Robust Scheduling of Smart Appliances in Active Apartments With User Behavior Uncertainty

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***Abstract*—In this paper, we propose a robust approach for scheduling of smart appliances and electrical energy storages (EESs) in active apartments with the aim of reducing both the electricity bill and the CO_2 emissions. The proposed robust formulation takes the user behavior uncertainty into account so that the optimal appliances schedule is less sensitive to unpredictable changes in user preferences. The user behavior uncertainty is modeled as uncertainty in the cost function coefficients. In order to reduce the level of conservativeness of the robust solution, we introduce a parameter allowing to achieve a trade-off between the price of robustness and the protection against uncertainty. Mathematically, the robust scheduling problem is posed as a multi-objective Mixed Integer Linear Programming (MILP), which is solved by using standard algorithms. The numerical results show effectiveness of the proposed approach to increase both the electricity bill and CO_2 emissions savings, in the presence of user behavior uncertainties. Mathematical insights into the robust formulation are illustrated and the sensitivity of the optimum cost in the presence of uncertainties is investigated. Although home appliances and EESs are considered in this work, we point out that the proposed scheduling framework is generally applicable to many use cases, e.g., charging and discharging of electrical vehicles in an effective way. In addition, it is applicable to various scenarios considering different uncertainty sources, different storage technologies and generic programmable electrical loads, as well as different optimization criteria.**

***Note to Practitioners*—This paper has been motivated by the problem of reducing electricity bill and CO_2 emissions related to the energy consumption in active apartments equipped with automation system, smart appliances, and EESs such as batteries. By considering users in the center of automation processes, it is fundamental for industrial practitioners to take uncertainties related to the user behavior into account. This means that users might decide to run appliances earlier or later than the optimal starting times computed by the automation system, based on given user preferences. Thus, neglecting the uncertainties related to the user behavior can considerably reduce the effectiveness of the optimal solution and lead to an actual expensive or not environment friendly schedule. To address this issue, we propose a robust optimization approach to the scheduling problem. The robust problem is posed as a MILP and solved by CPLEX (using the YALMIP MATLAB interface), which is a commercial implementation of a branch-and-bound algorithm. Having real energy consumption data from active apartments in the Stockholm Royal Seaport project, a numerical study is conducted in this paper. Simulation evaluations show that the proposed robust approach can improve both the CO_2 emissions and electricity bill savings to a large extent.**

***Index Terms*—Demand response, mixed-integer linear programming, multi-objective robust optimization, robust scheduling of smart appliances, user behavior uncertainty.**

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NOMENCLATURE

In the following, superscript k means time slot k.

Continuous decision variables:

p_{ij}^k Energy of phase j of appliance i.

p_G^k Exchanged power with the grid.

b_s^k Level of energy stored in the EES.

b_c^k Power exchange for charging the EES.

b_d^k Power exchange for discharging the EES.

Binary decision variables:

x_{ij}^k Indicates whether an energy phase is being processed or not.

s_{ij}^k Indicates whether an energy phase is already finished.

x_c^k Indicates whether the EES is charging.

x_d^k Indicates whether the EES is discharging.

c_t^k Indicates the transition time slots to start charging.

d_t^k Indicates the transition time slots to start discharging.

User input parameters:

TP_i^k Preferable finishing operation time of appliance i.

M Maximum deviation from optimal starting time.

λ Weighting parameter in the cost function.

Known (or computed based on user input) parameters:

N Number of appliances.

m Number of time slots in one day.

Δt Each time slot duration.

E^k Normalized electricity bill.

C^k Normalized CO_2 foot-print.

N_c Maximum charging and discharging cycles during one day.

C_λ^k Weighted sum of electricity tariff and CO_2 footprint.

$\tilde{C}_\lambda^k$ Uncertain weighted sum of electricity tariff and CO_2 footprint.

ϵ^k Uncertainty level.

Γ Protection (robustness) level.

I. Introduction

A. Motivation

RESIDENTIAL areas are responsible for nearly 40% of the energy consumption and CO_2 emission in developed countries. These areas are known to have significant potential for energy and cost savings, as well as load shifting (loads are classified as controllable (interruptible and non-interruptible) and uncontrollable), compared to industry and transportation [1]. Therefore, automation systems can be used to assist residents to take advantage of these potentials [2], [3]. Demand response (DR) has received increased attention in recent years since it can efficiently support load balancing and economical/environmental cost reduction [4], [5]. DR is commonly defined as changes in electricity use by consumers in response to changes in the electricity price over time [4], and help power markets set efficient energy prices, mitigate market power, improve economic efficiency, and increase safety [6]. Several studies have investigated the potential changes in residential electricity use under time-varying price rates by rescheduling smart (possible to control remotely) appliances [7]–[9]. The electricity use can also be sensitive to dynamic CO_2 intensity that is included in the demand response [10]. Thus, several works have focused on CO_2 emission factors and its potential impacts on the changes in household load profile, e.g., see [11], and proposed load management strategies accounting for both price and CO_2 information (e.g., see [3], [12]–[14] and the Stockholm Royal Seaport project[1]). To achieve energy and cost savings and have effective DR policies, home appliances are required to be smart and have the ability of being switched on or off remotely and in response to price and CO_2 signals. Furthermore, electrical energy storage (EES) technologies [15] can be integrated with DR policies to store energy and release it when it is more convenient [3], [16], [17]. Thus, in this work, we consider active apartments (which are apartments where effective DR policies are enabled through the integration of smart appliances, scheduling algorithms, energy management systems, and information exchange over wireless communication technologies), to be equipped with also EES. We then take advantage of the modeling capabilities and the computational advances of Mixed Integer Linear Programming (MILP) algorithms for stating a scheduling problem for smart home appliances and EESs that aims at achieving a trade off between electricity costs and CO_2 emission savings.

[1][Online]. Available: http://www.stockholmroyalseaport.com/en/

However, effective DR policies for automated active apartments equipped with smart appliances and EESs have to face several sources of uncertainty.

B. Literature Review

Uncertainties in DR strategies may cause considerable deterioration in the expected outcomes (electricity bill and CO_2 emission savings), and should be taken into account. Some of these uncertainties are related to the DR signals, in which electricity tariff and CO_2 footprint are subject to real-time amendment or forecasting errors. Coping with these uncertainties has been investigated in previous studies [6], [18]–[20]. Other sources of uncertainty handled in the literature are related to the user behavior. In [21], optimization of energy demand in residential areas is carried out for different occupant behavior scenarios (i.e., occupancy patterns or operation schemes). The authors propose a mechanisms that will provide building designers with solutions that are robust against these unknown occupant behaviors. An everyday energy-related behavior in 57 Swedish homes has been studied in [22]. The majority of residents in that work showed different energy-related behaviors, which indicates that strategies to influence the behaviors are required.

One should notice that, by considering the multi-user behavior and applying multi-strategies to influence the behavior, it is still unrealistic to assume perfect knowledge of users' energy need. An optimization-based real-time residential load management algorithm has been proposed in [23], which takes into account uncertainties related to the power consumption and starting time of uncontrollable loads, in order to minimize the energy payment for each user. In addition, the authors of [16] propose an energy efficient scheduling algorithm taking into account the uncertainty in appliances energy consumption. The novelty in that work is the introduced energy consumption adaptation variable, which is used to model the stochastic energy consumption patterns for various household appliances.

Based on our knowledge, in the literature only uncertainties related to the predicted starting time of uncontrollable appliances, or uncertainties related to the amount of consumed energy by each appliance have been considered. It is important to also take into account the customer behavior uncertainty in scheduling of controllable appliances, which is our focus.

C. Statement of Contribution

The minimum value of the cost function achieved by solving the scheduling problem might increase too much if the users decide to run appliances earlier or later than the optimal starting times computed by the automation system.

The main contribution of this paper is to cope with the uncertainty in the customer behavior for scheduling non-interruptible controllable appliances. It is known that the uncertainty can be handled by stochastic programming and robust approach, and stochastic programming generally requires higher computational burden (e.g., see numerical evaluations in [6], which indicates that the scenario based stochastic approach introduces higher computational burden than the robust approach). However a robust approach is more computationally appealing, it can lead to a conservative and potentially more expensive scheduling of appliances. This shortcoming can be effectively handled by the approach described in [24] and [25], which is adopted in this study. In this paper, uncertainty in the customer behavior is considered as a disturbance on the control signal (starting time of appliances), and we map this disturbance to the uncertainty in the cost function coefficients. After this mapping, the robust optimization approach described in [24] and [25] is applied, which considers a tunable degree of robustness, and it can decrease the price of robustness [26].

Another contribution here is the numerical study based on real energy consumption data from active apartments. We investigate the impact of DR policies on electricity price and CO_2 savings in the presence of user behavior uncertainties. Simulation results show that the proposed robust scheduling algorithm increases the electricity bill saving and CO_2 saving compared to the non-robust one, when variability in the tariff (uncertainty level) is high.

We point out that the proposed scheduling framework is applicable to scenarios with various uncertainty sources, storage technologies, generic programmable electrical loads, as well as different optimization criteria. The remainder of this paper is organized as follows.

D. Outline

Section II presents the problem formulation and describes how the proposed scheduling framework can be applied to relevant practical use cases. To cope with the uncertainty in the customer behavior in scheduling of controllable appliances, Section III proposes a robust scheduling algorithm for smart appliances and EESs. Section IV presents numerical results and discusses performance of the proposed robust approach in terms of sensitivity analysis, electricity price, and CO_2 emission savings. Finally, Section V provides conclusions and suggestions for future studies.

II. Problem Formulation and Modeling

The aim of scheduling smart appliances in active apartments is to reduce the electricity bill and CO_2 emission. As it was mentioned in [3] and [14], there exists a tradeoff between electricity costs and CO_2 emission in certain countries including Sweden. During the daytime, Sweden utilizes its relatively clean energy sources such as hydro power plants and nuclear power plants, while during nighttime it imports relatively inexpensive but CO_2 intense energy from Denmark, Germany and Poland whose primary energy source is combustive fuel power plants [14]. Thus, to minimize electricity bills and CO_2 emission at the same time, different methods have been proposed for optimal scheduling of appliances to deal with this possible conflict and tradeoff. Weighted sum and ε-constrained approaches are two of these methods that have mostly been used in the literature [3], [13], [14], [27]. Thus, the problem is a multi-objective minimization of electricity bill and CO_2 emission, and there exist constraints on the user preferences and operation process of appliances. This means that we are concerned with the scheduling of a number of home appliances in a certain period of time, in which the user can specify precedence relations between certain appliances (user preferences). In this scheduling framework, the operation process of an appliance is divided into a set of subtasks (energy phases) of the appliance operation (i.e. movement, pre-heating, and heating, for washing machines [13]) and it is considered that, once an energy phase starts, it must continue until it is finished. In addition, there exists a flexible delay between energy phases of each appliance that is modeled by a minimum and maximum delay time (constraints on the operation process of appliances). In this paper, we formulate the smart home appliances scheduling problem with EES in a mixed-integer linear programming (MILP) framework, as was discussed in [3], which considers the minimum electricity cost and CO_2 emission, and satisfies technical operation constraints of smart appliances and EES, and consumer preferences. To extend the formulation proposed in [3], uncertainties related to the customary running of appliances have been considered here, and a robust MILP framework is proposed. Electricity tariff and CO_2 footprint signals are assumed to be piecewise constant, and the MILP scheduling problem is solved using CPLEX (using the YALMIP MATLAB interface [28]), which is a commercial implementation of a branch-and-bound algorithm.

A. Optimal Scheduling of Appliances and EESs

In the mathematical formulation for scheduling of smart appliances in [3], the appliances execution period is discretized into m uniform time slots Δt (e.g. $\Delta t = 10$ minutes per slot). The number of appliances considered for scheduling is denoted by N, and n_i for $i = 1, 2, \ldots, N$, denotes the number of uninterruptible energy phases for each appliance. The energy assigned to energy phase j of appliance i during the whole period of time slot k is denoted by p_{ij}^k. In addition, auxiliary binary decision variables (x_{ij}^k) are required to indicate whether a particular energy phase is being processed or not. Moreover, two other sets of binary decision variables are needed to model the decision problem. One is denoted as s_{ij}^k, with a value of one indicating that, in appliance i, energy phase j is already finished by time slot k. The other set is denoted as t_{ij}^k. These decision variables are used to indicate whether, at time slot k, appliance i is making a transition between running phase $j-1$ to j. In that paper, to minimize the electricity bill and CO_2 emission, a multi-objective optimization problem is proposed subject to the following constraints.

The constraint that is enforced to make sure that the energy phases fulfill their energy requirement is as

$$\sum_{k=1}^{m} p_{ij}^k = ER_{ij}, \quad \forall i, j \tag{1}$$

where ER_{ij} is the energy requirements for energy phase j in appliance i. To determine that the lower and upper power limitation being assignment to the phase are satisfied, during time slot k, the constraint

$$\underline{p}_{ij}^{k} x_{ij}^{k} \leq p_{ij}^{k} \leq \overline{p}_{ij}^{k} x_{ij}^{k} \quad \forall i, j, k \tag{2}$$

is enforced, and the $\underline{p}_{ij}^{k}$ and $\overline{p}_{ij}^{k}$ are the lower and upper limits. Also, the power safety constraint can be imposed as

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}^{k} \leq \overline{P}^{k} \quad \forall k. \tag{3}$$

$\overline{P}^{k}$ is the upper limit of the total energy assigned at time slot k. The limits on energy phases process time are imposed as

$$\underline{T}_{ij} \leq \sum_{k=1}^{m} x_{ij}^{k} \leq \overline{T}_{ij} \quad \forall i, j, \tag{4}$$

where the $\underline{T}_{ij}$ and $\overline{T}_{ij}$ are the lower and upper limits of the number of time slots for energy phase j in appliance i to be processed. To satisfy the sequential processing of the energy phases of an appliance and also sequential operation between appliances, the following constraints are imposed respectively

$$\begin{aligned} x_{ij}^{k} &\leq s_{i(j-1)}^{k} && \forall i, k, \ \forall j = 2, \ldots, n_i \\ x_{ij}^{k} &\leq s_{\tilde{i} n_{\tilde{i}}}^{k} && \forall k \end{aligned} \tag{5}$$

where $\tilde{i}$ is the index of the appliance which must be finished before the appliance with i index can start running. To make sure that the energy phases are uninterruptible the following constraint is imposed:

$$\begin{aligned} x_{ij}^{k} &\leq 1 - s_{ij}^{k} && \forall i, j, k \\ x_{ij}^{k-1} - x_{ij}^{k} &\leq s_{ij}^{k} && \forall i, j \ \forall k = 2, \ldots, m \\ s_{ij}^{k-1} &\leq s_{ij}^{k} && \forall i, j \ \forall k = 2, \ldots, m. \end{aligned} \tag{6}$$

To increase the benefits from DR signals, delays between energy phases are considered to be flexible in the smart appliances. This gives the smart appliances the capability of flexible electricity consumption to help the consumers to reduce electricity bill and CO_2 emission. To count the number of time slots spent between the energy phases in an appliance and impose lower and upper limits (which are technical specifications of each appliance and are provided by companies) on these numbers, the constraints

$$t_{ij}^{k} = s_{i(j-1)}^{k} - \left(x_{ij}^{k} + s_{ij}^{k} \right) \ \forall i, j, \ \forall k = 2, \ldots, n_i \tag{7}$$

$$\underline{D}_{ij} \leq \sum_{k=1}^{m} t_{ij}^{k} \leq \overline{D}_{ij} \ \forall i, \ \forall j = 2, \ldots, n_i \tag{8}$$

are considered, where $\underline{D}_{ij}$ and $\overline{D}_{ij}$ are between-phase delay lower and upper bounds, respectively. Finally, to meet the household preferences and finishing a particular appliance within a specified time interval, the constraint

$$x_{ij}^{k} \leq TP_i^{k} \quad \forall i, j, k \tag{9}$$

is enforced, and TP_i^{k} is the time preference interval.

To include an EES in this framework, the following set of constraints is defined in [3].

The level of energy stored in the EES at time slot k, should always satisfy the lower ($\underline{b}_s$) and upper ($\overline{b}_s$) limitations

$$\underline{b}_s \leq b_s^{k} \leq \overline{b}_s, \quad \forall k, \tag{10}$$

where b_s^{k} is the state of charge (SOC) of the EES in time slot k. Moreover, to meet the lower and upper limitations on power exchanged with the EES when it is charging or discharging during time slot k, the two constraints

$$0 \leq b_c^{k} \leq \overline{b}_c^{k} x_c^{k}, \quad 0 \leq b_d^{k} \leq \overline{b}_d^{k} x_d^{k} \quad \forall k \tag{11}$$

are enforced, in which the auxiliary binary decision variables x_c^{k} and x_d^{k} indicate whether the EES is charging or discharging in time slot k, respectively. The power exchanged with the EES during time slot k is denoted by b_c^{k} (or b_d^{k}) when the EES is charging (or discharging). In addition, the constraint

$$x_c^{k} + x_d^{k} \leq 1 \quad \forall k \tag{12}$$

should be satisfied to make sure that the EES is not charging and discharging at the same time slot. To take the state of health of EESs into account, the total number of charging and discharging cycles during a day should be limited to a determined number N_c, and the constraints

$$\begin{aligned} x_c^{k} - x_c^{k-1} &\leq c_t^{k} \quad \forall k = 2, \ldots, m \\ x_d^{k} - x_d^{k-1} &\leq d_t^{k} \quad \forall k = 2, \ldots, m \\ \sum_{k=1}^{m} c_t^{k} + d_t^{k} &\leq N_c \end{aligned} \tag{13}$$

should be satisfied, where the binary decision variables c_t^{k} and d_t^{k} determine the transition time slots to start charging and discharging, respectively. The dynamic system constraint

$$b_s^{k} = \alpha b_s^{k-1} + \eta_c b_c^{k-1} - \eta_d b_d^{k-1} \quad \forall k = 2, \ldots, m \tag{14}$$

describes the evolution of energy stored in the EES, in which the α is a constant stored energy degradation in each sampling interval, and η_c and η_d are efficiencies accounting for the losses during charging and discharging. To satisfy the power balance in the system, the constraint

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}^{k} + b_c^{k} - b_d^{k} = p_G^{k} \quad \forall k \tag{15}$$

is enforced, where the exchanged power with the grid is denoted by p_G^{k}, and it should satisfy lower and upper limitations

$$\underline{p}_G^{k} \leq p_G^{k} \leq \overline{p}_G^{k} \quad \forall k \tag{16}$$

where the lower limit is negative to allow energy selling to the grid. Finally, it is reasonable to assume that the initial and the final energy levels (b_s^{0} and b_s^{T} respectively) in the EES are the same, since the final energy level is also the initial condition for the next day scheduling. Hence, the following equality constraint on the initial and final SOC is enforced

$$b_s^{0} = b_s^{T}. \tag{17}$$

Moreover, the initial level should be sufficiently high to allow a flexible use of the EES: in this study, we assume $b_s^0 = \underline{b}_s + ((\overline{b}_s - \underline{b}_s)/2)$. One can also consider b_s^0 as a variable, corresponding to the measured energy level of the EES at the beginning of the day. Now the proposed multi-objective optimization problem of jointly scheduling smart appliances and EES could be written as

$$
\begin{aligned}
\underset{\substack{p,x,s,t,b_s,\\ b_c,b_d,x_c,x_d,\\ c_t,d_t,p_G}}{\text{minimize}} \quad & \sum_{k=1}^{m} C_\lambda^k \, p_G^k \\
\text{subject to} \quad & \text{constraints } (1),(2),(4)-(9),(10)-(17) \\
& \lambda \in [0,1], \\
& p_{ij}^k \in \mathbb{R}, \quad \forall i,j,k \\
& x_{ij}^k, s_{ij}^k \in 0,1, \quad \forall i,j,k \\
& t_{ij}^k \in 0,1, \quad \forall i,k, \quad \forall j = 2,\ldots,n_i \\
& b_s^k, b_c^k, b_d^k, p_G^k \in \mathbb{R}, \quad \forall k \\
& x_c^k, x_d^k, c_t^k, d_t^k \in 0,1, \quad \forall k
\end{aligned}
\tag{18}
$$

which is called nominal problem (NOM) in this paper. In the objective function, the weighted sum of electricity tariff and CO_2 footprint $((1-\lambda)E^k + \lambda C^k)$ is denoted by C_λ^k, and p_G^k is the total energy exchanged by the grid at time slot k. Note that the cost function is parameterized by the weighting parameter $\lambda \in [0,1]$ (that would be chosen by end-users), in which $\lambda = 0$ implies end-users only care about the electricity bill, while for $\lambda = 1$ they only take CO_2 emission into account. Thus, by changing the parameter λ from 0 to 1, and solving the minimization problem in (18), the convex hull for the Pareto curve [29] of our multi-objective minimization problem would be generated. The following normalizations are applied in (18) to yield

$$
E^k = \frac{e^k}{\max(e^1, e^2, \ldots, e^m)} \quad C^k = \frac{c^k}{\max(c^1, c^2, \ldots, c^m)} \tag{19}
$$

where e^k and c^k denote the electricity bill and CO_2 foot-print for time slot k respectively and based on given 24-h ahead tariff curves (which are piecewise constant).

B. Use Cases

Here, we describe how the proposed scheduling framework can be applied to relevant practical use cases and capture relevant real-world scenarios.

1) Power Consumption of Active Apartments: One application area of the proposed scheduling framework is active apartments. To show the effectiveness of the proposed approach (to increase electricity bill and CO_2 emission savings), we consider the active apartments in the residential area in the Stockholm Royal Seaport (SRS) project in this paper. Since some real data of electricity consumption from SRS project are available, a numerical study based on that is done in this paper (see Section IV). Utility companies and the automated active apartments are the players in this study. In each of those apartments, effective DR policies (which are defined by the utilities) are enabled through the integration of necessary components including smart appliances, ESS, scheduling algorithm and energy management system. In addition to the DR signals (which are sent by utilities), information exchange over wireless communication technologies among the components is part of flow in this study. Precisely, the proposed scheduling algorithm take as inputs the day-ahead electricity tariff and CO_2 footprint [which are released by utility companies and are normalized in (19)], and also household time preferences (to be applied in (9)). Then to minimize the electricity bill and CO_2 emission of electricity consumption of an active apartment, an operation scheduling for the smart appliances and EES should be generated by the algorithm to minimize the cost function in (18). This operation scheduling should meet the energy constraints of smart appliances [the constraints from (1)–(3)], the time constraints of smart appliances [the constraints from (4)–(9)], and the operational constraints of the EES [the constraints from (10)–(17)].

2) Power Consumption of Electric Vehicles (EVs): Another application area of our mathematical framework is transportation, which is the other major contributor to energy use. Transportation increases green house gas in the atmosphere and is one of the largest fossil fuel users in the world [30]. Thus, EVs have the potential of reducing fuel consumption and CO_2 emission, and optimal scheduling for charging and discharging the batteries in the EVs is a key to integrate large numbers of them in the smart grid. By optimal scheduling, EVs could function as distributed generation and energy storage, supply loads, and smooth the unpredictable renewable generation (e.g. wind and solar energy). The same formulation discussed in this section is applicable for the EVs, which can be considered as chargeable batteries. The formulation discussed in this section can be applicable for EVs with a slight modification considering that an EV may drive during some periods in a day. Thus, to integrate EVs in the automation systems, time preferences for EV batteries are to be modeled in the problem formulation [similar to the time preferences introduced for smart appliances in (9)]. Hence, the following constraints have to be added to the constraint defined from (10) –(14), the constraints

$$
x_c^k \leq T_{Ec}^k, \quad x_d^k \leq T_{Ed}^k \quad \forall k \tag{20}
$$

where T_{Ec}^k and T_{Ed}^k characterize time preference intervals for charging and discharging of EV battery respectively. By considering users in the center of automation process for energy consumption of active apartments and EVs, it is important for industrial practitioners to take uncertainties related to the user behavior into account (see Section III).

3) Carbon Pricing: In addition to the application areas of our framework discussed above, another relevant application area concerns environmental-related taxes and carbon pricing. The global increase in emissions raises the need of designing an effective set of environmental-related taxes that effectively reduce the global energy-related CO_2 emissions, which should be based on the carbon content of fossil fuels that are purchased and consumed. In this context, carbon pricing is a central issue. Current prices put on carbon by means of taxes or emissions trading systems in developed countries, including Sweden, are generally much lower than those needed to limit the global average temperature increase. Governments should therefore take measures to reduce the entire carbon footprint rather than their territorial emissions; namely, an optimal policy for global pollutants like CO_2 must consider the implications of international trade [31], [32]. Further, since households have generally a relevant impact on the carbon footprint, changing household consumption patterns is central to achieving sustainable development and incentivize substantial behavioral adjustments to be successful in the climate change challenge. Consider Sweden, for instance. In Sweden, the majority of the impact on the carbon footprint is caused through households (76%) [33]. In the current institutional Swedish setting, a low carbon lifestyle is not sufficiently rewarded. Besides, the biggest portion, 43%, of the total costs for electricity currently paid by the Swedish consumer are environmental taxes (i.e., an energy tax and a quota obligation assigned to the electricity end-users for renewable electricity) and a value added tax; however, these environmental taxes for households account for other external effects than CO_2 emissions, such as noise, congestion and road wear from traffic [34].

In this scenario, our framework can provide useful insights into behavioral adjustments for households and a more effective carbon pricing, which accounts for the global damage of emissions. In the following, we show how the objective function of the problem NOM can be modified to include carbon pricing. Consider, for instance, that the carbon price is set as e_{CO_2} Euro per kg of CO_2 emitted; hence, the emission cost per kWh of exchanged power at each time slot k is $e_c^k = e_{\mathrm{CO}_2} e^k$ Euro. The objective function of (18) can be slightly modified to include the environmental taxes as $((1-\lambda)E_c^k + \lambda C^k)p_G^k$, where E_c^k and C^k are obtained by normalizing respectively e_c^k and c^k with respect to the total electricity price per kWh for the consumers at time slot k. By doing so, lambda can be interpreted as the percentage of the total cost associated to the carbon content of the electricity consumed; thus, our framework can give indications of how to incentivize a desired user behavior.

III. Robust Optimal Scheduling of Smart Appliances and EESs

In the optimization problem (NOM) discussed in the previous section, there exist several sources of uncertainty. For example, electricity tariff and CO_2 footprint are subject to real-time amendment or forecasting errors, and it is unrealistic to assume perfect knowledge of users' energy need. Thus, the minimum value of the cost function achieved by solving NOM problem, might increase too much according if the user behavior deviates from the forecasted one.

In this section we apply some robust optimization techniques to the scheduling problem of smart appliances and EESs in order to produce robust solutions which are in a sense immune against bounded uncertainty.

In the recent works that focus on reducing the monetary expense of customers, CO_2 emission, and peak-to-average ratio of the system based on scheduling of appliances, the uncertainties related to customary energy consumption are not considered, which is however quite important. That means, by running the optimization scheduling algorithm, the automation system will achieve optimal points of running appliances, but there exists uncertainty in the user behavior, and they might run the appliances earlier or later. Thus, in the MILP problem that we are faced with, the uncertainty is on the decision variables, while in the literature the coefficients of the inequality constraints are assumed to have uncertainty [24]–[26]. The idea here is to map the uncertainty on the decision variable to an equivalent uncertainty in the weighted sum tariff, which is illustrated in Fig. 1. As should be clear from this figure, deviating from starting times x_1 and x_2 by at most M time slots $(M\Delta t)$, turn into a variability of the tariff by at most Δy_1 and Δy_2, respectively. The parameter M can be defined based on the empirical model of the users, by having the historical data related to the uncertainties in their behaviors. This means deviating from the optimal start time of appliances, would affect the cost function, and could be equivalently considered as variability of the tariff curve that depends on the behavior of the curve in the neighborhood of starting time. Thus, the variability in the tariff curve is considered as the uncertainty in C_λ^k, and similar to what is done in [25] the NOM problem in (18) can be expressed in a generalized way as follows:

$$\begin{aligned} \underset{\substack{p,x,s,t,b_s,\\ b_c,b_d,x_c,x_d,\\ c_t,d_t,p_G}}{\text{minimize}} \quad & \sum_{k=1}^{m}\left(\widetilde{C}_\lambda^k \sum_{i=1}^{N}\sum_{j=1}^{n_i} p_{ij}^k + C_\lambda^k\left(b_c^k - b_d^k\right)\right) \\ \text{subject to} \quad & \text{the constraints in (18)} \end{aligned} \tag{21}$$

in which the uncertain data range in the interval

$$\left|\widetilde{C}_\lambda^k - C_\lambda^k\right| \le \epsilon^k \left|C_\lambda^k\right| \quad \forall k \tag{22}$$

where the parameter $\epsilon^k \ge 0$ is an uncertainty level at time k. To apply the following robust method, ϵ^k is needed to be defined properly. As was mentioned, deviating from the optimal start time of appliances, would affect the cost function in accordance to the behavior of the tariff curve in the neighborhood of starting time. Thus, in this paper, ϵ^k for time slot k, is given as

$$\epsilon^k = \frac{\max\left(C_\lambda^k, \sum\limits_{i=k-M}^{k+M} \frac{C_\lambda^i}{2M+1}\right) - C_\lambda^k}{\max\left(C_\lambda^k, \sum\limits_{i=k-M}^{k+M} \frac{C_\lambda^i}{2M+1}\right)} \quad \forall k. \tag{23}$$

which is a function of the tariff curve within an interval of $\pm M\Delta t = \pm 120$ min in the neighborhood of time slot k. Fig. 2 shows that the more variation we have in the tariff curve in the neighborhood of a time slot, the larger the ϵ^k we have for that time slot. The uncertain parameter $\widetilde{C}_\lambda^k$ in (22) appears as linear coefficient in the above inequality constraint and the robust optimization technique which is described in the following can be applied for that. Here, it is assumed that the scheduling of the EES is done by the automation system in the active apartments and only scheduling of the smart appliances is faced with uncertainty. The reason is that, the automation system recommends users when to run the appliances, but they can choose to ignore the recommendation and run the appliances earlier or later, but the EES would be scheduled by the automation system. One should notice that, when the exchanged power limitation in (16) is low, then customer scheduling will also affect the battery scheduling, as they are dependent regarding the (15). This is not the case for Sweden, and, as the upper limitation in (16) is sufficiently high, the EES

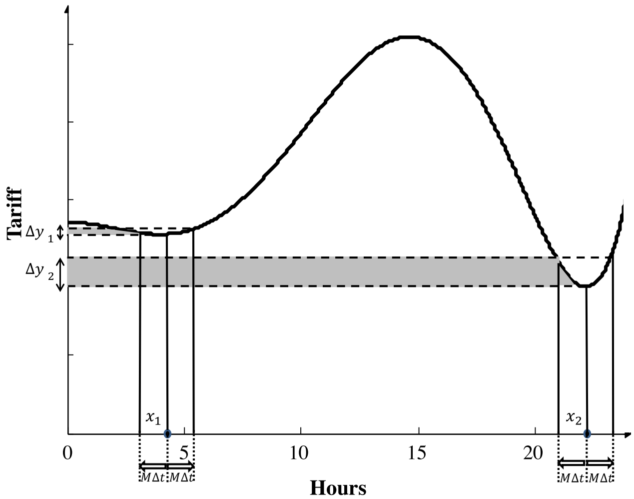

Fig. 1. Mapping the user behavior uncertainty to tariff uncertainty.

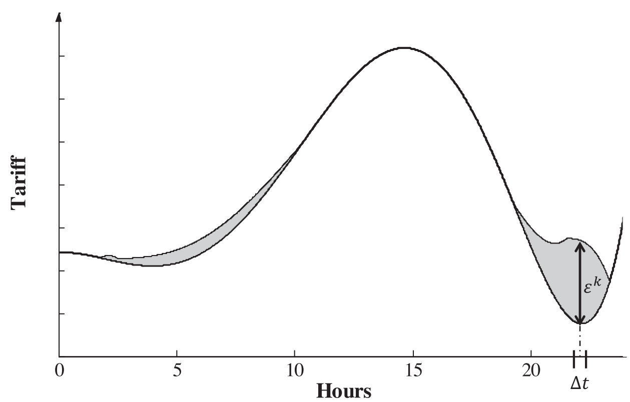

Fig. 2. ϵ^k behavior for different time slot.

will charge when the weighted sum tariff is low and discharge when it is high and is independent of appliances scheduling.

As is proved in [25], by using the worst-case values of the uncertain parameter $\widetilde{C}_\lambda^k$, the problem in (21) can be written equivalently as

$$\begin{aligned} \underset{\substack{p,x,s,t,b_s,\\ b_c,b_d,x_c,x_d,\\ c_t,d_t,p_G}}{\text{minimize}} \quad & \sum_{k=1}^{m} C_\lambda^k\, p_G^k + \sum_{k=1}^{m} \epsilon^k\, C_\lambda^k \left(\sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}^k \right) \\ \text{subject to} \quad & \text{the constraints in (18)} \end{aligned} \tag{24}$$

which is called robust optimization problem (ROB) in this paper. A concern with this approach is that it might be too conservative, i.e. it produces solutions whose objective function value is much worse than the nominal one. Effectively, when considering uncertain parameters we provide a robust solution that is feasible in all scenarios that uncertain parameters variations could define, which are all the possible tariff curves in Fig. 2; however, this comes at the cost of a degradation of the objective value, which could be excessive as some of the uncertain scenarios rarely occur. This increase in cost over the nominal solution is the so called *price of robustness* [26]. In order to prevent too conservative solutions, we follow the approach proposed in [6], [24], [35]. We then formulate an optimization problem where the degree of uncertainty can be regulated by a parameter denoted by Γ. The aim of the proposed approach is to compute schedules that are insensitive against the variation of at most Γ time slots. By varying Γ, the level of conservatism of the solution, and then the increase in cost, can be controlled. The authors in [24] and [26] prove that, even when more than Γ elements vary, the robust solution will be feasible with highprobability.

Problem ROB is then modified such that the weighted sum tariff can be uncertain in at most Γ time slots as follows:

$$\begin{aligned} \underset{\substack{p,x,s,t,b_s,\\ b_c,b_d,x_c,x_d,\\ c_t,d_t,p_G,q,z}}{\text{minimize}} \quad & \sum_{k=1}^{m} C_\lambda^k\, p_G^k + z\,\Gamma + \sum_{k=1}^{m} q^k \\ \text{subject to} \quad & \text{the constraints in (18)} \\ & \epsilon^k\, C_\lambda^k \left(\sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}^k \right) \le z + q^k \quad \forall k \\ & 0 \le q^k \quad \forall k \\ & 0 \le z \end{aligned} \tag{25}$$

which is called flexible robust problem $(\mathrm{ROB} - \Gamma)$. The parameter Γ is also defined as *protection level* of the schedule cost against uncertainty in the user behavior. This parameter can be defined based on the empirical model of the users, by having the historical data related to the uncertainties in their behaviors. Problem $\mathrm{ROB} - \Gamma$ stays a MILP and can be still solved by commercial solvers such as CPLEX.

Notice that, in the $\mathrm{ROB} - \Gamma$ problem, by having $\Gamma = 0$ the problem turns to the NOM problem and represents the most optimistic case, and the influence of user behavior uncertainty on the cost variations is completely ignored. On the other hand, by having $\Gamma = m$, user behavior uncertainty at all time slots will be considered for possible cost variations, which is the most conservative case ([24] and [26]) and the problem is equivalent to the ROB problem.

We remark that the proposed framework can be generally applied to other scenarios where different sources of uncertainty and different optimization criteria must be considered. For instance, ϵ^k can represent the variation from the day-ahead price at time slot k in the real-time energy market, while different optimization criteria can account for the user comfort or the demand peak reduction.

A. Mathematical Insights Into the Protection Level Γ

Here we will provide some insights into the robust formulation in (25). In particular, we aim at understanding the effect on the robust schedule of increasing or decreasing the protection level Γ defined in the previous section.

1) Model of Cost Uncertainty: As described in [24], we assume that each entry $\widetilde{C}_\lambda^k$, $k = 1, \ldots, m$ takes values in $[C_\lambda^k, C_\lambda^k + d^k]$, where $d^k = \varepsilon^k C_\lambda^k$ represents the variation from the nominal cost coefficient, C_λ^k. We allow the possibility to have $d^k = 0$, since ε^k can be zero for some $k = 1, \ldots, m$. We remind that, in this study, cost variations model the uncertainty in the user behavior.

As in [24], the parameter Γ previously introduced controls the protection level for the objective function against cost variations.

Let $K = \{k|\varepsilon_k > 0\}$; Γ is assumed to be integer and takes values in $[0, |K|]$, where 0 indicates the nominal solution and $|K|$ the most conservative solution. Generally, Γ represents a tradeoff between the level of conservativeness and the cost of the robust solution: the higher is Γ, the less sensitive is the solution to cost variations at the cost of a higher nominal cost.

In the next section, we will investigate more into detail the robust counterpart of problem (24) in order to understand the effect on the robust schedule of increasing or decreasing the protection level Γ.

In the following, vectors are denoted by bold letters.

2) Interpreting the Robust Counterpart of the Scheduling Problem: In the RO methodology, the best solution which is feasible for any realization of the data uncertainty in the given set is computed through the solution of the robust counterpart optimization problem. In our study, the robust counterpart of the scheduling problem (18) can be written as

$$\begin{aligned} \underset{\substack{p,x,s,t,b_s,\\ b_c,b_d,x_c,x_d,\\ c_t,d_t,p_G}}{\text{minimize}} \quad & \sum_{k=1}^{m} C_\lambda^k\, p_G^k + \beta(\boldsymbol{p}, \Gamma) \\ & \text{subject to the constraints in (18),} \end{aligned} \tag{26}$$

where $\beta(\boldsymbol{p}, \Gamma)$ is the protection function of the objective, $\rho^k := \sum_{i=1}^{N}\sum_{j=1}^{n_i} p_{ij}^k$ and $\boldsymbol{p} := [\rho^1, \ldots, \rho^m]'$. To solve the robust counterpart optimization problem, we will show how to convert the objective function of the problem (26) to a linear one by following the approach in [24] and resorting to the duality.

The protection function equals the objective function of the following linear optimization problem:

$$\begin{aligned} \beta(\boldsymbol{p}, \Gamma) = \underset{\boldsymbol{z}^0}{\text{maximize}} \quad & \sum_{k\in K} d^k |\rho^k| z^{0k} \\ \text{subject to} \quad & \sum_{k\in K} z^{0k} \le \Gamma \\ & 0 \le z^{0k} \le 1 \quad \forall k \in K. \end{aligned} \tag{27}$$

Notice that ρ^k is nonnegative in our study, hence $|\rho^k| = \rho^k$; in the following we will drop the absolute value.

Subsequently, we consider the dual problem of (27), which is then the primal.

We recall that in a dual problem a variable is introduced for each constraint in the primal so that the number of variables in the dual is equal to the number of constraints in the primal. Then the variable z is associated to the first constraint of (27), which involve the protection level Γ as right-hand side, and a variables q^k is associated to each constraint defining the upper bound on z^{0k}. Notice that the dual variables z and q^k are the ones introduced in problem $\mathrm{ROB} - \Gamma$.

Consider then the dual of the problem (27) as

$$\begin{aligned} \underset{z,\boldsymbol{q}}{\text{minimize}} \quad & \sum_{k\in K} q^k + \Gamma z \\ \text{subject to} \quad & z + q^k \ge d^k \rho^k \\ & 0 \le z,\ 0 \le q^k\ \forall k \in K. \end{aligned} \tag{28}$$

Substituting into problem (26), we obtain that problem (26) is equivalent to problem (25). We refer the reader to [24] for further details.

We now aim at gaining some insights into the optimal value of the protection function and how increasing the protection level Γ affects the robust solution.

We start with some definitions and assumptions.

Given a vector $\boldsymbol{p}_*$, let $\boldsymbol{z}_*^0$ be the optimal primal solution and $(z_*, \boldsymbol{q}_*)$ the optimal dual solution for problems (27) and (28) respectively (under non-degeneracy, the primal and dual optimal solutions are unique. In case of multiple optima, an unique optimal point can be selected by the help of appropriate tie-break rules, e.g., the lexicographic order [36]).

Without loss of generality, we assume that the indices are ordered in such that $d^1\rho_*^1 \ge d^2\rho_*^2 \ge \ldots \ge d^{|K|}\rho_*^{|K|}$. Further, assume that there are $n \ge 0$ time slots corresponding to the same value of the cost variation due to the uncertainty level, which we denote by $\bar{d}\bar{\rho}$.

Define the following sets of indices for time steps $k \in K$:

$$\begin{aligned} \mathrm{I}_i &:= \{i, i+1, \ldots, i+n\} \\ \mathrm{I}_1 &:= \{1, 2, \ldots, i-1\} \\ \mathrm{I}_{|K|} &:= \{i+n+1, \ldots, |K|\}, \end{aligned}$$

with $|\mathrm{I}_1| \le \Gamma$. Notice that the set I_1 contains the time steps k with the highest values of $d^k\rho_*^k$, while the set I_i is the set of time steps with the same value of the cost variation, defined above as $\bar{d}\bar{\rho}$.

We will now compute the optimal values of the primal variables in $\boldsymbol{z}_*^0$ and the dual variables in $(z_*, \boldsymbol{q}_*)$.

Notice that the dual variable z measures how the primal objective function will change if the Γ increases. If increasing Γ the value of the objective function changes, the corresponding dual, z, is positive. On the other hand, if when the primal problem is solved, the constraint with Γ is not active, this means that increasing Γ is not going to improve the objective function; hence $z = 0$. If $z > 0$, increasing Γ would be beneficial; this means that the corresponding constraint should be active at optimality. This relationship between dual variables and constraints in the primal must satisfy the complementary conditions, which mathematically state what has been explained above. At optimality, the complementary conditions must hold $\forall k \in K$.

1) $z_*^{0k}\,(z_* + q_*^k - d^k\rho_*^k) = 0$.
2) $z_*\,(\Gamma - \sum_{k\in K} z_*^{0k}) = 0$.
3) $q_*^k\,(1 - z_*^{0k}) = 0$.

Consider the case when $n > 1$. From the complementary conditions, we can derive the optimal solution of the primal and dual problems (27) and (28) as

$$\begin{aligned} &q_*^k \ge 0,\ z_* = d^k\rho_*^k - q_*^k && \forall k \in \mathrm{I}_1 \\ &q_*^k = 0,\ z_* = \bar{d}\bar{\rho} && \forall k \in \mathrm{I}_i \\ &q_*^k = 0,\ z_* \ge d^k\rho_*^k && \forall k \in \mathrm{I}_{|K|} \\ &z_*^{0k} = 1 && \forall k \in \mathrm{I}_1 \\ &z_*^{0k} \in (0,1) && \forall k \in \mathrm{I}_i \\ &z_*^{0k} = 0 && \forall k \in \mathrm{I}_{|K|}. \end{aligned}$$

Notice that, if $n = |\mathrm{I}_i| > 0$, there will be multiple optimal solutions, since any combination such that $\sum_{k\in \mathrm{I}_i} z^{0k} = \Gamma - |\mathrm{I}_1|$ is an optimal solution of problem (26), corresponding to the same value of the objective function.

The optimal values of z_* and q_*^k are then

$$\begin{aligned} z_* &= \bar{d}\bar{p} \\ q_*^k &= d^k \rho_*^k - z_* \quad \forall k \in I_1. \end{aligned}$$

If $n \leq 1$, at optimality there are clearly not time steps such that $z_*^{0k} \in (0,1)$. This means that we need only two sets of indices: *i*) $\mathrm{I}_1 := \{1, 2, \ldots, \Gamma\}$, containing Γ time steps k with the highest values of $d^k \rho_*^k$; *ii*) $\mathrm{I}_{|K|} := \{\Gamma+1, \ldots, |K|\}$. In this case, $z_* = \max_{k \in \mathrm{I}_{|K|}} d^k \rho_*^k$.

Summarizing, at optimality, given the optimal appliances power assignment $\boldsymbol{p}_*$, the optimal value of the protection function in (24) is

$$\beta(\boldsymbol{p}_*, \Gamma) = (\Gamma - |\mathrm{I}_1|)\, z_* + \sum_{k \in \mathrm{I}_1} d^k \rho_*^k \tag{29}$$

where $z_* = \max_{k \in \mathrm{I}_{|K|} \cup \mathrm{I}_i} d^k \rho_*^k$ and $|\mathrm{I}_1|$ is the cardinality of set I_1.

From the discussion above and, in particular, from (29), we can draw some conclusions about the effect of changing the protection level Γ on the robust solution, which are given here.

- As Γ grows, z_* decreases and $\sum_{k \in \mathrm{I}_1} q_*^k$ increases, since the number of time steps with $q_*^k > 0$, i.e., $|\mathrm{I}_1|$, become larger. This means that the robust optimal solution is affected more and more by the uncertain cost profile $\widetilde{C}_\lambda^k$ and less by the nominal cost profile C_λ^k, $\forall k \in K$. Then, the power assignment is generally shifted from time steps k with the lowest values of C_λ^k to time steps with lower values of $\widetilde{C}_\lambda^k$, mainly where variations are small or zero, despite the nominal tariff C_λ^k is higher.
- When the protection level Γ is small, the solution is less robust against cost variations. This entails that the optimal value of z is strictly positive and it can be used to assign power to time steps with low nominal tariffs and still high values of cost variations. When Γ increases, the optimal solution of problem (28) is required to be less and less sensitive to cost variations: hence, a larger number of q_*^k are to be strictly positive and a larger amount of power is assigned to time steps with small or zero variations. It can be interesting to notice that, in cases when $\widetilde{C}_\lambda^k$ and C_λ^k have similar profiles $\forall k \in K$, the nominal and the robust schedules get closer as Γ grows. In this cases, having a high protection level does not bring any benefit.
- If Γ is larger than the number of time steps k when $d^k \rho_*^k > 0$, $z_* = 0$ and the set I_1 collects all of the time steps k such that $d^k \rho_*^k > 0$. In this case, the robust solution of problem (25) does not depend on Γ and stays constant as Γ grows.
- For a certain value of Γ, the constraint with Γ in (27) is not active and then $z = 0$. This occurs when a protection is required for a number of time slots larger than the number of time slots when it is convenient to have a positive cost variation, which entails that it is convenient to buy or sell energy from/to the grid despite a positive uncertainty level. Since the constraints on the overall energy requirements, the power limits and the process times do not depend on Γ and stay the same both in the nominal and in the robust formulations, the number of time slots with a positive cost variation associated to the optimal nominal power schedule can provide a rough estimation of this value of Γ. Increasing Γ further will not change the solution of the robust optimization problem ROB $-\ \Gamma$.

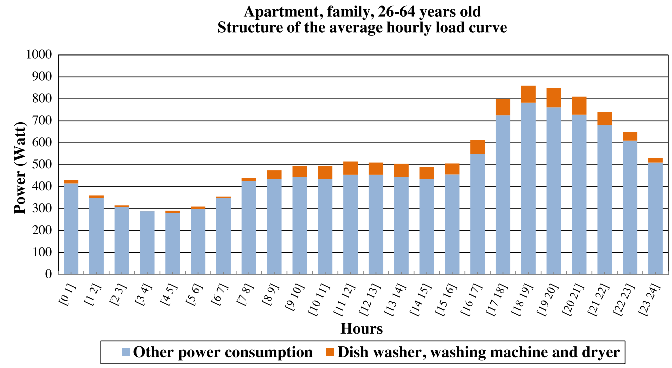

Fig. 3. Estimated average hourly power consumption of two active apartments for appliances versus other consumptions (for March 2013–January 2014).

IV. Simulation Results

Here, impact of the automation system on the electricity bill and CO_2 savings, under user behavior uncertainty, is investigated through simulations. To do this investigation, it is necessary to have the information related to hourly energy consumption in apartments without automation system, and the portion of household appliances in this energy consumption.

Within the SRS project, which is a new and environmentally sustainable city district being built in Stockholm, actual hourly energy consumption of two active apartments is available from March 2013 to January 2014. These data have been kindly provided by Fortum Corporation. As mentioned in [3], to determine the hourly power consumption of household appliances versus other consumptions, a comparison with previous works is done [37].

Based on this comparison, estimated average hourly power consumption of appliances versus other consumptions for the two active apartments is shown in Fig. 3. Two points are important to be noted here: one of them is that the loads in this figure have not been optimally scheduled and is related to the consumption before using automation system. The other one is the total amount of energy being used in one day in the active apartments, which is approximately 9.9 kWh on average, and in comparison with the apartments that were studied in [37] (consuming 12.6 kWh on average) has decreased more than 20%. This reduction in power consumption is reasonable based on the modern home appliances that are used in the active apartments.

Based on the energy consumption of those two active apartments and previous works [3], [37], a numerical study of 10000 active apartments (which is close to the number of active homes being built in the SRS project) for the evaluation of DR programs and prediction of bill and CO_2 savings of households under user behavior uncertainty is considered in this paper.

In this numerical study, effectiveness of the proposed ROB $-\ \Gamma$ approach is shown in the simulation results. User time preferences are uncertain, which means for each appliance the starting time, and consequently end time, are supposed to vary within an interval of $M\Delta t = \pm 120$ min from their nominal value. Then we apply both the nominal and the RO approaches and we compare the computed schedules in terms of costs and sensitivity to variations of the time preferences from their nominal values. The nominal schedule is computed by solving the problem $\mathrm{ROB}-\Gamma$ with $\Gamma=0$. The robust scheduling problem is solved with different choices of the parameter Γ in order to find the best schedule accounting for an uncertain user behavior with a reasonably small increase in cost compared to the nominal schedule. All of the problems in this paper are solved by CPLEX.

In the simulations, the technical specifications of the smart appliances (e.g., dishwasher, washing machine, and dryer) have been extracted from [13]. Moreover, as it was mentioned in [3], hourly price tariffs for June 2013 are downloaded from Nordpool website.[2] In addition, the SVK website[3] provides us with electricity generation by fuel type data, electricity import, and electricity export for 2013. Hourly CO_2 foot print curves can be computed based on these data [10]. In addition, by investigating the available batteries for the houses equipped with solar panels,[4] the following specifications (the same as [3]) is applied for battery consideration in this work.

- Storage capacity: 1700 Wh.
- Maximum power exchange: 1000 W.
- Maximum Depth Of Discharge (DOD): 30%.
- Stored energy degradation (α): negligible.
- Charging and discharging efficiency: 90%.
- Maximum charging and discharging cycles: Five (per day).

To generate scenarios for simulating user behavior uncertainties in the proposed robust approach, a sampling method can be used, in which the starting time of the first energy phase of each appliance (t_{i1}^k) is considered as a variable or input, that is allowed to vary within an interval of $t_{i1}^* \pm M\Delta t$. Here, the t_{i1}^* is the optimum starting time for the first energy phase of ith appliance scheduled by $\mathrm{ROB}-\Gamma$ approach. The most common sampling method is indisputably the pure Monte Carlo, mainly because of its simplicity [38]. However, as the number of samples are limited because of the computational time, this method is known to have poor space filling properties, and leaving large unsampled regions. In this paper, the Latin hypercube sampling (LHS) method [39] which is an extension of stratified sampling is utilized to generate the scenarios. The LHS method ensures that each of the input variables has all of its range represented, and partition it into the equally probable intervals. In this method, a LHS of size Y (number of partitions for each input, which is the number of time slots within the interval of $t_{i1}^* \pm M\Delta t$, that is equal to $2M+1$) with W number of inputs (each input is the start time of one of the appliances in this work, and by having three appliances, the number of inputs is three here), is obtained from a random selection of Y values (one per stratum) for each input. Thus we achieve W Y-tuples that form the W columns of the $Y\times W$ matrix of scenarios generated by LHS, that means the ith row of this matrix contains one of the partition for each input variable and will correspond to the ith scenario [40].

[2][Online]. Available: http://www.nordpoolspot.com/

[3][Online]. Available: http://www.svk.se/

[4][Online]. Available: http://www.voltaicsystems.com/blog/all/

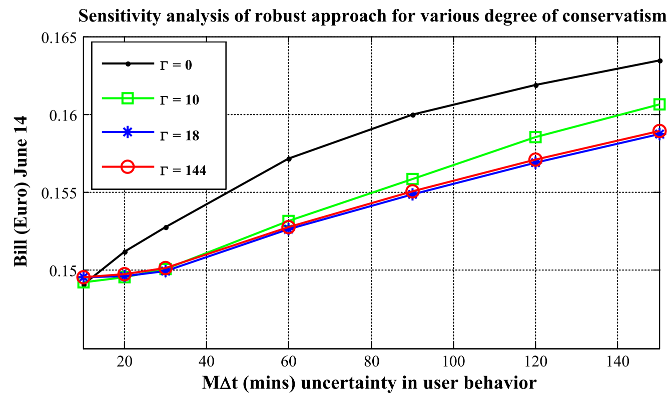

Fig. 4. Sensitivity analysis of $\mathrm{ROB}-\Gamma$ approach for various degree of conservatism (Γ). By having $\Gamma=0$, the problem turns to the NOM problem and represents the most optimistic case in which user behavior uncertainty is completely ignored (high sensitivity to uncertainty), and $\Gamma=144$ is related to the most conservative approach and the problem is equivalent to the ROB problem.

A. Sensitivity Analysis of Robust Approach

Here, the impact of the robust approach on the electricity bill and CO_2 emission savings, in the presence of user behavior uncertainty, is investigated. By applying the $\mathrm{ROB}-\Gamma$ approach, the sensitivity of the electricity bill with respect to the uncertainty $(M\Delta t)$ and degree of robustness (Γ), for June 14, 2013, in Sweden is depicted in Fig. 4 in which $\Delta t =$ 10 min. The figure shows that the robust schedules outperform the nominal schedule, i.e. the one corresponding to $\Gamma=0$ in terms of costs in presence of user uncertainty. In particular, the best uncertain cost could be achieved when $\Gamma = 18$ and having a value of $\Gamma > 18$ does not bring any benefit in terms of costs; this is because the variable z is zero when $\Gamma = 18$, which implies that increasing the protection level does not change the solution.

For the sensitivity analysis, different values of M have been discussed here, while in the future by having historical data related to the user behavior and the uncertainties, it would be possible to determine the related M for each user and subsequently defining ϵ^k more precisely.

Day-ahead and uncertain tariff, and also scheduling of appliances by applying $\mathrm{ROB}-\Gamma$ approach with $\Gamma=0$ and $\Gamma=18$, for $M=12$, are depicted in Fig. 5. As it is shown in this figure, the scheduled dryer has been shifted from the evening to the morning to avoid the possible occurrence (in the presence of user behavior uncertainty) with the high price of electricity between 22:00 and 23:00. The number of binary variables, continuous variables and constraints (which are the most important indicators in MILP problem), in the $\mathrm{ROB}-\Gamma$ $(\Gamma=18)$ problem, are 6624, 3027, and 22461, respectively. It takes 1.57 s to solve this problem by CPLEX in MATLAB R2014b.

B. Impact of DR Signals on the Electricity Bill and CO_2 Saving in the Robust Approach

As was discussed in previous sections, DR signals provide costumers an opportunity to save electricity bill and CO_2 emission by shifting consumption and using batteries. To illustrate potential future benefits of automation systems in active houses, 10 000 apartments are considered as a numerical study in [3].

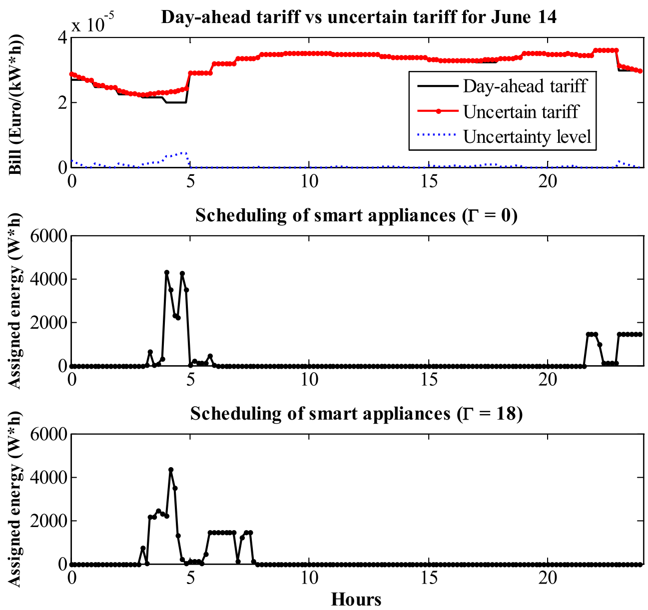

Fig. 5. Scheduling of appliances by applying $\mathrm{ROB} - \Gamma$ approach with $\Gamma = 0$ (turns to NOM problem, and user behavior uncertainty is completely ignored) and $\Gamma = 18$ (in which dryer has been shifted from the evening to the morning to avoid the possible occurrence with the high price of electricity between 22:00 and 23:00, in the presence of user behavior uncertainty).

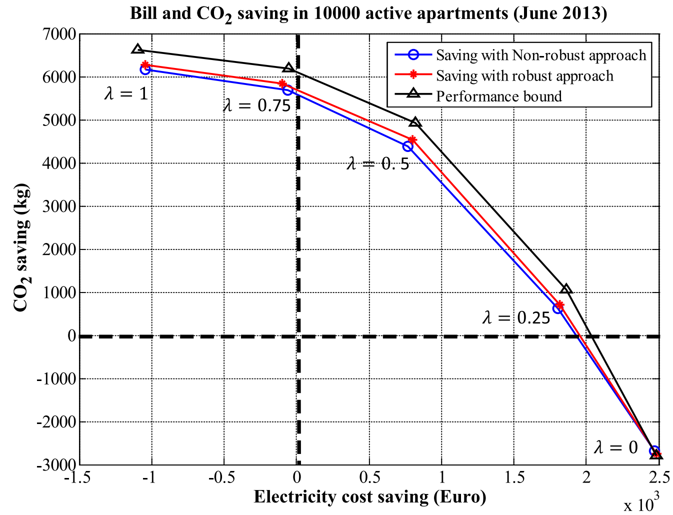

Fig. 6. Impact of robust approach on bill and CO_2 savings in 10 000 active apartments with user behavior uncertainty (convex hull for the Pareto curve ([29]) of the multi-objective minimization problem).

For that numerical study, three different scenarios including reference apartments (without automation system), test apartment (equipped with automation system), and test apartment with battery (equipped with automation system and battery) have been taken into account and compared with each other. Throughout that comparison, average hourly power consumption data from the mentioned two real active apartments without automation system (Fig. 3) is used and is considered as the average hourly power consumption of the reference apartment. For each scenario in [3], the number and types of the smart appliances that are running in one day in those 10 000 apartments, is calculated from the technical specification, average hourly power consumption data from the two real active apartments in the day, and considering the fact that 4% of energy consumption is devoted to the washing machine and dryer and 4% for the dishwasher.

In [3], average hourly power consumption curves related to smart appliances and battery, and the total bill and CO_2 savings in these 10000 apartments for the test apartment and test apartment with battery, is shown for June 2013. In that work, it has been shown that, caring only about the electricity price by consumers, causes CO_2 emission to be increased. So, considering the CO_2 intensity signal can help avoid carbon emissions increases, and it should also be taken into account. In addition, simulation results in that study show optimized use of the battery can further increase daily cost saving and CO_2 emission reduction.

In this paper, by considering the both CO_2 intensity and electricity tariff signals, the effect of robust scheduling on bill and CO_2 emission savings in the automated apartment equipped with batteries is investigated. Taking into account the impact of user time preferences on the load shift, the scheduling of appliances have been computed for time preferences between 08:00 and 24:00 hours. Note that this time interval has been chosen based on Fig. 3, which shows that families in active apartments are more interested to run their household appliances within this period. In Fig. 6, impact of $\mathrm{ROB} - \Gamma$ approach on bill and CO_2 savings for 10 000 apartments (with user behavior uncertainty consideration) in June 2013 is investigated. In this figure, savings with the nonrobust approach (there exists uncertainty, but the NOM approach has been applied), with robust approach (there exists uncertainty, but $\mathrm{ROB} - \Gamma$ approach has been applied), and performance bound (there is no uncertainty on user behavior and is the case that discussed in [3], that we can have maximum saving) are compared for different attitude ($\lambda = 0 \ldots 1$) of users toward electricity cost and CO_2 savings. Despite of having relatively small variability in the electricity tariff and CO_2 foot-print signals (low uncertainty level) in June 2013, the simulation results show, that the proposed robust scheduling algorithm increases CO_2 emissions and the electricity bill savings (in the presence of user behavior uncertainty) in comparison with nonrobust approach.

As it is mentioned previously, and is also shown in this figure, there exists a tradeoff between electricity costs and CO_2 emission in certain countries including Sweden. This means, the more caring about the electricity price, the more CO_2 emission is produced. Thus, the automation system will provide the users with the cost of electricity and CO_2 emission for different choice of λ (e.g., for $\lambda = 0, 0.25, 0.5, 0.75, 1$), and they can decide which one is of their interest. For example, by choosing λ in the middle range (e.g., for $\lambda = 0.25, 0.5$ in Fig. 6) for the case under study, users will have both bill and CO_2 emission savings. By choosing $\lambda \geq 0.75$ (or $\lambda \leq 0.25$), despite the CO_2 (or bill) saving, electricity cost (or CO_2 emission) increases.

Simulations are all done on a 64-bit Windows system with an Intel Core i7-3770, 3.40 GHz and 16.0 GB of RAM, in MATLAB R2014b. Simulation results show that computational time difference for solving the NOM problem and $\mathrm{ROB} - \Gamma$ problem (for the same number of appliances with the same characteristics, and same user input) is negligible. In [3], computation time for solving the NOM problem, for different scenarios (with increasing number of appliances, in which each appliance has five energy phases) is investigated. In that work, a comparison with related papers that apply DP and MCA for scheduling of smart appliances has been studied.

V. Conclusion and Future Studies

This paper proposes a new robust approach for scheduling smart home appliances and batteries. The novel robust optimal scheduling formulation is posed as a multi-objective MILP, which not only aims to decrease the CO_2 emissions and the electricity bill, but also takes the uncertain behavior of users into account. A numerical study for 10 000 active apartments in the Royal Seaport project was investigated for assessing the impacts of DR signals on load shifting and bill and CO_2 saving factors. This assessment is based on different attitudes of users toward the environmentally and economic benefits (different λ) and considering customary scheduling of smart appliances. That means, by running the scheduling algorithm, the automation system will achieve optimal times for running appliances, but there exists uncertainty on the user behavior, and they might run the appliances earlier or later. Since these uncertainties may cause considerable distortion to the optimal solution, it is important to take them into account. Simulation results show, despite of having relatively smooth curve for electricity tariff and CO_2 footprint signals (low uncertainty level) in most of the days (in June 2013), the proposed robust scheduling algorithm increases CO_2 emissions and the electricity bill savings (in the presence user behavior uncertainty). Thus, having more volatility in these signals will increase uncertainty and make this method more important. It has also been shown that, the more consumers care about the electricity price, the more CO_2 emission is produced (for λ close to one). An optimized use of the battery can further increase daily cost saving and CO_2 emission reduction. As a future study, the other uncertainties such a huge load shifting by using automation system in a large number of apartments (which causes the real-time tariff to vary more from the day-ahead one) could be taken into account, and interpreted as the level of uncertainties (ϵ^k). In addition, as it was discussed, only 8% of power consumption of the active apartments is devoted to the smart appliances, and almost half of it is related to the lightning, heating and cold appliances. Thus, by taking these consumptions into account in automated systems, the bill and CO_2 savings could be significantly increased.

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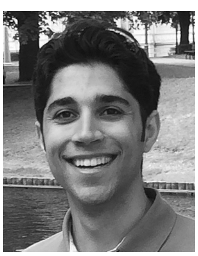

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