem (16) with a shared ESS. This sum is computed as $G^{\mathrm{TOT},0} = \sum\limits_{k=1}^{T} c_k \left(\sum\limits_{a=1}^{N} \sum\limits_{i=1}^{N^a} \sum\limits_{j=1}^{n^i} p_k^{ija,0} \right)$.
The following problem is solved for initializing the aggregator:

$$\begin{array}{ll} \min & \sum_k c_k p_k^{\mathrm{grid},0} \\ \text{s.t.} & \text{constraints } (10) - (15) \\ & p_k^{\mathrm{app},0} + b_k^c - b_k^d = p_k^{\mathrm{grid},0} \\ & \underline{p}_k^{\mathrm{grid}} \leq p_k^{\mathrm{grid},0} \leq \overline{p}_k^{\mathrm{grid}} \\ & \sum\limits_{k=1}^{T} p_k^{\mathrm{app},0} = E^{\mathrm{app}}. \end{array} \tag{18}$$

The aggregated energy profile computed through Problem (18) is the best possible since it accounts only the total energy required to run all the appliances in the network of apartments, without considering user preferences and technical constraints on the energy assignment. Thus the optimal value of Problem (18) is a lower bound on the optimal solution of the problem (16) with a shared ESS. Once all the apartment solve the corresponding Problem (17), they send the computed optimal energy profile to the aggregator, which calculates the difference between the aggregated energy profiles obtained at apartment and aggregator levels as follows:

$$\Delta p_k^{\mathrm{AGG},0} = p_k^{\mathrm{app},0} - \left(\sum_{a=1}^{N} \sum_{i=1}^{N^a} \sum_{j=1}^{n^i} p_k^{ija,0} \right).$$

This difference is sent to the apartments as shift request signal and the algorithm proceeds according the steps described in Algorithm 1.

Before describing the iterations of the proposed distributed algorithm, we formulate the problems to be solved at apartment and aggregator levels, to be done after initialization.

Problem at apartment level: The problem at apartment level a at iteration l is formulated as follows:

$$\begin{array}{ll} \min & \sum\limits_{k=1}^{T} c_k \left(\sum\limits_{i=1}^{N^a} \sum\limits_{j=1}^{n^i} p_k^{ija,l} + \beta \tilde{p}_k^{\mathrm{a},l} \right) \\ \text{s.t.} & \text{constraints } (1) - (9) \\ & \sum\limits_{j=1}^{n^i} p_k^{ij,l} = \sum\limits_{j=1}^{n^i} p_k^{ij,l-1} + \delta p_k^{\mathrm{a},l} \\ & |\frac{\Delta p_k^{\mathrm{AGG},l}}{N}| - \tilde{p}_k^{\mathrm{a},l} \leq |\delta p_k^{\mathrm{a},l}| \leq |\frac{\Delta p_k^{\mathrm{AGG},l}}{N}| \end{array} \tag{19}$$

where the decision variable $\delta p_k^{\mathrm{a},l}$ models the differences in the energy profile between two consecutive iterations. The variable $\delta p_k^{\mathrm{a},l}$ has the same sign of $\Delta p_k^{\mathrm{AGG},l}$, which is the energy shift request signal sent by the aggregator at iteration l. Notice that the unmet share of the energy shift requested by the aggregator at time slot k, $\tilde{p}_k^{\mathrm{a},l}$, is penalized in the objective function with a factor greater than energy prices by at least 2 order of magnitude. Unmet energy shift can be needed mainly to avoid constraint violation in Problem (19).

Problem at aggregator level: The problem at aggregator level at iteration l is formulated as follows:

$$\begin{array}{ll} \min & \sum_k c_k p_k^{\mathrm{grid},l} \\ \text{s.t.} & \text{constraints } (10) - (15) \\ & p_k^{\mathrm{app},l} + b_k^c - b_k^d = p_k^{\mathrm{grid},l} \\ & p_k^{\mathrm{app},l} = \sum\limits_{a=1}^{N} \sum\limits_{i=1}^{N^a} \sum\limits_{j=1}^{n^i} p_k^{ija,l} \\ & \underline{p}_k^{\mathrm{grid}} \leq p_k^{\mathrm{grid},l} \leq \overline{p}_k^{\mathrm{grid}}. \end{array} \tag{20}$$

The shift request signal at iteration l is computed as follows:

$$\Delta p_k^{\mathrm{AGG},l} = p_k^{\mathrm{app},0} - \left(\sum_{a=1}^{N} \sum_{i=1}^{N^a} \sum_{j=1}^{n^i} p_k^{ija,l} \right)$$

Computation of cost benefits due to ESS: The overall profit at the end of the scheduling horizon at each iteration l is: $G^{\mathrm{TOT},l} = G^{\mathrm{TOT},0} - \sum_k c_k p_k^{\mathrm{grid},l}$. The cost benefits are equally shared among the apartments, however a penalty is assigned to the unmet energy shift requested by the aggregator. The profit at apartment level a at iteration l is then computed as: $G^{\mathrm{a},l} = \max(\frac{G^{\mathrm{TOT},l}}{N} - \sum_k c_k \tilde{p}_k^{\mathrm{a},l}, 0)$.

Steps of the distributed algorithm : The steps of the proposed algorithm are detailed in 1.

Algorithm 1 Distributed algorithm

1: Initialization and computation of $\Delta p_k^{\mathrm{AGG},0}$, $\forall k$
2: **for** $l = 1, 2, \dots, \mathrm{MaxIteration}$ **do**
3: each apartment solves Problem (19)
4: each apartment sends to aggregator the computed power profiles
5: aggregator solves Problem (20)
6: aggregator computes $G^{\mathrm{TOT},l}$
7: each apartment a computes $G^{\mathrm{a},l}$
8: if $G^{\mathrm{a},l} < G^{\mathrm{a},l-1}$, apartment a accepts the energy profile, otherwise $p_k^{ija,l} = p_k^{ija,l-1}$, $\forall i, j, k$
9: if all apartments accept, stop, otherwise compute $\Delta p_k^{\mathrm{AGG},l}$ and repeat

Redistribution strategy: An improvement in the solution obtained by Algorithm 1 at each iteration can be achieved by trying to redistribute the unmet energy shift request from the Aggregator among the apartments. Hence, the profiles of the total positive and negative unmet energy per time slot are computed respectively as $\tilde{p}_k^{+,l} = \sum\limits_{a=1}^{N} \tilde{p}_k^{+\mathrm{a},l}$ and $\tilde{p}_k^{-,l} = \sum\limits_{a=1}^{N} \tilde{p}_k^{-\mathrm{a},l}$. An additional step is to be included in Algorithm 1 between step 3 and 4. The redistribution is achieved by solving the following problem starting from the apartment level 1:

$$\begin{array}{ll} \min & \sum_{k=1}^{T} c_k \left(\sum_{i=1}^{N^a} \sum_{j=1}^{n^i} p_k^{ija,l} - \gamma(\delta p_k^{+\mathrm{a},l} + \delta p_k^{-\mathrm{a},l}) \right) \\ \text{s.t.} & \text{constraints } (1) - (9) \\ & \sum_{j=1}^{n^i} p_k^{ija,l} = \sum_{j=1}^{n^i} \bar{p}_k^{ija,l} + \delta p_k^{+\mathrm{a},l} - \delta p_k^{-\mathrm{a},l} \\ & 0 \leq \delta p_k^{+\mathrm{a},l} \leq \tilde{p}_k^{+,l} \\ & 0 \leq \delta p_k^{-\mathrm{a},l} \leq \tilde{p}_k^{-,l}. \end{array} \tag{21}$$

where $\sum_{j=1}^{n^i} \bar{p}_k^{ija,l}$ is the energy per time slot computed at iteration l at step 3. The total unmet energy per time slot is then updated by subtracting $\delta p_k^{+\mathrm{a},l}$ and $\delta p_k^{-\mathrm{a},l}$ from $\tilde{p}_k^{+,l}$ and $\tilde{p}_k^{-,l}$ respectively. Problem (21) is solved then for the next apartments until either there is still unmet energy shift or all the apartments have been asked for redistribution.

Properties of the distributed algorithm: Algorithm 1 has the following desirable properties:

- **feasibility of the solution**: at the initialization step, bounds on the optimal value of Problem (16) are computed. Clearly, the optimal schedules computed at the initialization step are not feasible solutions of the centralized problem. After the initialization, during each iteration of Algorithm 1, feasible solutions are obtained: this is guaranteed by the procedure defined by the algorithm. Effectively, during a generic iteration, the energy profiles sent by the apartments and included in (20) as given aggregated load satisfying all the appliances constraints and user preferences, as defined by Problem (19); on the other hand, the ESS schedule computed by solving (20) fulfills all the technical and operational constraints concerning the ESS and the interaction with the distribution grid. Every time the energy profiles at apartment level are computed based on energy shift requests from the aggregator, an updated ESS schedule is computed based on the resulting aggregated energy profile. By doing so, the solution computed at each iteration satisfies all the constraints formulated in the centralized problem (16);
- **suboptimality of the solution**: as mentioned above, at the initialization step a lower and an upper bound on the optimal value of Problem (16) are computed. Subsequently, at each iteration of Algorithm 1, the solution steps towards the optimal solution of the centralized problem. This is ensured by two aspects of the procedure: *i)* an apartment accepts an update on it energy use profile only if its local objective function, which includes also ESS-related benefits, decreases; *ii)* the ESS schedule has to account for the energy profiles computed at the apartment level, which certainly leads to a value of the objective function at the aggregator level greater than the one computed at the initialization step. However, the algorithm provides a suboptimal solution since there are no guarantees that the optimal solution is reached when the algorithm terminates;
- **fair allocation of profits**: the ESS-related profits at the end of the scheduling horizon are equally divided among the apartments, so are the energy shift requests. Further, an incentive mechanism is considered: users are penalized for the unmet energy shift request and rewarded for taking on a share of the total unmet energy shift requested by the aggregator. We will include a mathematical proof of this third property in an extended version of this study.

We remark that infeasibility can occur at aggregator level during a generic iteration. This can be prevented by modifying Problem (20) and replacing the constraint on $p_k^{\mathrm{app},l}$ with the following constraint:

$$p_k^{\mathrm{a},l} - \Delta p_k^{\mathrm{AGG},l} \leq p_k^{\mathrm{app},l} \leq p_k^{\mathrm{a},l} + \Delta p_k^{\mathrm{AGG},l},$$

where $p_k^{\mathrm{a},l} = \sum_{a=1}^{N} \sum_{i=1}^{N^a} \sum_{j=1}^{n^i} p_k^{ija,l}$ and $\Delta p_k^{\mathrm{AGG},l}$ is opportunely weighted in the objective function.

IV. Motivation example and preliminary results

Scheduling problem for a network of apartments, which are sharing an ESS, is formulated in the (16). From (16) One may notice that p_k^{grid} (which is the power exchange with the grid at PCC) is simply the power consumption of the apartments plus power exchange(charging/discharging) whit the ESS. In the real power network, the p_k^{grid} is limited within upper and lower limits, to protect the network from overload. In this problem, aggregators goal is to minimize the electricity consumption cost for whole the system, and in the most optimistic case the SHAs in the apartments will be scheduled (while satisfying their constraints) when the price of electricity is minimum. Also, ESS will charge when the price is low and discharge when the price is high, to make the most possible profit out of the grid. This optimistic case will result in the optimal solution for the problem as far as the p_k^{grid} is within the power limitation bound for all the times during the day, and in this case we can say that scheduling of appliances in the apartments could be done separately from the ESS scheduling (the scheduling is decoupled). This is not always the case, and by scheduling of the SHAs and charging of the ESS to be happened at the same time (when the price of electricity is low), the p_k^{grid} violates the power limitations at some points during the day. In this case, the overload should be shifted to the other times by the aggregator, in which either users should change their desired scheduling or the ESS scheduling should change. In this sense, SHAs and ESS scheduling are coupled with each other and the optimization algorithm in the aggregator level should find an optimal solution, by joint scheduling of SHAs and ESS.

Motivation example: In this example, apartments A and B (number of apatments in Fig. 1 is two) share an ESS, and whole the system is connected to the grid at PCC. The scheduling of the shared storage, and also appliances in these apartments are shown in Fig. 2, for two different cases,i) the boundaries on power exchange with the grid at PCC are not limiting (in the left side of the figure), and ii) the power

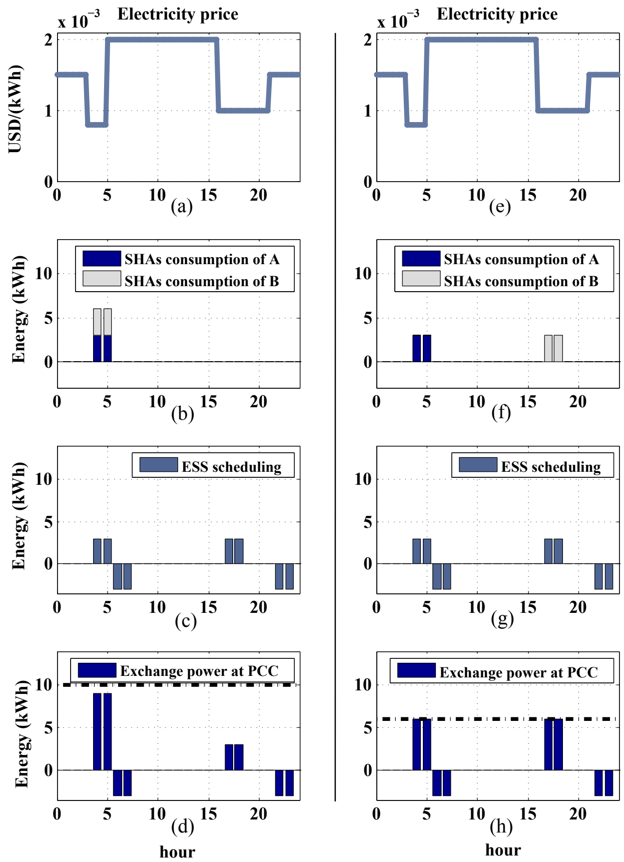

Fig. 2. Scheduling of appliances (in the apartments A and B) and the shared ESS in two different cases, i) the boundaries on power exchange with the grid at PCC are not limiting (in the left side), and ii) the power exchange at PCC is more limited (in the right side).

exchange at PCC is limited in a narrow bound (in the right side). As it is shown in the left side of this figure, by having an upper limitation on the p_k^{grid} to be high enough (in this case 10kW), the schedulung of SHAs in the both apartments A and B and also ESS charging will be scheduled when the electricity price has the lowest value (between 3:00 and 5:00 am). Therefore, in this case the ESS charging and the SHAs can be scheduled in a decoupled fashion, and the maximum total power exchange (9kW between 3:00 and 5:00 am) will not violate the power limitation. SHAs and ESS scheduling, and the total power consumption are illustrated in the parts (b), (c), and (d),respectively.

On the other hand, in the case that the power exchange limitation (6kW) is lower enough, aggregator cannot keep the same scheduling for SHA and ESS, otherwise a deviation from power limitation will happen at PCC (between 3:00 and 5:00 am). In this case, aggregator should manage for overload shifting from 3:00-5:00 am to another times of the day, either through negotiation with the apartments to shift their SHAs and incentivise them with monetary profit, or by re-scheduling the ESS charging/discharging. In the first solution scenario, shifting the SHAs consumption from the lowest price time period (3:00-5:00 am) to the other low price period (16:00-21:00), will cause a small increase in electricity bill. This is because of the small difference between the electricity price in these two period. In the second solution scenario, if the ESS-charging happens to shift from 3:00-5:00 am time duration, it will not be able to discharge in 5:00-7:00 am, and will cause a big effect on profit making. Thats because of the difference between the electricity price in these two period, which is noticeable. Thus, the first solution scenario for this coupled case is more money affordable, and parts (f) and (g) of the figure show the proper scheduling of the SHAs and ESS. This scenario causes no violation from the power limitation (see parts (h)). Therefore, it is necessary for aggregator to apply an optimal operation strategy, through coordinating with apartments, to schedule the SHAs and the ESS, and deal with the coupling cases. In addition, by applying a centralized approach, the calculation time would not be reasonable when the number of apartments increases. Therefore, essence of having a distributed scheduling approach is obvious.

Preliminary results: In order to evaluate the proposed distributed framework, we present preliminary results obtained by applying the Algorithm 1 to a microgrid system comprising 4 active apartments with 3 smart appliances each: a dishwasher, a washing machine and a dryer. We consider a piecewise constant electricity tariff signal extracted from Nordpool website. We include in the numerical evaluation the hourly energy use due to sources of electricity consumption other than household appliances.

The shared ESS has the following technical features:

- Storage capacity: 20000Wh
- Maximum power exchange: 8000W
- Maximum Depth Of Discharge(DOD): 30%
- Stored energy degradation (α): negligible
- Charging and discharging efficiency: 90%
- Maximum charging and discharging cycles: 5 (per day).

We then apply both Problem (16) and Algorithm 1 to the system under consideration and compute the corresponding schedules of the appliances, the shared ESS and the interaction with the grid. Figure 3 depicts the comparison between the solution computed by solving the centralized problem and the solutions obtained by the proposed distributed algorithm at iteration 1 and 6, which is the last iteration in this particular case study. We can notice that, as Algorithm 1 (the distributed approach) is iterated, the solutions get closer to the optimal one (the solution resulting from solving the centralized problem). The total electricity cost of the optimal solution is 1.200 USD while the electricity cost resulting from the final iteration of Algorithm 1 is 1.215 USD, hence only 1.3% higher. On the other hand, the computational time of the centralized problem (16) was 745 sec while the proposed distributed algorithm computes the solution in 7.29 sec, hence the computational time has decreased by two orders of magnitude. The MILP problems were solved using CPLEX with the YALMIP MATLAB interface [18].

Further studies will focus on conducting extensive simulations and investigate the potential improvements in the algorithm performance brought by the redistribution strategy described in Section III.

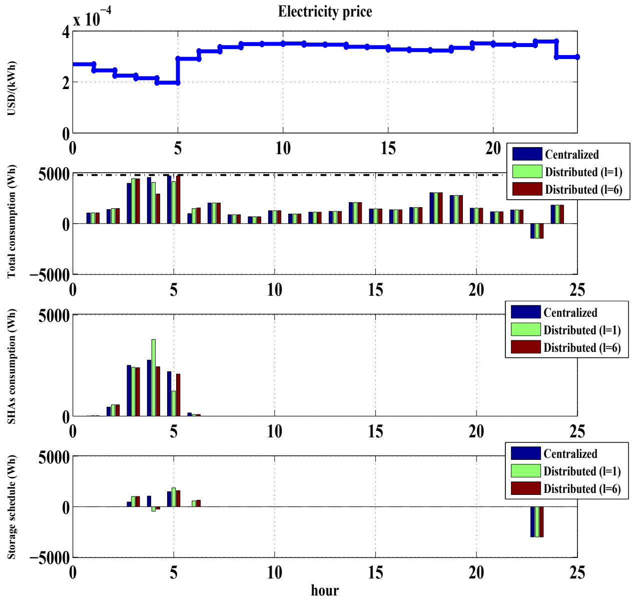

Fig. 3. Comparison between the optimal solution of the centralized problem (16) and the solution computed by Algorithm 1 at iterations 1 and 6 (the last iteration).

V. Conclusion and Future Studies

Over the last decade, storage devices have become one of the important components in smart grid for peak demand shaving, voltage imbalances mitigation, and consumers′ electricity bill reduction. Due to the high cost of ESSs, it could be convenient to consumers to deploy and share them in a cooperative manner. In this work we propose an iterative distributed approach to solve the problem of coordinating a set of smart appliances located in a network of apartments sharing an ESS such that each household can profit from the use of the ESS while technical and operational constraints, as well as user preferences, are satisfied. The problem of coordinating the shared resources among the consumers is complicated by a fairness requirement, i.e., storage will equally benefit consumers according to their flexible loads. The novel distributed scheduling algorithm proposed in this paper has the following properties *i)* provides a feasible solution to the centralized scheduling problem; *ii)* allocates fairly ESS-related profits among the users; *iii)* requires limited messages to be exchanged between each consumer and the aggregator, and no message passing among the consumers, to keep consumers' privacy, and (iii) is suitable for online optimization-based control scheme, such as MPC. Numerical results show that the computed solution is close to the optimal one computed by a centralized problem. Although home appliances and EESs are considered in this work, we point out that the proposed framework can be also extended to scenarios considering different uncertainty sources, different storage technologies and generic programmable electrical loads, as well as different optimization criteria.

As a future study, uncertainties on such a huge load shifting by using automation system in a large number of apartments (which causes the real-time tariff to vary from the day-ahead one) will be taken into account.

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