# Partial and full attitude synchronization of multiple underactuated spacecraft

Ziyang Meng, Dimos V. Dimarogonas, and Karl H. Johansson

***Abstract*—We study leaderless attitude synchronization problem of multiple underactuated spacecraft in this paper. We adopt the special parameterizations of attitude proposed by Tsiotras et al. (1995) to describe attitude kinematics, which has been shown to be very convenient for control of underactuated axisymmetric spacecraft with two control torques. It is assumed that angular velocity commands are possible and this paper is confined to the kinematic level. We first propose a partial attitude synchronization controller that is based on the exchange of each spacecraft's information with local neighbors. Under a necessary and general connectivity assumption and by use of an appropriate Lyapunov function, we show that the attitudes of spacecraft converge to a fixed or time-varying synchronization trajectory. Then, full attitude synchronization of multiple underactuated spacecraft is considered, where the discontinuous distributed algorithm is proposed. Simulations are given to validate the theoretical results and indicate several interesting observations.**

***Index Terms*—distributed attitude synchronization, underactuated spacecraft**

## I. INTRODUCTION

Synchronization of multi-agent systems has received a great amount of attentions recently due to its broad applications in power network [1], biological network [2], social network [3], mechanical network [4] and so on. Different protocols were proposed for different agent models, including general linear system [5], nonlinear system [6], Lagrangian system [7], mobile robotic system [8] and so on.

In this work, we exemplify the agent dynamics as attitude of a rigid body and study attitude synchronization problem of multiple spacecraft. Relevant works include [9], [10], [11], [12], [13], [14]. In particular, attitude synchronization problem of a group of rotating and translating rigid bodies was studied in [10], where a ring communication topology was considered. Attitude direction cosine matrix was used in [13] to construct an attitude synchronization algorithm while the extension to the case of directed switching communication topologies was also given. The authors of [9] proposed a cooperative attitude tracking protocol such that the follower spacecraft track a time-varying leader spacecraft using relative attitude and relative angular velocity information. A passivity-based group orientation approach was introduced in [11] to solve distributed attitude alignment problem, where the inertial frame information is not assumed to available to the spacecraft. Similar problem was also considered in [12], where a standing assumption is that the states of the leader spacecraft are only available to a subset of follower spacecraft and the follower spacecraft only have local information exchange. In addition, the attitude containment problem was considered in [14] and the influence of communication delay between different spacecraft was studied in [15].

The authors are with ACCESS Linnaeus Centre, School of Electrical Engineering, Royal Institute of Technology, Stockholm 10044, Sweden. Email: {ziyangm, dimos, kallej}@kth.se.

In this paper, we focus on attitude synchronization problem of multiple underactuated spacecraft. In particular, we consider axisymmetric spacecraft with two control torques and assume that angular velocity commands are possible. We adopt the special parametrization of attitude given in [16] to describe attitude kinematics and propose attitude synchronization algorithms. We show that attitude synchronization is achieved under the proposed algorithms. Several interesting phenomena are also observed regarding the steady trajectories of the attitudes.

## II. BACKGROUND AND PRELIMINARIES

### A. Graph theory

Using graph theory, we can model the communication topology among spacecraft in the formation. A graph $\mathcal{G}$ consists of a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a finite nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of ordered pairs of nodes. An edge $(i, j)$ denotes that nodes $j$ can obtain information from $i$. All the neighbors of node $i$ are denoted as $\mathcal{N}_i := \{j | (j, i) \in \mathcal{E}\}$, where we assume that $i \notin \mathcal{N}_i$.

A directed path in a directed graph is a sequence of arcs of the form $(i_1, i_2), (i_2, i_3), \ldots$. If there exists a path from node $i$ to $j$, then node $j$ is said to be reachable from node $i$. $\mathcal{G}$ is said to be strongly connected if each node is reachable from any other node.

### B. Switching communication topology and joint connectivity

The distributed algorithm relies on the use of communication unit for each agent. One common issue of equipping communication unit is the possible communication link failure. Therefore, it is necessary to consider the validness of the proposed algorithms for the case of switching communication topology. We associate the switching communication topology with a time-varying graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$, where $\sigma : [0, +\infty) \to \mathcal{P}$ is a piecewise constant function and $\mathcal{P}$ is finite set of all possible graphs. $\mathcal{G}_{\sigma(t)}$ remains constant for $t \in [t_\varsigma, t_{\varsigma+1})$, $\varsigma = 0, 1, \ldots$ and switches at $t = t_\varsigma$, $\varsigma = 1, \ldots$. In addition, we assume that $\inf_\varsigma (t_{\varsigma+1} - t_\varsigma) \geq \tau_d > 0$, $\varsigma = 1, \ldots$, where $\tau_d$ is a constant known as dwell time [17]. The joint graph of $\mathcal{G}_{\sigma(t)}$ during time interval $[t_1, t_2)$ is defined by $\mathcal{G}_{\sigma(t)}([t_1, t_2)) = \bigcup_{t \in [t_1, t_2)} \mathcal{G}(t) = (\mathcal{V}, \bigcup_{t \in [t_1, t_2)} \mathcal{E}(t))$. Moreover, $j$ is a neighbor of $i$ at time $t$ when $(j, i) \in \mathcal{E}_{\sigma(t)}$, and $\mathcal{N}(\sigma(t))$ represents the set of agent $i$'s neighbors at time $t$.

**Definition 2.1.** *$\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected if there exists a constant $T > 0$ such that $\mathcal{G}([t, t + T))$ is strongly connected for any $t \geq 0$.*

### C. The $(w,z)$ attitude parametrization

Since we are interested in the control of underactuated axisymmetric spacecraft, we use a special parametrization to represent the attitude of a spacecraft. This parametrization is based on a pair of a complex w and a real coordinate $z$, which is proposed in [16].

The $(\mathrm{w},z)$ parametrization can be derived from attitude direction cosine matrix $R=[R_{pq}]\in\mathbb{R}^{3\times 3}$ by using the following relationship (see Lemma 1 of [18]):

$$\mathrm{w}=\frac{R_{23}}{1+R_{33}}-\mathbf{j}\frac{R_{13}}{1+R_{33}},$$

$$\cos(z)=\frac{1}{2}((1+|\mathrm{w}|^2)\mathrm{trace}(R)+|\mathrm{w}|^2-1),$$

where $\mathbf{j}=\sqrt{-1}$, $\overline{\mathrm{w}}$ denotes the complex conjugate of a complex number $\mathrm{w}\in\mathbb{C}$, and $|\mathrm{w}|^2=\mathrm{w}\overline{\mathrm{w}}$ denotes square of modulus of w.

Based on the above relationship, we can use $(\mathrm{w},z)$ to describe attitude coordinates from now on. Consider $n$ spacecraft with attitude $(\mathrm{w}_i,z_i)$, $\forall i=1,2,\ldots,n$. The kinematic equation of each spacecraft is described by

$$\dot{\mathrm{w}}_i=-\mathbf{j}\omega_{i3}^*\mathrm{w}_i+\frac{\omega_i}{2}+\frac{\overline{\omega_i}}{2}\mathrm{w}_i^2, \tag{2a}$$

$$\dot{z}_i=\omega_{i3}^*+\mathrm{Im}(\omega_{\mathrm{i}}\overline{\mathrm{w}_{\mathrm{i}}}), \tag{2b}$$

where $\mathrm{w}_i=\mathrm{w}_{i1}+\mathbf{j}\mathrm{w}_{i2}$ and $\omega_i=\omega_{i1}+\mathbf{j}\omega_{i2}$. In this paper, we focus on angular velocity commands and assume that only angular velocity $\omega_i$ can be manipulated. Also, for an axisymmetric spacecraft, $\omega_{i3}^*$, $\forall i=1,2,\ldots,n$, remains constant for all $t\geq 0$ although the torque input about this axis is zero.

In this case, only two-axis stabilization of pointing is possible for the general case when at least one $\omega_{i3}^*$, $i\in\mathcal{V}$ is nonzero.

In addition, for the special case when $\omega_{i3}^*\equiv 0$, for all $i\in\mathcal{V}$, the three-axis stabilization of pointing is possible, where the kinematic equations become

$$\dot{\mathrm{w}}_i=\frac{\omega_i}{2}+\frac{\overline{\omega_i}}{2}\mathrm{w}_i^2, \tag{3a}$$

$$\dot{z}_i=\mathrm{Im}(\omega_{\mathrm{i}}\overline{\mathrm{w}_{\mathrm{i}}}). \tag{3b}$$

**Remark 2.1.** *We know that every three-dimensional parameterizations will cause singularity. Therefore, only almost global attitude control is possible in this paper and we restrict the discussion on the corresponding kinematic parameters.*

## III. Partial attitude synchronization of multiple underactuated spacecraft

In this section, we focus on the kinematic (2a) and study the partial attitude synchronization problem, where the synchronization manifold $\mathcal{W}$ is defined as $\mathcal{W}=\{(\mathrm{w}_1,z_1,\ldots,\mathrm{w}_n,z_n):\mathrm{w}_1=\cdots=\mathrm{w}_n\}$.

We first show that partial attitude norm synchronization can be achieved for arbitrary constant $\omega_{i3}^*$, for all $i\in\mathcal{V}$ under very mild communication condition, where the norm synchronization manifold $\mathcal{W}_1$ is defined as $\mathcal{W}_1=\{(\mathrm{w}_1,z_1,\ldots,\mathrm{w}_n,z_n):|\mathrm{w}_1|=\cdots=|\mathrm{w}_n|\}$. We then show that attitude synchronization can be achieved for the special when $\omega_{i3}^*\equiv 0$, for all $i\in\mathcal{V}$ and observe some interesting behaviors on the convergence of attitude trajectories.

The following attitude synchronization algorithm is proposed for the $i$th spacecraft

$$\omega_i=-\sum_{j\in\mathcal{N}_i(\sigma(t))}a_{ij}(t)(\mathrm{w}_i-\mathrm{w}_j), \tag{4}$$

where $a_{ij}(t)>0$ is the weight of arc $(j,i)$ for $i,j\in\mathcal{V}$ at $t$. We also assume that $a_{ij}(t)$ satisfies the following condition:

**Assumption 3.1.** *There exists constants $a^*>0$ and $a_*>0$ such that for all $i,j\in\mathcal{V}$, $a_*\leq a_{ij}(t)\leq a^*$, $t\in\mathbb{R}^+$.*

**Theorem 3.1.** *Suppose that Assumption 3.1 holds and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. For multiple underactuated spacecraft kinematic* (2a)*, algorithm* (4) *guarantees that partial attitude norm synchronization is achieved with respect to $\mathcal{W}_1$. In particular, it follows that $\lim_{t\to\infty}|w_i(t)|=w^*$, for all $i\in\mathcal{V}$, where $w^*$ is a positive constant.*

*Proof:* The proof is omitted due to length limitation and is available upon request. ∎

### A. Discussion on the special case when $\omega_{i3}^*\equiv 0$, for all $i\in\mathcal{V}$

In this section, we focus on the special case when the angular velocity of uncontrollable axis $\omega_{i3}^*$ remain zero for all $i\in\mathcal{V}$. We show that in addition to partial attitude norm synchronization, partial attitude synchronization is also achieved with respect to $\mathcal{W}$.

**Proposition 3.1.** *Suppose that Assumption 3.1 holds and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. For multiple underactuated spacecraft kinematic* (2a) *with $\omega_{i3}^*\equiv 0$, for all $i\in\mathcal{V}$, algorithm* (4) *guarantees that partial attitude synchronization is achieved with respect to $\mathcal{W}$. In particular, it follows that $\lim_{t\to\infty}(w_i(t)-w_j(t))=0$, for all $i,j\in\mathcal{V}$.*

*Proof:* The proof is omitted due to length limitation and is available upon request. ∎

### B. Discussion on the different steady trajectories for different cases of $\omega_{i3}^*$

We notice that Theorem 3.1 only claims that the norms of all spacecraft's partial attitudes reach synchronization. There is no affirmative assertion on the pattern of the steady trajectory when multiple spacecraft reach partial attitude synchronization. It turns out that the uncontrollable angular velocity $\omega_{i3}^*$ plays important role in forming the pattern of the steady trajectory. We next show that different selections on $\omega_{i3}^*$ produce different steady trajectories using simulations.

*1) Case I (nonidentical $\omega_{i3}^*$):* We first consider the case that angular velocities $\omega_{i3}^*$ are nonidentical for all $i\in\mathcal{V}$. In particular, we consider that there are four spacecraft ($n=4$) in the group. The weight $a_{ij}$ is chosen to be 1 when $(j,i)\in\mathcal{E}$. The communication graph $\mathcal{G}$ switches between $\mathcal{G}^1$ (Fig. 1) and $\mathcal{G}^2$ (Fig. 2) at time instants $t_\varrho=\varrho$, $\varrho=0,1,\ldots$.

Fig. 3 shows the trajectories of $\mathrm{w}_{i1}$ and $\mathrm{w}_{i2}$ for all $i=1,2,3,4$. We see that attitude synchronization is achieved

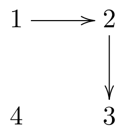


**Fig. 1.** The communication graph $\mathcal{G}^1$

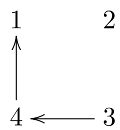


**Fig. 2.** The communication graph $\mathcal{G}^2$

while the final attitudes of all the spacecraft converge to zero. This is an interesting observation since intuitively, a weakly coupled and strongly heterogeneous network may not display coherent behavior. However, the simulation result shows that the final trajectories of all the agents converges to the origin as if each agent is commanded with an absolute damping. This also presents the coherent behavior.

*2) Case II (identically nonzero $\omega_{i3}^*$):* We next consider the the case that angular velocities $\omega_{i3}^*$ are identically nonzero for all $i \in \mathcal{V}$. Fig. 4 shows the trajectories of $\mathrm{w}_{i1}$ and $\mathrm{w}_{i2}$ for all $i = 1, 2, 3, 4$. We see that attitude synchronization is achieved while the final attitudes of all the spacecraft converge to time-varying bounded curves.

*3) Case III (all zero $\omega_{i3}^*$):* We finally consider the the case that angular velocities $\omega_{i3}^*$ are zeros for all $i \in \mathcal{V}$. Fig. 5 shows the trajectories of $\mathrm{w}_{i1}$ and $\mathrm{w}_{i2}$ for all $i = 1, 2, 3, 4$. We see that attitude synchronization is achieved while the final attitudes of all the spacecraft converge to a nonzero constant. This is similar to the coherent behavior presented for the consensus of multiple single integrators.

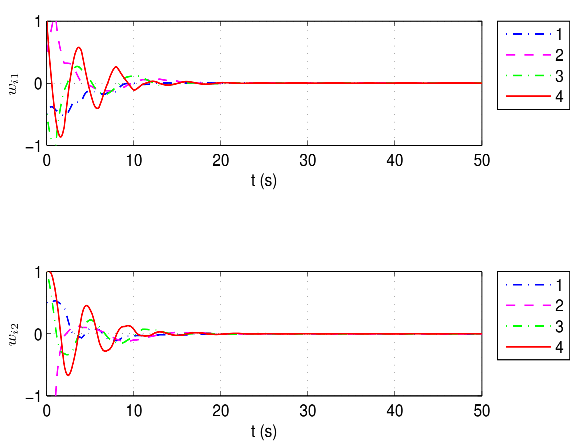


**Fig. 3.** Attitude trajectories for the case of nonidentical $\omega_{i3}^*$

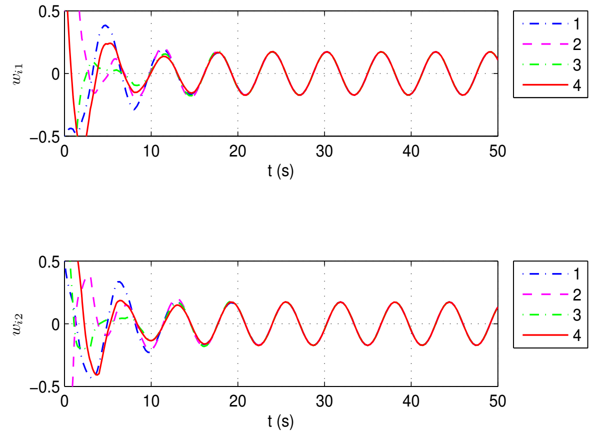


**Fig. 4.** Attitude trajectories for the case of identically nonzero $\omega_{i3}^*$

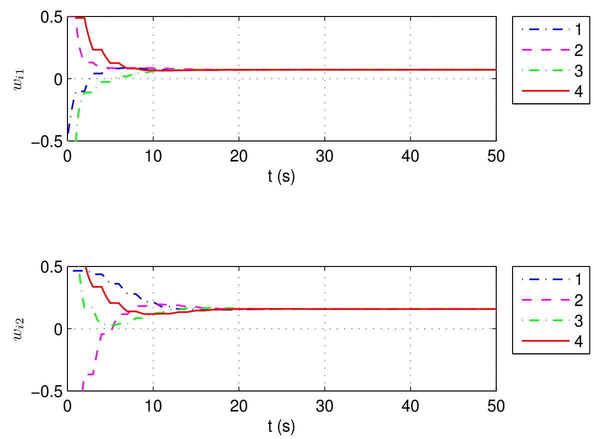


**Fig. 5.** Attitude trajectories for the case of all zero $\omega_{i3}^*$

## IV. Full Attitude Synchronization of Multiple Underactuated Spacecraft

In this section, we focus on the kinematic (3) and study full attitude synchronization problem, where the synchronization manifold $\mathcal{P}$ is defined as $\mathcal{P} = \{(\mathrm{w}_1, z_1, \dots, \mathrm{w}_n, z_n) : z_1 = \cdots = z_n, \mathrm{w}_1 = \cdots = \mathrm{w}_n\}$.

The following attitude synchronization algorithm is proposed for the $i$th spacecraft

$$\omega_i = -\gamma \mathrm{w}_i - \mathbf{j}\frac{\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(t)(z_i - z_j)}{\overline{\mathrm{w}_i}}, \qquad (5)$$

where $a_{ij}(t) > 0$ is the weight of arc $(j, i)$ for $i, j \in \mathcal{V}$ at $t$.

**Theorem 4.1.** *Suppose that Assumption 3.1 holds and $\mathcal{G}_{\sigma(t)}$ is uniformly jointly strongly connected. Also assume that $\mathrm{w}_i(0) \neq 0$, for all $i \in \mathcal{V}$. For multiple underactuated spacecraft (3),*

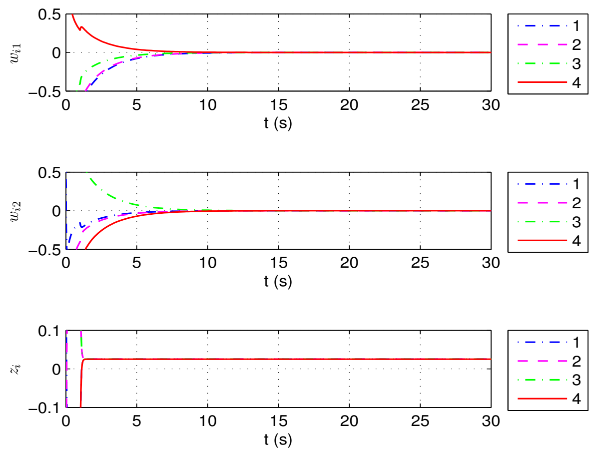


**Fig. 6.** Attitude trajectories of system (3) under algorithm (5)

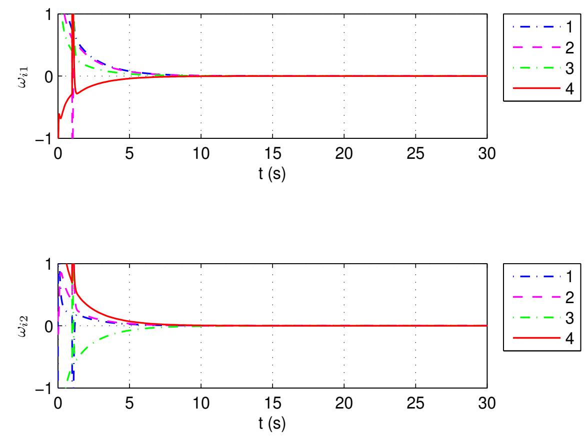


**Fig. 7.** Trajectories of control inputs of system (3) under algorithm (5)

*algorithm* (5) *guarantees that full attitude synchronization is achieved with respect to $\mathcal{P}$ if $\gamma$ is chosen sufficiently small. In particular, it follows that*

- $w_i(t) \neq 0$, *for all $i \in \mathcal{V}$ and for all $t \geq 0$.*
- $\lim_{t\to\infty} w_i(t) = 0$, *for all $i \in \mathcal{V}$.*
- $\lim_{t\to\infty}(z_i(t) - z_j(t)) = 0$, *for all $i, j \in \mathcal{V}$.*
- *The control input $\omega_i$ is bounded for all $i \in \mathcal{V}$.*

*Proof:* The proof is omitted due to length limitation and is available upon request. ■

### A. Simulation verification

The communication topology is chosen same as the one given in Section III-B. The control gain $\gamma$ is chosen as $\gamma = 0.1$. Figs. 6 and 7 show, respectively, the trajectories of $w_{i1}$, $w_{i2}$, and $z_i$ and the control inputs $\omega_{i1}$ and $\omega_{i2}$, for all $i = 1, 2, 3, 4$. We see that attitude synchronization is achieved while the control inputs are bounded.

## V. Concluding Remarks

We study leaderless attitude synchronization problem of multiple underactuated spacecraft in this paper. A special parametrization of attitude is used to describe attitude kinematics, which is particularly useful for control of underactuated axisymmetric spacecraft with two control torques. We propose a partial attitude synchronization protocol and a full attitude synchronization protocol, respectively. Simulation results are given to validate the theoretical results and several interesting phenomena regarding the steady trajectories of the attitudes are observed.

## References


[1] F. Dorfler, M. Chertkov, and F. Bullo, "Synchronization in complex oscillator networks and smart grids," *Proceedings of the National Academy of Sciences*, vol. 110, no. 6, pp. 2005–2010, 2013.

[2] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[3] C. Altafini, "Consensus problems on networks with antagonistic interactions," *IEEE Transactions Automatic Control*, vol. 58, no. 4, pp. 935–946, 2013.

[4] Z. Lin, M. Broucke, and B. Francis, "Local control strategies for groups of mobile autonomous agents," *IEEE Transactions on Automatic Control*, vol. 49, no. 4, pp. 622–629, 2004.

[5] L. Scardovi and R. Sepulchre, "Synchronization in networks of identical linear systems," *Automatica*, vol. 45, no. 11, pp. 2557–2562, 2009.

[6] L. Moreau, "Stability of multi-agent systems with time-dependent communication links," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169–182, 2005.

[7] S.-J. Chung and J.-J. E. Slotine, "Cooperative robot control and concurrent synchronization of Lagrangian systems," *IEEE Transations on Robotics*, vol. 25, no. 3, pp. 686–700, 2009.

[8] J. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Transactions on Robotics and Automation*, vol. 19, no. 6, pp. 933–941, 2003.

[9] M. VanDyake and C. Hall, "Decentralized coordinated attitude control of a formation of spacecraft," *J. Guidance, Control and Dynamics*, vol. 29, no. 5, pp. 1101–1109, 2006.

[10] S. Nair and N. E. Leonard, "Stable synchronization of mechanical system networks," *American Institute of Mathematical Sciences*, vol. 2, no. 4, pp. 595–624, 2007.

[11] H. Bai, M. Arcak, and J. T.-Y. Wen, "Rigid body attitude coordination without inertial frame information," *Automatica*, vol. 44, no. 12, pp. 3170–3175, 2008.

[12] W. Ren, "Distributed cooperative attitude synchronization and tracking for multiple rigid bodies," *IEEE Transactions on Control Systems Technology*, vol. 18, no. 2, pp. 383–392, 2010.

[13] A. Sarlette, R. Sepulchre, and N. E. Leonard, "Autonomous rigid body attitude synchronization," *Automatica*, vol. 45, no. 2, pp. 572–577, 2009.

[14] D. V. Dimarogonas, P. Tsiotras, and K. J. Kyriakopoulos, "Leader-follower cooperative attitude control of multiple rigid bodies," *Systems and Control Letters*, vol. 58, no. 6, pp. 429–435, 2009.

[15] E. Jin, X. Jiang, and Z. Sun, "Robust decentralized attitude coordination control of spacecraft formation," *Systems & Control Letters*, vol. 57, no. 7, pp. 567–577, 2008.

[16] P. Tsiotras, M. Corless, and J. M. Longuski, "A novel approach to the attitude control of axisymmetric spacecraft," *Automatica*, vol. 31, no. 8, pp. 1099–1112, 1995.

[17] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59–70, 1999.

[18] P. Tsiotras and J. Luo, "Control of underactuated spacecraft with bounded inputs," *Automatica*, vol. 36, no. 8, pp. 1153–1169, 2000.