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Event-triggered Control for Multi-Agent Systems

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Abstract— Event-driven strategies for multi-agent systems are motivated by the future use of embedded microprocessors with limited resources that will gather information and actuate the individual agent controller updates. The control actuation updates considered in this paper are event-driven, depending on the ratio of a certain measurement error with respect to the norm of a function of the state, and are applied to a first order agreement problem. A centralized formulation of the problem is considered first and then the results are extended to the decentralized counterpart, in which agents require knowledge only of the states of their neighbors for the controller implementation.

I. INTRODUCTION

Decentralized control of large scale multi-agent systems is currently facilitated by recent technological advances on computing and communication resources. Several results concerning multi-agent cooperative control have appeared in recent literature involving agreement or consensus algorithms [17], [11],[20], formation control [5], [4], [7], [2] and distributed estimation [18],[23].

One of the most important aspects in the implementation of distributed algorithms is the communication and controller actuation schemes. A probable future design may equip each agent with a small embedded micro-processor, who will be responsible for collecting information from neighboring nodes and actuating the controller updates according to some ruling. The scheduling of the actuation updates can be done in a time-driven or an event-driven fashion. The first case involves the traditional approach of sampling at pre-specified time instances, usually separated by a specific period. In real applications, the embedded processors are resource-limited, and thus an event-triggered approach seems more favorable. In addition, a proper design can also preserve desired properties of the ideal continuous state-feedback system, such as stability and convergence. A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [3]. Stochastic event-driven strategies have appeared in [19],[12]. In this paper, we use the deterministic event-triggered strategy introduced by P. Tabuada in [24]. Similar results on deterministic event-triggered feedback control have appeared in [26],[25],[10],[16],[1].

In [24], the control actuation is triggered whenever a certain error becomes large enough with respect to the norm of the state. It is assumed that the nominal system is Input-to-State stable [21],[22] with respect to measurement errors. Motivated by this, in [6] we provided event-triggered control strategies for a class of cooperative control algorithms, namely those that can be reduced to a first order agreement problem [17], which has been proven to be ISS [14]. In [6], knowledge of the initial average of the states was required by the agents in order to implement the control strategy. The motivation of the current paper is to relax this assumption. In particular, no knowledge of the initial average is required. We consider both the cases of centralized and decentralized event-triggered multi-agent control. In the first case, we show that there exists a strictly positive lower bound on the time between two consecutive actuation updates. In the decentralized case, each agent is equipped with its own embedded microprocessor that can gather only neighboring information. We show that continuous evolution is enforced at each time instant for at least one agent and also provide a minimum lower bound for it; thus ensuring that the overall switched system does not reach an undesired accumulation point, i.e., it does not exhibit *Zeno* behavior [13]. The results are depicted through simulated examples.

The remainder of the paper is organized as follows: Section II presents some necessary background and discusses the problem treated in the paper. The centralized case is discussed in Section III while Section IV presents the decentralized counterpart. Some examples are given in Section V while Section VI includes a summary of the results of this paper and indicates further research directions.

II. BACKGROUND AND PROBLEM STATEMENT

In this section we first review some related results on algebraic graph theory [9] that are used in the paper and proceed to describe the problem in hand.

A. Algebraic Graph Theory

For an undirected graph G with N vertices the *adjacency matrix* $A = A(G) = (a_{ij})$ is the $N \times N$ matrix given by $a_{ij} = 1$, if $(i,j) \in E$, where E is the set of edges, and $a_{ij} = 0$, otherwise. If there is an edge $(i,j) \in E$, then i,j are called *adjacent*. A *path* of length r from a vertex i to a vertex j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. For $i = j$, this path is called a *cycle*. If there is a path between any two vertices of the graph G, then G is called *connected*. A connected graph is called a *tree* if it contains no cycles. The *degree* d_i of vertex i is defined as the number of its

neighboring vertices, i.e. $d_i = \{\#j : (i,j) \in E\}$. Let Δ be the $n \times n$ diagonal matrix of d_i's. Then Δ is called the *degree matrix* of G. The (combinatorial) *Laplacian* of G is the symmetric positive semidefinite matrix $L = \Delta - A$. For a connected graph, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is the vector of ones, $\mathbf{1}$. We denote by $0 = \lambda_1(G) \leq \lambda_2(G) \leq \ldots \leq \lambda_N(G)$ the eigenvalues of L. If G is connected, then $\lambda_2(G) > 0$.

B. System Model

The system considered consists of N agents, with $x_i \in \mathbb{R}$ denoting the state of agent i. Note that the results of the paper are extendable to arbitrary dimensions. We assume that agents' motion obeys a single integrator model:

$$\dot{x}_i = u_i, i \in \mathcal{N} = \{1, \ldots, N\} \tag{1}$$

where u_i denotes the control input for each agent.

Each agent is assigned a subset $N_i \subset \{1, \ldots, N\}$ of the rest of the team, called agent i's *communication set*, that includes the agents with which it can communicate. The undirected *communication graph* $G = \{V, E\}$ of the multi-agent team consists of a set of vertices $V = \{1, ..., N\}$ indexed by the team members, and a set of edges, $E = \{(i,j) \in V \times V | i \in N_j\}$ containing pairs of vertices that correspond to communicating agents.

C. Problem Statement

The agreement control laws in [8], [17] were given by

$$u_i = -\sum_{j \in N_i} (x_i - x_j) \tag{2}$$

and the closed-loop equations of the nominal system (without quantization) were $\dot{x}_i = -\sum_{j \in N_i} (x_i - x_j,)$, $i \in \{1, \ldots, N\}$, so that $\dot{x} = -Lx$, where $x = [x_1, \ldots, x_N]^T$ is the stack vector of agents' states and L is the Laplacian matrix of the communication graph. For a connected graph, all agents' states converge to a common point, called the "agreement point", which coincides with the average $\frac{1}{N}\sum_i x_i(0)$ of the initial states.

Note that the model (1),(2) has been shown to capture the behavior of other multi-agent control problems apart from the agreement problem. For example, it was shown in [7] that a class of formation control problems can be reduced to a first order agreement one with an appropriate transformation.

In this paper, we redefine the above control formulation to take into account event-triggered strategies. Consider the system (1). Both centralized and decentralized event-triggered cooperative control are treated. The control formulation and problem statement for each case are described in the sequel.

1) Centralized Event-triggered Cooperative Control: For each $i \in \mathcal{N}$, and $t \geq 0$, introduce a (state) measurement error $e_i(t)$. Denote the stack vector $e(t) = [e_1(t), \ldots, e_N(t)]^T$. The discrete time instants where the events are triggered are defined when a condition $f(e(t), x(t)) = 0$ holds. The sequence of event-triggered executions is denoted by: $t_0, t_1, \ldots$. As noted above, each t_i is defined by

$f(e(t_i), x(t_i)) = 0$, for $i = 0, 1, \ldots$. To the sequence of events $t_0, t_1, \ldots$ corresponds a sequence of control updates

$$u(t_0), u(t_1), \ldots$$

Between control updates the value of the input u is held constant and equal to the last control update, i.e.,:

$$u(t) = u(t_i), \forall t \in [t_i, t_{i+1}) \tag{3}$$

and thus the control law is *piecewise constant* between the event times $t_0, t_1, \ldots$.

The centralized cooperative control problem treated in this paper can be stated as follows: "derive control laws of the form (3) and event times $t_0, t_1, \ldots$ that drive system (1) to an agreement point."

2) Decentralized Event-triggered Cooperative Control: In the decentralized case, there is a separate sequence of events $t_0^k, t_1^k, \ldots$ defined for each agent k according to $f_k(e_k(t_i^k), \sum_{j \in N_k}(x_i(t_i^k) - x_j(t_i^k))) = 0$, for $k \in \mathcal{N}$ and $i = 0, 1, \ldots$. Hence a separate condition encoded by the function $f_k(e_k(t_i^k), \sum_{j \in N_k}(x_i(t_i^k) - x_j(t_i^k)))$ triggers the events for agent $k \in \mathcal{N}$. The update condition is distributed in the sense that each agent requires knowledge of its own measurement error and the relative states of its neighboring agents in order to verify this condition.

The decentralized control law for k is updated both at its own event times $t_0^k, t_1^k, \ldots$, as well as at the last event times of its neighbors $t_0^j, t_1^j, \ldots, j \in N_k$. Thus it is of the form

$$u_k(t) = u_k(t_i^k, \bigcup_{j \in N_k} t_{i'(t)}^j), \tag{4}$$

where $i'(t) \triangleq \arg \min_{l \in \mathbb{N}: t \geq t_l^j} \{t - t_l^j\}$.

The decentralized cooperative control problem can be stated as follows: "derive control laws of the form (4), and event times $t_0^k, t_1^k, \ldots$, for each agent $k \in \mathcal{N}$ that drive system (1) to an agreement point."

III. CENTRALIZED APPROACH

Consider the event-triggered multi-agent control problem described previously. We assume that the control law can be actuated only at discrete instances of time instead of being a continuous feedback. These instances are triggered when the measurement error of the state variable reaches a certain threshold. In the case treated in this section, the centralized event-triggered control scheme is considered. The decentralized case is treated in the next section.

Following the notation given in the previous section, the state measurement error is defined by

$$e(t) = x(t_i) - x(t), i = 0, 1, \ldots \tag{5}$$

for $t \in [t_i, t_{i+1})$. The choice of t_i encoded by the function f will be given in the sequel. The proposed control law in the centralized case has the form (3) and is defined as the event-triggered analog of the ideal control law (2):

$$u(t) = -Lx(t_i), t \in [t_i, t_{i+1}) \tag{6}$$

The closed loop system is then given by

$$\dot{x}(t) = -Lx(t_i) = -L(x(t) + e(t)) \tag{7}$$

Denote by $\bar{x}(t) = \frac{1}{N}\sum_i x_i(t)$ the average of the agents' states. Using the fact that the graph is undirected, the time derivative of $\bar{x}(t)$ is then given by

$$\dot{\bar{x}} = \frac{1}{N}\sum_i \dot{x}_i = -\frac{1}{N}\sum_i \sum_{j \in N_i}(x_i(t) - x_j(t))$$
$$-\frac{1}{N}\sum_i \sum_{j \in N_i}(e_i(t) - e_j(t)) = 0$$

so that $\bar{x}(t) = \bar{x}(0) = \frac{1}{N}\sum_i x_i(0) \equiv \bar{x}$, i.e., the initial average remains constant.

A candidate ISS Lyapunov function [22] for the closed-loop system 7 is:

$$V = \frac{1}{2}x^T L x$$

We have

$$\dot{V} = x^T L \dot{x} = -x^T LL(x + e) = -\|Lx\|^2 - x^T LLe$$

so that

$$\dot{V} \leq -\|Lx\|^2 + \|Lx\|\|L\|\|e\|$$

Enforcing e to satisfy

$$\|e\| \leq \sigma \frac{\|Lx\|}{\|L\|} \tag{8}$$

with $\sigma > 0$, we get

$$\dot{V} \leq (\sigma - 1)\|Lx\|^2$$

which is negative for $\sigma < 1$ and $\|Lx\| \neq 0$.

Thus, the events are triggered when:

$$f(e, x) \triangleq \|e\| - \sigma \frac{\|Lx\|}{\|L\|} = 0 \tag{9}$$

This choice of f is of course motivated by the analysis above in order to guarantee convergence to an agreement point. The event times are thus defined by $f(e(t_i), x(t_i)) = 0$, for $i = 0, 1, \ldots$. At each t_i, the control law is updated according to (6):

$$u(t_i) = -Lx(t_i)$$

and remains constant, i.e., $u(t) = -Lx(t_i)$ for all $t \in [t_i, t_{i+1})$. Once the control task is executed the error is reset to zero, since at that point we have $e(t_i) = x(t_i) - x(t_i) = 0$ for the specific event time so that (8) is enforced.

The proposed control policy attains a strictly positive lower bound on the inter-event times. This is proven in the following theorem:

Theorem 1: Consider system $\dot{x} = u$ with the control law (6),(9) and assume that the communication graph G is connected. Suppose that $0 < \sigma < 1$. Then for any initial condition in $\mathbb{R}^N$ the inter-event times $\{t_{i+1} - t_i\}$ implicitly

defined by the rule (9) are lower bounded by a strictly positive time τ which is given by

$$\tau = \frac{\sigma}{\|L\|(1 + \sigma)}$$

Proof: We will compute the time derivative of $\frac{\|e\|}{\|Lx\|}$:

$$\frac{d}{dt}\frac{\|e\|}{\|Lx\|} = -\frac{e^T \dot{x}}{\|e\|\|Lx\|} - \frac{(Lx)^T L\dot{x}}{\|Lx\|^2}\frac{\|e\|}{\|Lx\|}$$
$$\leq \frac{\|e\|\|\dot{x}\|}{\|e\|\|Lx\|} + \frac{\|\dot{x}\|\|L\|\|e\|}{\|Lx\|^2}$$
$$= \left(1 + \frac{\|L\|\|e\|}{\|Lx\|}\right)\frac{\|\dot{x}\|}{\|Lx\|}$$
$$\leq \left(1 + \frac{\|L\|\|e\|}{\|Lx\|}\right)\frac{\|Lx\| + \|Le\|}{\|Lx\|}$$
$$\leq \left(1 + \frac{\|L\|\|e\|}{\|Lx\|}\right)^2$$

Using the notation

$$y = \frac{\|e\|}{\|Lx\|}$$

we have

$$\dot{y} \leq (1 + \|L\|y)^2$$

so that y satisfies the bound

$$y(t) \leq \phi(t, \phi_0)$$

where $\phi(t, \phi_0)$ is the solution of

$$\dot{\phi} = (1 + \|L\|\phi)^2, \phi(0, \phi_0) = \phi_0$$

Hence the inter-event times are bounded from below by the time τ that satisfies

$$\phi(\tau, 0) = \frac{\sigma}{\|L\|}$$

The solution of the above differential equation is given by

$$\phi(\tau, 0) = \frac{\tau}{1 - \tau\|L\|}$$

so that

$$\tau = \frac{\sigma}{\|L\|(1 + \sigma)}$$

and the proof is complete. $\diamond$

Using the extension of La Salle's Invariance Principle for hybrid systems [15], the following Corollary regarding the convergence of the closed-loop system is now evident:

Corollary 2: Consider system $\dot{x} = u$ with the control law (6),(9) and assume that the communication graph G is connected. Suppose that $0 < \sigma < 1$. Then all agents converge to their initial average, i.e.,

$$\lim_{t \to \infty} x_i(t) = \bar{x} = \frac{1}{N}\sum_i x_i(0)$$

for all $i \in \mathcal{N}$.

Proof: Since $\dot{V} \leq (\sigma - 1)\|Lx\|^2$, by Theorem IV.1 in [15], we have that $\lim_{t \to \infty} Lx(t) = 0$. Since G is connected, the latter corresponds to the fact that all elements of x are equal at steady state, i.e., $\lim_{t \to \infty} x_i(t) = x^*$. Since the initial average remains constant we have $x^* = \bar{x} = \frac{1}{N}\sum_i x_i(0)$ at steady state. $\diamond$

IV. DECENTRALIZED APPROACH

In the centralized case, all agents had to be aware of the global measurement error e in order to enforce the condition (8). In this section, we consider the decentralized counterpart. In particular, each agent now updates its own control input at event times it decides based on information from its neighboring agents. The event times for each agent $i \in \mathcal{N}$ are denoted by $t_0^i, t_1^i, \ldots$. We will follow the structure described at the end of Section II to define the functions $f_i, i \in \mathcal{N}$ according to which the event times for agent i are defined.

The measurement error for agent i is defined as

$$e_i(t) = x_i(t_k^i) - x_i(t), t \in [t_k^i, t_{k+1}^i) \qquad (10)$$

The decentralized control strategy for agent i is now given by:

$$u_i(t) = -\sum_{j \in N_i} \left(x_i(t_k^i) - x_j(t_{k'(t)}^j) \right) \qquad (11)$$

where

$$k'(t) \triangleq \arg \min_{l \in \mathbb{N}: t \geq t_l^j} \left\{ t - t_l^j \right\}$$

Thus for each $t \in [t_k^i, t_{k+1}^i)$, $t_{k'(t)}^j$ is the last event time of agent j. Hence, each agent takes into account the last update value of each of its neighbors in its control law. The control law for i is updated both at its own event times $t_0^i, t_1^i, \ldots$, as well as at the event times of its neighbors $t_0^j, t_1^j, \ldots, j \in N_i$.

Note that this definition of k' implies $x_j(t_{k'(t)}^j) = x_j(t) + e_j(t)$. We thus have

$$\dot{x}_i(t) = -\sum_{j \in N_i} \left(x_i(t_k^i) - x_j(t_{k'(t)}^j) \right) =$$
$$= -\sum_{j \in N_i} (x_i(t) - x_j(t)) - \sum_{j \in N_i} (e_i(t) - e_j(t))$$

Hence in this case we also have $\dot{\bar{x}} = 0$ for the agents' initial average.

Denote now $Lx \triangleq z = [z_1, \ldots, z_N]^T$ and consider again

$$V = \frac{1}{2} x^T L x$$

Then

$$\dot{V} = x^T L \dot{x} = -x^T L(Lx + Le) = -z^T z - z^T Le$$

From the definition of the Laplacian matrix we get

$$\dot{V} = -\sum_i z_i^2 - \sum_i \sum_{j \in N_i} z_i (e_i - e_j)$$
$$= -\sum_i z_i^2 - \sum_i |N_i| z_i e_i + \sum_i \sum_{j \in N_i} z_i e_j$$

Using now the inequality $|xy| \leq \frac{a}{2} x^2 + \frac{1}{2a} y^2$, for $a > 0$, we can bound $\dot{V}$ as

$$\dot{V} \leq -\sum_i z_i^2 + \sum_i a|N_i| z_i^2$$
$$+ \sum_i \frac{1}{2a} |N_i| e_i^2 + \sum_i \sum_{j \in N_i} \frac{1}{2a} e_j^2$$

where $a > 0$.

Since the graph is symmetric, by interchanging the indices of the last term we get

$$\sum_i \sum_{j \in N_i} \frac{1}{2a} e_j^2 = \sum_i \sum_{j \in N_i} \frac{1}{2a} e_i^2 = \sum_i \frac{1}{2a} |N_i| e_i^2$$

so that

$$\dot{V} \leq -\sum_i (1 - a|N_i|) z_i^2 + \sum_i \frac{1}{a} |N_i| e_i^2$$

Assume that a satisfies

$$0 < a < \frac{1}{|N_i|} \qquad (12)$$

for all $i \in \mathcal{N}$. Then, enforcing the condition

$$e_i^2 \leq \frac{\sigma_i a(1 - a|N_i|)}{|N_i|} z_i^2 \qquad (13)$$

for all $i \in \mathcal{N}$, we get

$$\dot{V} \leq \sum_i (\sigma_i - 1)(1 - a|N_i|) z_i^2$$

which is negative definite for $0 < \sigma_i < 1$.

Thus for each i, an event is triggered when

$$f_i \left(e_i, \sum_{j \in N_i} (x_i - x_j) \right) \triangleq e_i^2 - \frac{\sigma_i a(1 - a|N_i|)}{|N_i|} z_i^2 = 0 \qquad (14)$$

where $z_i = \sum_{j \in N_i} (x_i - x_j)$. The update rule (14) holds at the event times t_k^i corresponding to agent i:

$$f_i \left(e_i \left(t_k^i \right), \sum_{j \in N_i} (x_i(t_k^i) - x_j \left(t_k^i \right)) \right) = 0$$

with $k = 0, 1, \ldots$ and $i \in \mathcal{N}$. At an event time t_k^i, we have

$$e_i(t_k^i) = x_i(t_k^i) - x_i(t_k^i) = 0$$

and thus, condition (13) is enforced.

It should be emphasized that the condition (14) is verified by agent i only based on information of each own and neighboring agents' information.

A similar theorem regarding the inter-event times holds in the decentralized case as well:

Theorem 3: Consider system $\dot{x}_i = u_i, i \in \mathcal{N} = \{1, \ldots, N\}$ with the control law (11) and update ruling (14), and assume that G is connected. Suppose that $0 < a < \frac{1}{|N_i|}$ and $0 < \sigma_i < 1$ for all $i \in \mathcal{N}$. Then for any initial condition in $\mathbb{R}^N$, and any time $t \geq 0$ there exists at least one agent $k \in \mathcal{N}$ for which the next inter-event interval is strictly positive.

Proof: Assume that (14) holds for all $i \in \mathcal{N}$ at time t. If it doesn't hold, then continuous evolution is possible since at least one agent can still let its absolute measurement error increase without resetting (10). Hence assume that at t all errors are reset to zero. We will show that there exists at least one $k \in \mathcal{N}$ such that its next inter-event interval is bounded from below by a certain time $\tau_D > 0$.

Denoting

$$k = \arg\max_i |z_i|$$

and considering that $|e_i| \leq \|e\|$ holds for all i, we have

$$\frac{|e_k|}{N|z_k|} \leq \frac{\|e\|}{\|z\|}$$

so that

$$\frac{|e_k|}{|z_k|} \leq N\frac{\|e\|}{\|z\|} = N\frac{\|e\|}{\|Lx\|}$$

From the proof of Theorem 1 and the control update rule (14), we deduce that the next inter-event interval of agent k is bounded from below by a time τ_D that satisfies

$$N\frac{\tau_D}{1 - \tau_D \|L\|} = \frac{\sigma_k a(1 - a|N_k|)}{|N_k|}$$

so that

$$\tau_D = \frac{\sigma_k a(1 - a|N_k|)}{N|N_k| + \|L\|\sigma_k a(1 - a|N_k|)}$$

and the proof is complete. $\diamond$

We should note that the result of this Theorem is more conservative than the centralized case, since it only guarantees that there are no accumulation points and continuous evolution is viable at all times instants. Using now La Salle's Invariance Principle for Hybrid Systems [15], the following convergence result is straightforward:

Corollary 4: Consider system $\dot{x} = u$ with the control law (11),(14) and assume that the communication graph G is connected. Then all agents converge to their initial average, i.e.,

$$\lim_{t \to \infty} x_i(t) = \bar{x} = \frac{1}{N}\sum_i x_i(0)$$

for all $i \in \mathcal{N}$.

Proof: By virtue of Theorem 3, the closed-loop switched system does not exhibit Zeno behavior. The rest of the proof is identical to that of Corollary 2. $\diamond$

V. EXAMPLES

The results of the previous Sections are depicted through computer simulations.

Consider a network of four agents whose Laplacian matrix is given by

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

We consider both the centralized and the decentralized framework. Four agents start from random initial conditions and evolve under the control law (6),(9) in the first case, and the control law (11),(14) in the second case. In the centralized case, we have set $\sigma = 0.65$, and $\sigma_1 = \sigma_2 = 0.55$, $\sigma_3 = \sigma_4 = 0.75$ and $a = 0.2$ for the decentralized control example. Figure 1 shows the evolution of $\|Lx(t)\|$ in both cases in time. The bottom solid line shows the evolution in the centralized and the top dotted line in the decentralized

case. It can be seen that the system converges in both frameworks.

Figure 2 shows the evolution of the error norm in the centralized case. The solid line represents the evolution of the error $\|e(t)\|$. This stays below the specified state-dependent threshold $\|e\|_{max} = \sigma\dfrac{\|Lx\|}{\|L\|}$ which is represented by the dotted line in the Figure. The existence of a minimum inter-event time is clearly visible in this example.
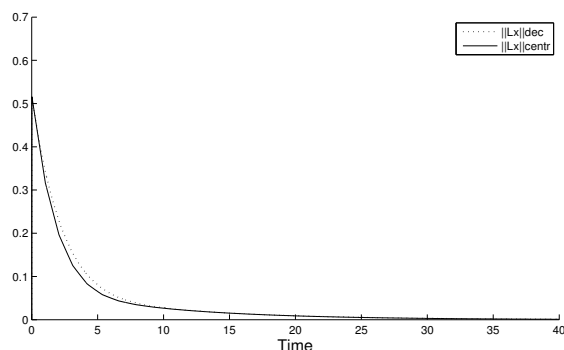

Fig. 1. Four agents evolve under (6),(9) in the centralized case, and the control law (11),(14) in the decentralized case. Convergence to the initial average is achieved in both cases.
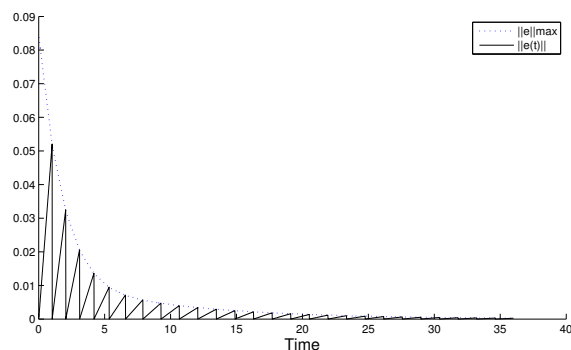

Fig. 2. Evolution of the error norm in the centralized case. The solid line represents the evolution of the error norm $\|e(t)\|$, which stays below the specified state-dependent threshold $\|e\|_{max} = \sigma\dfrac{\|Lx\|}{\|L\|}$ which is represented by the dotted line in the Figure.

The next two figures depict how condition (13) is realized in the decentralized case for agents 1,3. In particular, the solid line in Figure 3 shows the evolution of $|e_1(t)|$. This stays below the specified state-dependent threshold given by (13) $|e_1|_{\max} = \sqrt{\dfrac{\sigma_1 a(1 - a|N_1|)}{|N_1|}} z_1$ which is represented by the dotted line in the Figure. The same holds for agent 3 as shown in Figure 4 where the solid line represents $|e_3(t)|$ which also stays below the specified state-dependent threshold given by (13) $|e_3|_{\max} = \sqrt{\dfrac{\sigma_3 a(1 - a|N_3|)}{|N_3|}} z_3$, represented by the dotted line in the Figure.
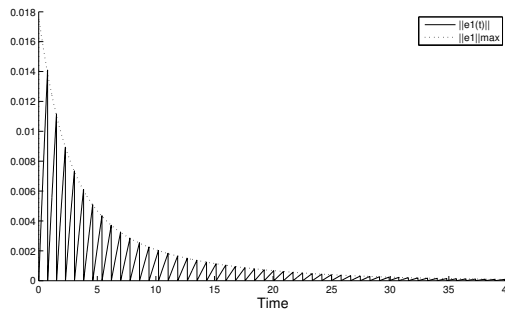
Fig. 3. Four agents are controlled by (11),(14) in the decentralized case. Condition (13) is depicted in the this case for agent 1. The solid line shows the evolution of $|e_1(t)|$. This stays below the specified state-dependent threshold given by (13) $|e_1|_{\max} = \sqrt{\frac{\sigma_1 a (1-a|N_1|)}{|N_1|}} z_1$ which is represented by the dotted line.
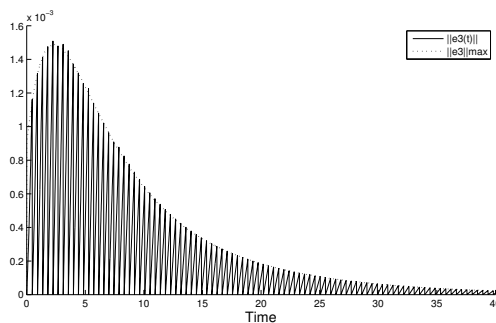

Fig. 4. Condition (13) is depicted in the this case for agent 3. The solid line shows the evolution of $|e_3(t)|$. This stays below the specified state-dependent threshold given by (13) $|e_3|_{\max} = \sqrt{\frac{\sigma_3 a (1-a|N_3|)}{|N_3|}} z_3$ which is represented by the dotted line.

VI. CONCLUSIONS

We considered event-driven strategies for multi-agent systems. The actuation updates were event-driven, depending on the ratio of a certain measurement error with respect to the norm of a function of the state. A centralized formulation of the problem was considered first and then the results were extended to the decentralized counterpart, in which agents required knowledge only of the states of their neighbors for the controller implementation. The results of the paper were supported through simulated examples. Future work will focus on the application of the framework to other cooperative multi-agent control tasks.

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