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Distributed Contention Resolution in Broadcast Control Systems

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Abstract— **The flexibility of a control system can be improved by closing the loop over a wireless network. In these systems, however, the controller is generally assigned to the plant a priori. In this paper, a novel distributed control architecture called the broadcast control system is introduced. In this system the plant broadcasts its state to multiple controllers, where each controller makes a local decision to send back the control input. This allows the plant to use different controllers while moving. Moreover, the control performance can be optimized by exploiting the diversity of the wireless links. A contention resolution phase is introduced, which dynamically assigns a controller to the plant at runtime based on the wireless link quality. The coordinate descent algorithm is proposed as an effective way to optimize the thresholds of the contention resolution phase. The subproblem structure of the objective function is used to prove the convergence of the optimization algorithm. Numerical results show that the control performance can be increased by adding more controllers to the system, which is particularly effective when the link quality is poor.**

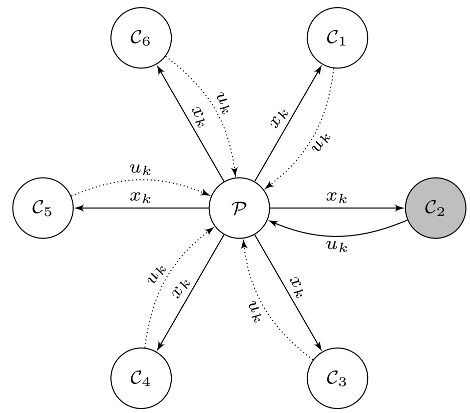

Figure 1. The plant $\mathcal{P}$ broadcasts its state x_k to all neighboring controllers $\mathcal{C}_1, \ldots, \mathcal{C}_6$. After a contention resolution phase one of the controllers sends back the control input u_k.

I. Introduction

In wireless networked control systems the control loop is closed over a wireless network. These systems have advantages in terms of an increased flexibility and a reduction in cost. This flexibility is however not always optimally exploited, since the controller is often paired with the plant a priori. This can be especially problematic when the plant is mobile, since the channel to the fixed controller will fluctuate. Furthermore, the design of wireless networked control systems is generally challenging due to the stochastic nature of the wireless channel [1].

These issues are addressed in this paper by the introduction of a novel distributed control architecture called the broadcast control system. The system setup is shown in Figure 1, and consists of a plant and multiple controllers. In this system the used controller is dynamically assigned to the plant, where the wireless link quality is used to determine the best controller to use. At every time step the plant broadcasts its state to all controllers within range. Ideally, only the controller with the best link quality sends back the corresponding control input. To this end, a contention resolution phase is introduced, which assigns each controller to a tournament round. Controllers with a good link quality get assigned to earlier rounds, which increases the probability that a controller with a good link quality sends back the control input. This strategy increases the flexibility of the control system, since the plant is no longer tied to a single controller. Furthermore, by exploiting the diversity of the wireless links the reliability of the communication is improved.

The broadcast control system can be motivated by a mobile robotics use case. Consider an automated factory in which mobile robots move around to perform tasks at different locations. These robots typically receive tasks from a centralized coordinator, where each robot executes the tasks fully autonomously. This requires every robot to be equipped with a complex control system, which increases the cost of the robot. One possible way to reduce this cost is to offload some of the control computations from the mobile robots to controllers deployed around the factory, where the measurements and control inputs are communicated over a wireless network. These wireless controllers can be more powerful and cost effective, since they do not have the same constraints as the embedded controllers. Furthermore, they enable the design of more complex control algorithms, while more efficient resource sharing can be achieved by allowing multiple robots to simultaneously use the same wireless controller. However, introducing wireless links into the control loop can have a negative impact on the control performance, particularly when the robots are mobile. The broadcast control system solution can be used to combat the negative effects of the wireless channel, while also providing the required flexibility for the robots to move around. Furthermore, the distributed nature of the control system makes the system more resilient to controller failures.

A different solution to the fixed controller assignment is proposed in [2], where the safety of the controller handover procedure is investigated. This procedure allows the control process to be handed over from one point of computation to

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another during runtime. A similar approach is taken in [3], where a real-time middleware is introduced that allows for the upgrade and migration of controllers. The problem of controller placement has been investigated in [4] and [5]. A control architecture is proposed in which the sensor and actuator communicate over a string of nodes separated by erasure channels. In [5] a scenario is investigated where each node can be either a control node or a relay node. The role of each node depends on the transmission outcomes, so the control architecture can adapt to changing network conditions. This problem is different from the problem studied in this paper, since the medium access issues are not investigated and the nodes need to be arranged in a string. The problem of medium access in networked control systems has been studied extensively [6] [7] [8] [9]. A medium access protocol with attention-based tournaments is introduced in [9], where dynamic priorities are assigned to data packets based on the attention they require. The contention resolution phase introduced in this paper shares similarities with this approach. The key difference is that in this paper the priorities are based on the link qualities, resulting in a contention resolution strategy based on thresholds. The term broadcast control is used in a multi-agent system setting in [10]. In this setting individual agents are not able to communicate with each other, but rely on a centralized controller to broadcast control commands to all agents.

In this paper the design of the contention resolution for a single plant is addressed, which is characterized by an important performance trade-off. On the one hand, introducing more controllers increases the diversity of the wireless links, which makes the system more robust. On the other hand, the introduction of more controllers increases the collision probability. The main contributions of this paper are: (i) the performance analysis of the contention resolution phase, (ii) the proposal of an effective optimization algorithm to maximize this performance, and (iii) the convergence proof of the proposed algorithm.

The remainder of this paper is organized as follows. Section II describes the considered problem in more detail. In Section III the optimization strategy to maximize the success probability of the contention resolution phase is analyzed. The performance of the proposed system is numerically evaluated in Section IV. Section V presents the conclusions and provides suggestions for future work.

II. Problem Formulation

Consider a broadcast control system consisting of a plant $\mathcal{P}$ and N controllers denoted by $\mathcal{C}_i, i \in \{1, \ldots, N\}$. Figure 1 illustrates the system when considering six controllers. The plant periodically broadcasts its state x_k to all N controllers. During a contention resolution phase each controller makes a local decision to send back the control input u_k. If the contention resolution is successful one of the controllers sends back the control input. In the remainder of this section the control system, communication system, and contention resolution are discussed in more detail, after which the problem statement is presented.

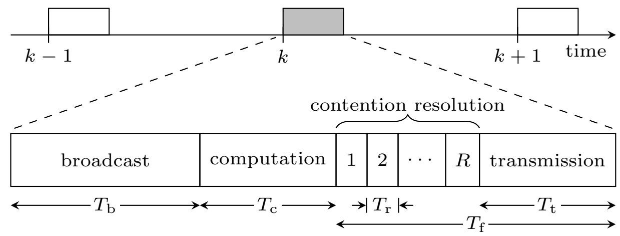

Figure 2. The frame structure allows the plant and controllers to communicate during each time step.

A. Control System

The plant is modeled as a continuous time linear system given by

$$\dot{x}(t) = A_\text{p} x(t) + B_\text{p} u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input, A_p is the system matrix, and B_p is the input matrix. Let the total delay from the start of the broadcast until the plant receives the control input be denoted by T. This delay is generally time-varying, but in this paper it is considered to be constant due to the design of the contention resolution phase. Let T_s denote the sampling period, which is assumed to be larger than T. Sampling using zero-order hold gives the following discrete time system

$$x_{k+1} = A x_k + B_0 u_k + B_1 u_{k-1}, \tag{2}$$

with $A = e^{A_\text{p} T_\text{s}}$, $B_0 = \int_0^{T_\text{s}-T} e^{A_\text{p} s}\, ds\, B_\text{p}$, $B_1 = \int_{T_\text{s}-T}^{T_\text{s}} e^{A_\text{p} s}\, ds\, B_\text{p}$, where k is the discrete time index [11].

Depending on the communication link quality, a subset of controllers will receive the broadcast from the plant. Each of these controllers makes a local decision to send back the corresponding control input. The successful reception of the control input by the plant depends on the quality of the communication channels, and the local decisions of the controllers. The control input is not received successfully if the packet collides, is lost, or is not sent by any of the controllers. Assume all controllers employ the same state feedback control law given by

$$u_k = -\theta_k K x_k, \tag{3}$$

where K is the feedback gain, and θ_k is a Bernoulli random variable representing the control packet reception. Let the distribution of θ_k be defined by $\Pr\{\theta_k = 0\} = 1 - p_\text{s}$ and $\Pr\{\theta_k = 1\} = p_\text{s}$, where p_s is the probability of a control packet being successfully received by the plant.

B. Communication System

Each time step k marks the start of a frame during which the plant and controllers can communicate over a time-varying wireless channel, as shown in Figure 2. In the remainder of this paper the index k will be used to refer to both the time step and its associated frame. During each frame the channel is modeled as a block fading channel, so the channel gain is assumed to be constant during each frame.

The channel gain is assumed to be independent and identically distributed between controllers and between frames, due to the large time duration between time steps. The channel gain is modeled by the path loss and the fading. The path loss h_{PL}^2 is assumed to be constant due to the fixed distance between the nodes. The random fading experienced by controller i in time step k is given by $h_{i,k}^2$, where the magnitude of $h_{i,k}$ is assumed to be a Rayleigh distributed process with unit mean. The instantaneous signal-to-noise ratio (SNR) of the channel between the plant and controller i in frame k is given by

$$\gamma_{i,k} = \frac{h_{\text{PL}}^2 h_{i,k}^2 P_{\text{tx}}}{\sigma^2}, \tag{4}$$

where P_{tx} is the fixed transmit power and σ^2 is the power of the noise process. Furthermore, the average SNR is defined as $\bar{\gamma} = h_{\text{PL}}^2 P_{\text{tx}}/\sigma^2$. The instantaneous SNR can then be written as $\gamma_{i,k} = \bar{\gamma} h_{i,k}^2$, which is exponentially distributed. Based on the instantaneous SNR a varying amount of information can be exchanged between the plant and the controller. A packet transmission of l bits in T_0 seconds over the wireless channel results in a transmission rate of $\rho = l/T_0$. Depending on the instantaneous SNR $\gamma_{i,k}$ of controller i in time step k the channel capacity is given by

$$c_{i,k} = W \log_2 \left(1 + \gamma_{i,k}\right), \tag{5}$$

where W is the bandwidth in Hz. Equation (5) provides an upper bound on the information rate at which the transmission can be performed successfully. Error free transmissions can be guaranteed only by choosing a bit rate lower than the channel capacity. Any choice of a bit rate higher than the channel capacity will imply unsuccessful transmission. This results in a required SNR threshold of $2^{\rho/W} - 1$ for which the packet is successful received.

C. Contention Resolution

Each frame is divided into a broadcast, a computation, a contention resolution, and a transmission phase, as shown in Figure 2. Furthermore, the contention resolution and transmission phases are collectively called the feedback phase. During the broadcast phase of duration T_{b} the plant broadcasts a packet containing the state of the plant with a size of l_{b} bits. This results in a required transmission rate of $\rho_{\text{b}} = l_{\text{b}}/T_{\text{b}}$, so the SNR threshold is given by $\Gamma_{\text{b}} = 2^{\rho_{\text{b}}/W} - 1$. During the computation phase of duration T_{c} the controllers compute the control input based on the received plant state. The contention resolution phase is used to make a decision about which controller sends back the control input to the plant, while the transmission phase is used for finishing the control packet transmission. The contention resolution phase is divided into R tournament rounds of duration T_{r}. If a tournament round is successful the assigned controller can start transmitting for a fixed duration of T_{t}, so the total delay is time-varying. In this paper, however, the total delay is assumed to be equal to the worst-case delay given by

$$T = T_{\text{b}} + T_{\text{c}} + T_{\text{f}}, \tag{6}$$

where the duration of the feedback phase is given by $T_{\text{f}} = RT_{\text{r}} + T_{\text{t}}$. Given a control packet size of l_{t}, a transmission rate of $\rho_{\text{t}} = l_{\text{t}}/T_{\text{t}}$ is required during the transmission phase for a successful transmission. As before, the required SNR threshold is given by $\Gamma_{\text{t}} = 2^{\rho_{\text{t}}/W} - 1$.

The design of the contention resolution phase is key to the performance of the broadcast control system. It aims to exploit the controller with the best instantaneous channel, while reducing the chances of contention. The instantaneous SNR $\gamma_{i,k}$ of the channel is measured during the broadcast phase. By assuming channel reciprocity this instantaneous SNR is used to assigned each controller i to a tournament round r. Let $\Gamma_{\text{max}} \geq \Gamma_1 \geq \Gamma_2 \geq \cdots \geq \Gamma_{R-1} \geq \Gamma_R \geq \Gamma_{\text{min}}$ denote these SNR thresholds, so the rounds are defined by the intervals $[\Gamma_{\text{max}}, \Gamma_1], [\Gamma_1, \Gamma_2], \ldots, [\Gamma_{R-1}, \Gamma_R]$. The threshold Γ_{min} represents the minimum SNR for the broadcast and control packets to be successfully received, while the upper bound Γ_{max} is introduced due to physical limitations. Let the threshold vector be defined by $\Gamma = [\Gamma_1, \ldots, \Gamma_R]$. All controllers monitor the channel to check for active transmissions. Controller i starts transmitting at the start of round r if $\gamma_{i,k}$ is inside the interval of the round and no transmissions have been detected prior to round r. If none of the controllers start transmitting during the contention resolution phase, a fixed controller is used for transmission. Controller i receives the plant broadcast in frame k only if $\gamma_{i,k} > \Gamma_{\text{b}}$, while a packet from the controller to the plant can only be received when $\gamma_{i,k} > \Gamma_{\text{t}}$. By requiring a minimum SNR

$$\Gamma_{\text{min}} = \max\{\Gamma_{\text{b}}, \Gamma_{\text{t}}\}, \tag{7}$$

both the broadcast and transmission phases are successful when the SNR is above Γ_{min}. This implies that the frame is not successful only when a packet collision occurs or when a fixed controller is chosen with an SNR below Γ_{min}.

D. Problem Statement

The performance of the control system greatly depends on the performance of the contention resolution. The main aim of this paper is therefore to optimize the success probability p_{s}, which is defined as the probability that the broadcast, contention resolution, and transmission phases are all successful. This aim is pursued by three objectives. The first objective is to derive an analytical expression for the success probability and formulate a suitable optimization problem. The second objective is to find an effective optimization algorithm to solve this problem. Furthermore, the convergence of this algorithm needs to be shown. The third objective is to numerically evaluate the performance of this system in terms of the success probability and the control performance. Of particular interest is the scaling behavior when the number of controllers increases.

III. Performance Optimization

In this section an optimization strategy is presented to optimize the contention resolution thresholds. First, an analytical expression for the success probability is derived. Second, the coordinate descent algorithm is shown to be a suitable candidate to maximize this probability, since it can exploit the subproblem structure of the objective function. Third, this structure is used to prove the convergence of the algorithm. Finally, the problem of optimizing the number of rounds is investigated.

A. Success Probability

A transmission starting in the first tournament round is successful if only one controller is assigned to this round and the remaining controllers are assigned to rounds $\{2, \ldots, R\}$. Thus, the success probability in the first round is given by

$$p_{\text{s},1} = N\left(F(\Gamma_{\text{max}}) - F(\Gamma_1)\right) F^{N-1}(\Gamma_1), \tag{8}$$

where $F(\cdot)$ denotes the cumulative distribution function (CDF) of $\gamma_{i,k}$, and $F^N(\cdot)$ is used to denote $(F(\cdot))^N$. In order for a transmission starting in tournament round $r \in \{2, \ldots, R\}$ to be successful two conditions need to be met: (i) no transmissions have taken place in previous tournament rounds, and (ii) only one controller is assigned to tournament round r. This results in the following expression for a successful transmission in tournament round $r \in \{2, \ldots, R\}$

$$p_{\text{s},r} = N\left(F(\Gamma_{r-1}) - F(\Gamma_r)\right) F^{N-1}(\Gamma_r). \tag{9}$$

If none of the controllers transmit during the tournament round, a fixed controller is chosen a priori for transmission. However, this fixed controller might not have an SNR above the threshold Γ_{min}, which results in the following success probability

$$p_{\text{s,f}} = \left(F(\Gamma_R) - F(\Gamma_{\text{min}})\right) F^{N-1}(\Gamma_R). \tag{10}$$

The overall success probability of the packet exchange during each frame is then given by

$$p_{\text{s}} = \sum_{r=1}^{R} p_{\text{s},r} + p_{\text{s,f}}. \tag{11}$$

In these expressions it is assumed that a contention resolution phase exists, which is not the case if $R = 0$ or $N = 1$. In both cases a fixed controller is chosen, so the success probability is given by $p_{\text{s}} = F(\Gamma_{\text{min}})$. In the remainder of this paper it is therefore assumed that $R \geq 1$ and $N \geq 2$.

B. Threshold Optimization

The thresholds Γ can be optimized by solving the following optimization problem

$$\begin{aligned} \text{maximize} \quad & p_{\text{s}}(\Gamma) \\ \text{subject to} \quad & \Gamma_{r-1} \geq \Gamma_r, \forall r \in \{1, \ldots, R\}, \\ & \Gamma_R \geq \Gamma_{\text{min}}, \\ & \Gamma_1 \leq \Gamma_{\text{max}}. \end{aligned} \tag{P1}$$

The constraints of this problem can be simplified in order to find an effective optimization strategy. The first constraint is not strictly necessary, since the resulting SNR thresholds can be sorted to satisfy this constraint. Furthermore, maximizing the objective function naturally leads to the constraint being satisfied. Next, let $v = \Gamma/\bar{\gamma}$ denote the normalized threshold vector, while the normalized upper and lower limits are given by $v_{\text{min}} = \Gamma_{\text{min}}/\bar{\gamma}$ and $v_{\text{max}} = \Gamma_{\text{max}}/\bar{\gamma}$. The second and third constraint can then be expressed by requiring that $v_r \in \mathcal{V}$ for all $r \in \{1, \ldots, R\}$ where $\mathcal{V} = [v_{\text{min}}, v_{\text{max}}]$, which can be written as $v \in \mathcal{V}^R$. Finally, the objective function can be rewritten by using the CDF of the exponential distribution and flipping the sign, which gives

Algorithm 1: Coordinate Descent

input : $f : \mathbb{R}_+^R \to [0, 1]$, objective function
$v^{(0)} \in \mathcal{V}^R$, initial thresholds
output : v^*, optimal thresholds

$j \leftarrow 0$
repeat
  for $r = 1$ **to** R **do**
    $v_r^{(j+1)} \leftarrow \underset{w \in \mathcal{V}}{\arg\min}\ f(v_1^{(j+1)}, \ldots, v_{r-1}^{(j+1)}, w, v_{r+1}^{(j)}, \ldots, v_R^{(j)})$
  end
  $j \leftarrow j + 1$
until convergence
return $v^{(j)}$

$$f(v) = \sum_{r=1}^{R} f_r(v) + f_{\text{f}}(v), \tag{12}$$

where

$$f_1(v) = N\left(e^{-v_{\text{max}}} - e^{-v_1}\right)\left(1 - e^{-v_1}\right)^{N-1}, \tag{13}$$

$$f_r(v) = N\left(e^{-v_{r-1}} - e^{-v_r}\right)\left(1 - e^{-v_r}\right)^{N-1}, \tag{14}$$

$$f_{\text{f}}(v) = \left(e^{-v_R} - e^{-v_{\text{min}}}\right)\left(1 - e^{-v_R}\right)^{N-1}. \tag{15}$$

So the simplified optimization problem is given by

$$\begin{aligned} \text{minimize} \quad & f(v) \\ \text{subject to} \quad & v \in \mathcal{V}^R. \end{aligned} \tag{P2}$$

This optimization problem is non-convex [12], which means that an effective optimization algorithm needs to be found. One approach would be to exploit the subproblem structure of the objective function. An algorithm that accomplishes this is called the coordinate descent algorithm [13] [14]. The cyclic coordinate descent algorithm is given by Algorithm 1, where $\mathbb{R}_+$ denotes the set of positive real numbers. This algorithm minimizes the objective function successively along coordinate directions. The coordinate descent algorithm thus generates a sequence $\{v^{(j)}\}$ with $v^{(j)} = (v_1^{(j)}, \ldots, v_R^{(j)})$, where j denotes the iteration number.

C. Convergence

Assume that the sequence generated by the coordinate descent algorithm has limit points. In the following proposition the results in [15] are used to show that these limit points are critical points of Problem (P2), where critical points are defined as points where the gradient of the objective function is zero.

Proposition 1. *Assume that the coordinate descent algorithm applied to Problem* (P2) *has limit points. Then, every limit* *point is a critical point of the problem if the function*

$$g(v_r) = (e^{-v_{r-1}} - e^{-v_r})(1 - e^{-v_r})^M + e^{-v_r}(1 - e^{-v_{r+1}})^M \tag{16}$$

is strictly quasiconvex on domain $\mathcal{V}$, where $v_{r-1} \in \mathcal{V}$, $v_{r+1} \in \mathcal{V}$, and $M = N - 1 \in \mathbb{N}_{\geq 1}$.

Proof. For $R \leq 2$ the convexity assumption given by Equation (16) is not necessary to prove that every limit point is a critical point of the problem, which is shown in [15]. However, for $R > 2$ Proposition 5 in [15] states that the coordinate descent algorithm converges to a critical point of Problem (P2) if $f(v)$ is componentwise strictly quasiconvex with respect to $R-2$ components of v. The objective function with respect to component v_r for $r \in \{2, \ldots, R-1\}$ can be written as

$$f(v_r) = Ng(v_r) + q(v_1, \ldots, v_{r-1}, v_{r+1}, \ldots, v_R), \tag{17}$$

where the function q consists of all the terms not depending on v_r. From the componentwise strictly quasiconvex definition it follows that $g(v_r)$ needs to be strictly quasiconvex on domain $\mathcal{V}$ for $v_{r-1} \in \mathcal{V}, v_{r+1} \in \mathcal{V}$ [15] [16]. □

What remains to be shown is the strict quasiconvexity of $g(v_r)$, which is proven in the following proposition for the case where $\mathcal{V} = \mathbb{R}_+$.

Proposition 2. *The function*

$$g(v_r) = (e^{-v_{r-1}} - e^{-v_r})(1 - e^{-v_r})^M + e^{-v_r}(1 - e^{-v_{r+1}})^M \tag{18}$$

is strictly quasiconvex on domain $\mathbb{R}_+$, where $v_{r-1} \in \mathbb{R}_+, v_{r+1} \in \mathbb{R}_+$, and $M = N - 1 \in \mathbb{N}_{\geq 1}$.

Proof. Consider the case where $v_{r-1} < v_{r+1}$, and the derivative of $g(v_r)$ with respect to v_r is given by

$$g'(v_r) = e^{-v_r}(1 - e^{-v_r})^M - e^{-v_r}(1 - e^{-v_{r+1}})^M + Me^{-v_r}(e^{-v_{r-1}} - e^{-v_r})(1 - e^{-v_r})^{M-1}. \tag{19}$$

Considering v_r on the interval $(0, v_{r-1})$ it holds that $g'(v_r) < 0$, since $e^{-v_r} > e^{-v_{r-1}}$ and $(1 - e^{-v_r}) < (1 - e^{-v_{r+1}})$. Similarly, considering v_r on the interval (v_{r+1}, ∞) it holds that $g'(v_r) > 0$. The function $g(v_r)$ is therefore strictly decreasing on the interval $(0, v_{r-1})$, and strictly increasing on the interval (v_{r+1}, ∞). Next, consider v_r on the interval $[v_{r-1}, v_{r+1}]$ and note that $g(v_{r-1}) = g(v_{r+1})$, and it holds that $g(v_r) < g(v_{r-1})$. From Rolle's Theorem it follows that there must exist at least one point $v_r^* \in (v_{r-1}, v_{r+1})$ for which $g'(v_r^*) = 0$ [17]. In the following it is shown that v_r^* is a unique minimum. Consider the condition $g'(v_r^* + \varepsilon) \geq 0$ given by

$$M\left(e^{-v_{r-1}} - e^{-(v_r^*+\varepsilon)}\right)\left(1 - e^{-(v_r^*+\varepsilon)}\right)^{M-1} + \left(1 - e^{-(v_r^*+\varepsilon)}\right)^M \geq \left(1 - e^{-v_{r+1}}\right)^M, \tag{20}$$

where strict equality is achieved for $\varepsilon = 0$. Strict inequality is achieved for $\varepsilon > 0$, since the left-hand side is strictly positive and strictly increasing in ε. A similar argument can be made for $g'(v_r^* + \varepsilon) \leq 0$, where strict inequality is achieved for $\varepsilon < 0$. Thus $g(v_r^* + \varepsilon)$ is strictly increasing for $\varepsilon > 0$ and strictly decreasing for $\varepsilon < 0$, so v_r^* is a unique minimum. The function is therefore strictly quasiconvex on the interval $[v_{r-1}, v_{r+1}]$, and consequently also on the interval $(0, \infty)$. The same reasoning can be used to show that the function is strictly quasiconvex when $v_{r-1} > v_{r+1}$. In the final case where $v_{r-1} = v_{r+1}$ the minimum is obtained at $v_r = v_{r-1}$, so it holds that $g'(v_{r-1}) = 0$. Furthermore, using a similar argument as before it is clear that the $g(v_r)$ is strictly increasing for $v_r > v_{r-1}$ and strictly decreasing for $v_r < v_{r-1}$. The function $g(v_r)$ is thus quasi-convex on the domain $\mathbb{R}_+$. □

The coordinate descent algorithm therefore converges and can be used to compute an optimal success probability vector Γ^* and the resulting optimal success probability p_{s}^*.

D. Rounds Optimization

In the presented threshold optimization problem the number of rounds R is an important design choice. Increasing the number of rounds generally reduces the contention at the cost of a higher delay. In the remainder of this section the problem of optimizing the number of rounds is investigated.

Consider the case where Γ_{b} and Γ_{t} can be chosen freely. The optimal choice is then given by $\Gamma_{\mathrm{b}} = \Gamma_{\mathrm{t}}$, since the minimum SNR is defined as $\Gamma_{\mathrm{min}} = \max\{\Gamma_{\mathrm{b}}, \Gamma_{\mathrm{t}}\}$. Assume further that the packet sizes of the broadcast and transmission phases are given by $l = l_{\mathrm{b}} = l_{\mathrm{t}}$, from which it follows that $T_{\mathrm{b}} = T_{\mathrm{t}}$. Furthermore, the feedback phase has a fixed duration of T_{f}, so the transmission duration can be computed as $T_{\mathrm{t}} = T_{\mathrm{f}} - RT_{\mathrm{r}}$. From this it follows that the minimum SNR is given by

$$\Gamma_{\mathrm{min}} = 2^{-\frac{l}{W(T_{\mathrm{f}} - RT_{\mathrm{r}})}} - 1, \tag{21}$$

which means that the resulting success probability p_{s}^* given Γ_{min} can be optimized with respect to $R \in \{1, \ldots, R_{\mathrm{max}}\}$. The maximum number of rounds R_{max} can be computed as the maximum R for which $T_{\mathrm{t}} > 0$. When choosing R it is clear that there exists a trade-off. On the one hand, increasing the number of rounds will reduce the number of collisions, which increases the overall success probability. On the other hand, increasing the number of rounds will decrease the transmission duration T_{t}, which reduces the overall success probability. The optimal solution p_{s}^* is therefore unimodal in $R \in \{1, \ldots, R_{\mathrm{max}}\}$, and the optimal number of rounds R^* can therefore be found using an appropriate search algorithm.

IV. Numerical Evaluation

In this section the performance of the proposed broadcast control system is numerically evaluated. As a reference the performance of using a fixed controller is considered. The system is analyzed in a low-power wireless setting, such as the IEEE 802.15.4 standard [18]. Assume the bandwidth is given by $W = 2\,\mathrm{MHz}$. An SNR gap of $8\,\mathrm{dB}$ is introduced

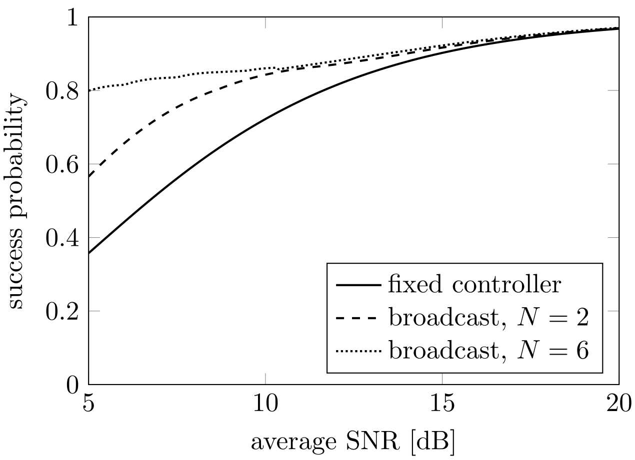

Figure 3. Performance of the broadcast control system for different number of controllers when optimizing the number of rounds, compared to the performance of a fixed controller.

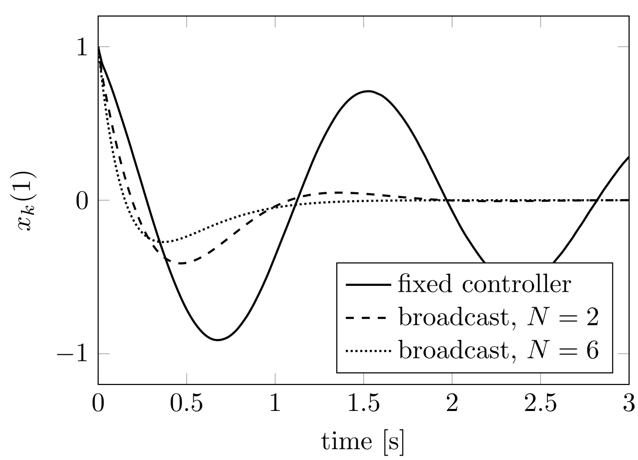

Figure 4. Averaged initial value response using either a fixed controller or the broadcast control system for different number of controllers. The responses are obtained by averaging over 10^6 Monte Carlo simulations.

which reduces the average SNR to model the channel estimation, modulation, and encoding losses. Furthermore, the maximum SNR is given by $\Gamma_{\max} = 50\,\mathrm{dB}$. The packet size is assumed to be the same for both the broadcast and transmission phase, and is chosen to be $300\,\mathrm{bytes}$. The duration of a tournament round is assumed to be $T_\mathrm{r} = 0.3\,\mathrm{ms}$, which is larger than the turnaround time of $192\,\mu\mathrm{s}$ for the IEEE 802.15.4 standard. The duration of the broadcast phase is chosen to be $T_\mathrm{b} = 2\,\mathrm{ms}$, and the computation time T_c is assumed to be zero. Finally, in the static scenario the links are assumed to be symmetric, so all links have the same average SNR.

Consider the scenario where the number of rounds is optimized according to Section III-D. To this end, let the duration of the control phase be $T_\mathrm{f} = 4\,\mathrm{ms}$, so the total delay T remains constant. Figure 3 shows the success probability p_s of the system as a function of the average SNR $\bar{\gamma}$. It is clear that the success probability can be improved significantly when increasing the number of controllers by exploiting the diversity of the wireless links. This is particularly beneficial when the SNR is low, since the performance gain by the added diversity outweighs the downside of more contention. Note that the non-smoothness is caused by the jumps in the optimal number of rounds.

Next, the performance of the control system is investigated when using the broadcast control strategy. Consider the following unstable second-order system given by

$$A_\mathrm{p} = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix},\ B_\mathrm{p} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{22}$$

which are the matrices of the plant model given by Equation (2). A state feedback controller is used given by $K = [15\ 40]$, and the initial state is given by $x_0 = [1\ 0]^\top$. The system is sampled with sampling time $T_\mathrm{s} = 20\,\mathrm{ms}$, the transmission time is given by $T_\mathrm{t} = 3\,\mathrm{ms}$, the number of rounds is set to $R = 10$, and the average SNR is chosen to be $\bar{\gamma} = 6\,\mathrm{dB}$. Figure 4 shows the averaged initial value response of the first state variable $x_k(1)$. It is clear that the performance of the control system suffers by using a fixed controller when the SNR is low, which is indicated by the slow settling time. By adding more controllers and employing the broadcast control strategy the control performance can be improved significantly.

The optimization strategy in this paper assumes a static scenario, where the average SNR remains constant and is the same for all controllers. When the plant is mobile the scenario becomes more involved, since the average SNR becomes time-varying and will not be the same for each controller. Extending the current optimization problem to handle the mobility of the plant is left for future work. However, using the static optimization problem for the mobile scenario can still result in a considerable performance gain. Consider the hexagon configuration shown in Figure 1. Each controller is assumed to have a distance of $50\,\mathrm{m}$ to the center of the hexagon. The plant moves inside these controllers along a circular trajectory around the origin with a radius of $40\,\mathrm{m}$. The average SNR is now dependent on the distance d between the plant and the controller, and is assumed to be given by $\bar{\gamma}_\mathrm{m}(d) = \left(\frac{d_0}{d}\right)^\alpha \bar{\gamma}_0$, where $\alpha = 2$ is the path loss coefficient at the reference distance $d_0 = 10$, and $\bar{\gamma}_0 = 15\,\mathrm{dB}$ is the reference SNR. However, a fixed $\bar{\gamma}$ is needed to compute the optimal thresholds, which now needs to be chosen as a design parameter. Figure 5 shows the success probability as a function of the position of the plant along the circular trajectory, where the success probability is computed using Monte Carlo simulations. The performance of the broadcast control system is shown for two choices of $\bar{\gamma}$. The performance is compared to using a fixed controller, which is the gray controller in Figure 1. It is clear that the fixed controller performs poorly when the plant is moving away from the controller. On average the broadcast control system performs better, since the controller assignment can vary every time step. However, the choice of $\bar{\gamma}$ is not trivial and requires further investigation.

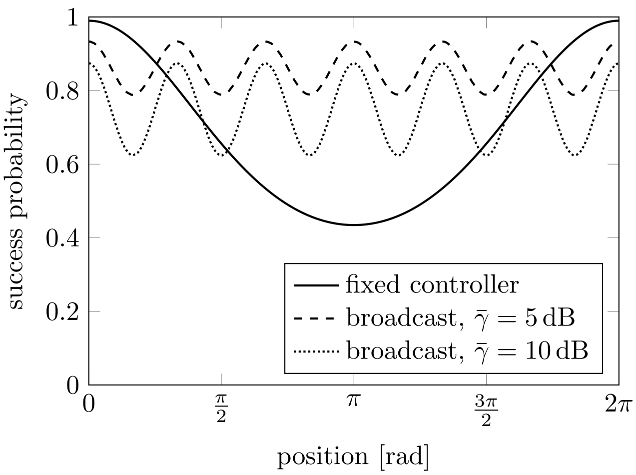

Figure 5. Performance of a plant moving along a circle using either a fixed controller or the broadcast control system for different choices of the average SNR.

V. Conclusions and Future Work

In this paper the broadcast control system was introduced. In this system the plant broadcasts its state to a group of controllers, where each controller makes a local decision to send back the corresponding control action. A contention resolution phase was introduced to reduce the collision probability. An optimization problem was proposed that optimizes the success probability by choosing the thresholds of the contention resolution phase. The coordinate descent algorithm was chosen as a suitable optimization algorithm, which was proven to converge to a critical point. Finally, numerical results showed that the proposed system performs well in comparison to choosing a fixed controller, in particular when the link quality is poor.

Many possibilities for future work exist. One interesting scenario is to consider multiple plants that can share multiple controllers. In order to model this, the results in this paper need to be extended to take into account packet collisions between multiple control loops. Furthermore, the design of the contention resolution thresholds in the mobile scenario is not trivial, and needs to be investigated further. The optimization problem in this paper could be modified to optimize the control performance directly, which would give more insights into the trade-off between success probability and delay.

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