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Simple conditions for L_2 stability and stabilization of networked control systems

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Abstract: The stability analysis and stabilization of networked control systems subject to data loss and time-varying transmission delays are explored. The stability result is based on quadratic separation and operator theory, which allows to capture the above phenomena into the single formalism of aperiodic sampling. The obtained stability condition is expressed through an LMI. The stabilization problem is a bit more involved due to the inherent structure of the obtained LMI. An approximation (dilation) is then proposed to obtain a more tractable LMI for stabilization. Several examples illustrate the effectiveness of the proposed approach.

Keywords: Networked Control Systems; Quadratic Separation; LMIs

1. INTRODUCTION

Networked control systems (NCS) [Hespanha et al., 2007, Heemels et al., 2009] is a wide class of physical systems controlled or interconnected through a network. The network has an important influence on the overall systems behavior by inducing delays, data loss and other constraints, like data and transmission channels with finite capacity. It is well-known that the network may deteriorate the system performance, so it should be considered in the analysis and the controller design.

Remote control of processes through wireless networks (Fig. 1) is a very important problem. In such a set-up, the controller receives data from the sensors through a network and sends back the control input. Such a problem has been widely studied in the literature, for instance in [Yu et al., 2004, Yue et al., 2004, Naghshtabrizi and Hespanha, 2006, Hespanha et al., 2007, Cloosterman et al., 2009, Heemels et al., 2009].

In this paper, the stability and L_2-gain analysis of linear systems remotely controlled by a state-feedback controller is considered. From the nominal expression of the sampled-data control law, the network effects are successively added to the control law in order to build an accurate model for the closed-loop system. It turns out that the control law behaves as if the 'sampling' was asynchronous and bounded from above by $\tau_{max} + (m+1)T_{max}$ where τ_{max}, m and T_{max} are the maximal propagation delay value, the number of consecutive dropouts and the maximal sampling period of the controller respectively. Problems related to asynchronous sampling have been studied in several papers with many different approaches: time-delay systems [Yu et al., 2004, Fridman et al., 2004], impulsive systems [Naghshtabrizi et al., 2008, Seuret, 2009], sampled-data techniques [Mirkin, 2007], robust techniques [Fujioka, 2009]. Robust techniques involving Integral Quadratic Constraints (IQCs) coupled with well-posedness techniques will be considered in this paper. To this aim, the system is then interpreted as an interconnection of uncertain and dynamical operators with an implicit algebraic expression. In order to consider the operators accurately for the stability analysis of the NCS, IQCs [Rantzer and Megretski, 1997] are employed to characterize them through their input/output behavior. The stability conditions, expressed as Linear Matrix Inequalities (LMIs), are then obtained using recent results on quadratic separation [Goh and Safonov, 1995, Iwasaki and Hara, 1998, Peaucelle et al., 2007]. Finally, an LMI-based stabilization result is obtained from a dilation of the stability conditions.

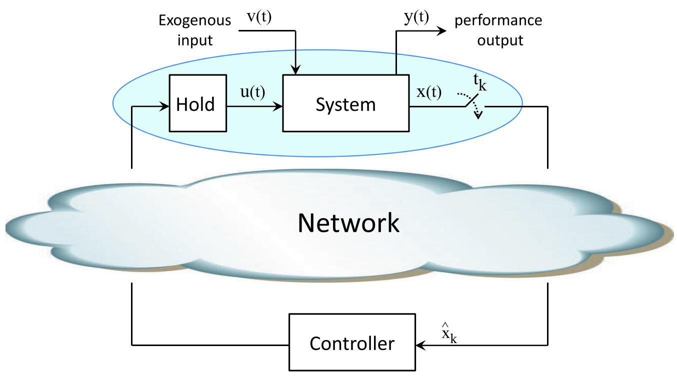

Fig. 1. Networked control system

The paper is structured as follows, Section 2 introduces preliminary results on quadratic separation, IQCs and the closed-loop system expression. Section 3 is devoted to the stability analysis of the NCS and some illustrative examples are given. Finally, in Section 4, a solution to the stabilization problem is derived.

Notations: Throughout the paper, the following notations are used. The set of L_2^n consists of all measurable functions $f : \mathbb{R}^+ \to \mathbb{R}^n$ such that the L_2 norm $\|f\|_{L_2} = \left(\int_0^\infty f^*(t)f(t)dt\right)^{1/2}$ is finite. When no ambiguity may occur, the superscript n will be omitted. The truncation operator $\mathbb{P}_T$ is defined as $\mathbb{P}_T(f) = f_T$ with $f_T(t) = f(t)$ when $t \leq T$ and 0 otherwise. The set L_{2e}^n denotes the extended set of L_2^n which consists of the functions whose time truncation lies in L_2^n. For two symmetric matrices, A and B, $A \succ (\succeq)\ B$ means that $A - B$ is positive (semi)definite. $M \in \mathbb{S}_{++}^n$ means that the matrix M an $n \times n$ symmetric positive definite matrix. $\mathsf{1_n}$ and $\mathsf{0_{m\times n}}$ denote the identity matrix of size n and zero matrix of size $m \times n$ respectively. $A_\perp$ is a full rank matrix spanning the null-space of A, i.e. $AA_\perp = \mathsf{0}$. For a square matrix A, $A^{\mathcal{S}}$ stands for the sum $A + A^T$.

2. PRELIMINARIES

2.1 Quadratic separation:

The original NCS in Fig. 1 can be interpreted as the interconnection in Fig. 2. This section provides a fundamental result on quadratic separation, as described in [Iwasaki and Hara, 1998, Peaucelle et al., 2007, Ariba and Gouaisbaut, 2009], which will be used as a basis for the stability analysis.

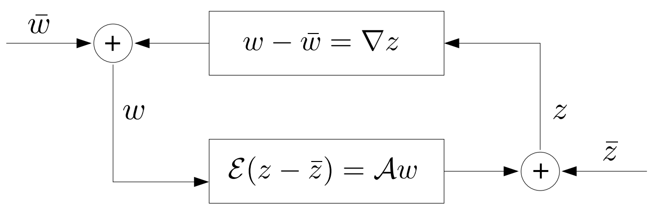

Fig. 2. Implicit feedback system.

Consider the feedback system in Fig. 2 where $\mathcal{E} \in \mathbb{R}^{n_c \times n_z}$, $\mathcal{A} \in \mathbb{R}^{n_c \times n_w}$ are constant matrices with $n_w = \dim(w)$, $n_z = \dim(z)$ and n_c is any integer greater than 0. The matrix ∇ is an $n_w \times n_z$ matrix of operators. Then we have the following theorem:

Theorem 1. The interconnected system of Fig. 2 is well-posed if there exists a symmetric matrix Θ satisfying both conditions

$$[\,\mathcal{E} \quad -\mathcal{A}\,]_\perp^T \Theta [\,\mathcal{E} \quad -\mathcal{A}\,]_\perp \succ \mathsf{0} \tag{1}$$

and

$$\langle \begin{bmatrix} \mathsf{1} \\ \mathbb{P}_T\nabla \end{bmatrix} u_T, \Theta \begin{bmatrix} \mathsf{1} \\ \mathbb{P}_T\nabla \end{bmatrix} u_T \rangle \leq 0 \tag{2}$$

for all $u \in L_{2e}$ and all $T > 0$.

Proof : The proof can be found in [Ariba et al., 2008, Peaucelle et al., 2009]. $\diamondsuit$

The above result considers the well-posedness of the interconnection, that is, that the loop signals w and z are uniquely defined by the input signals $\bar{w}$ and $\bar{z}$. When the operator ∇ consists of a dynamic operator, for instance an integral operator, then the interconnection becomes a dynamical system and in such case, well-posedness can be made equivalent to stability provided that the structure of the separator Θ is chosen accordingly. For more details, see e.g. [Iwasaki and Hara, 1998].

2.2 Networked control system model:

Let us consider the following LTI continuous-time process

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ev(t) \\ y(t) &= Cx(t) + Du(t) + Fv(t) \\ x(0) &= x_0 \end{aligned} \tag{3}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $v \in \mathbb{R}^p$, $y \in \mathbb{R}^q$ and $x_0 \in \mathbb{R}^n$ are the system state, the control input, the exogenous input, the controlled output and the initial condition.

Ignoring first the network presence, a sampling-based control law of the form

$$u(t) = Kx(t_k),\ t \in [t_k, t_{k+1}) \tag{4}$$

is considered where $\{t_k\}_{k\in\mathbb{N}}$ is an increasing sequence of time-instants, with not necessarily constant increments but bounded from above by T_{max}. The time varying propagation delay $\tau(t)$ is assumed to belong to $[0, \tau_{max}]$. It is also assumed that the process is driven by an event-based system [1], hence data loss can be easily incorporated in the control input simply by noting that a dropout will be reflected in an extension of the holding duration of the actuator input. This yields the following control input expression:

$$\begin{aligned} u(t) &= Kx(t_k) \\ t &\in [t_k + \tau_k, t_{k+1} + \tau_{k+1}) \\ t_{k+1} - t_k &\leq (1+m)T_{max} \\ \tau_k &\in [0, \tau_{max}] \end{aligned} \tag{5}$$

where $\tau_k = \tau(t_k)$, $k \in \mathbb{N}$ and m is the number of consecutive dropouts.

Remark 1. To derive the above result, we have exploited the fact that the controller is static and time-invariant. In such a case, the network effects on both forward and backward paths can be merged together as in a *one-channel feedback NCS* [Hespanha et al., 2007]. Hence, only the network effects on one path needs to be considered (e.g. the forward path) and thus the control law (5) can be considered without loss of generality. Note that this is however not the case when a time-varying or a dynamic controller is considered.

The quantity referred to as the *maximum allowable transfer interval* (MATI, [Walsh et al., 1999]) denoted by μ and defined as

$$\mu := (1+m)T_{max} + \tau_{max} \tag{6}$$

is very often used to compare the different methods. The term *communication outage* [Henriksson et al., 2009] is also employed to refer to the time interval during which sensors data and controller signals do not reach the controller and the actuator respectively. The set of

[1] that is the control input on the process side is updated only when a new data come, otherwise the previous data is maintained

admissible communication outages is parameterized by the MATI as

$$\mathscr{S}_\mu := \{(m,\tau,T) \in \mathbb{N} \times \mathbb{R}_+ \times \mathbb{R}_{++} :\ (1+m)T + \tau \leq \mu\}. \quad (7)$$

Using the complete model for the control input, the closed-loop system writes

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BKx(t_k) + Ev(t) \\ y(t) &= Cx(t) + DKx(t_k) + Fv(t) \\ x(0) &= x_0 \\ t &\in [t_k + \tau_k, t_{k+1} + \tau_{k+1}). \end{aligned} \quad (8)$$

with the conditions (5).

3. STABILITY ANALYSIS

3.1 NCS model:

In order to tackle the problem in the well-posedness framework, the following transformed equivalent model is used instead

$$\begin{aligned} \dot{x}(t) &= (A+BK)x(t) - BK\delta(t) + Ev(t) \\ y(t) &= (C+DK)x(t) - DK\delta(t) + Fv(t) \\ \delta(t) &= \Delta_{sh}[\dot{x}](t) \\ x(0) &= x_0 \\ t &\in [t_k + \tau_k, t_{k+1} + \tau_{k+1}) \end{aligned} \quad (9)$$

where the operator $\Delta_{sh}(\cdot)$ is defined as

$$\Delta_{sh}[\eta](t) = \int_{t_k+\tau_k}^{t} \eta(s)ds,\ t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}) \quad (10)$$

with $t_{k+1} - t_k + \tau_{k+1} - \tau_k \leq \mu$ with $\mu > 0$. The signals involved in the above system can be related through the dynamical expression

$$\underbrace{\begin{bmatrix} x(t) \\ \delta(t) \\ v(t) \end{bmatrix}}_{w(t)} = \underbrace{\begin{bmatrix} \mathcal{I}1_n & & \\ & \Delta_{sh}1_n & \\ & & \Delta_\gamma \end{bmatrix}}_{\nabla} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t) \\ y(t) \end{bmatrix}}_{z(t)}, \quad (11)$$

and the implicit algebraic expression

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t) \\ y(t) \end{bmatrix}}_{z(t)} = \underbrace{\begin{bmatrix} A+BK & -BK & E \\ 0 & 0 & 0 \\ C+DK & -DK & F \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{bmatrix} x(t) \\ \delta(t) \\ v(t) \end{bmatrix}}_{w(t)} \quad (12)$$

where $\mathcal{I}$ is the integral operator and Δ_γ is a virtual operator characterizing the L_2 gain of the transfer $v \to y$ (detailed further).

3.2 Characterization of the operators:

In this section, the IQCs defining the operators involved in ∇ are derived.

Lemma 2. The integration operator $\mathcal{I}$ is characterized by the IQC:

$$\Pi_1 := \langle \begin{bmatrix} 1_n \\ \mathcal{I}1_n \end{bmatrix} x_T, \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} \begin{bmatrix} 1_n \\ \mathcal{I}1_n \end{bmatrix} x_T \rangle \leq 0.$$

for all $x \in L_{2e}^n$ and for any matrix $P \in \mathbb{S}_{++}^n$.

Proof : Expanding the expression, we get $\forall T > 0, \forall x \in L_{2e}^n$, $(x_T = \mathbb{P}_T(x))$

$$\begin{aligned} \Pi_1 &= -2\int_0^{+\infty} x_T(t)^T P \int_0^t x_T(s)dsdt \\ &= -2\int_0^{+\infty} \frac{d}{dt}(\mathcal{I}x_T)^T P(\mathcal{I}x_T)dt \\ &= -\int_0^T x_T^T(s)dsP\int_0^T x_T(s)ds \leq 0 \end{aligned}$$

$\diamondsuit$

Lemma 3. The operator Δ_{sh} can be characterized by the IQC:

$$\langle \begin{bmatrix} 1_n \\ \Delta_{sh}1_n \end{bmatrix} x_T, \begin{bmatrix} -\frac{4}{\pi^2}\mu^2 S_1 & -S_2 \\ -S_2 & S_1 \end{bmatrix} \begin{bmatrix} 1_n \\ \Delta_{sh}1_n \end{bmatrix} x_T \rangle \leq 0.$$

for all $x \in L_{2e}^n$ and for any matrices $S_1, S_2 \in \mathbb{S}_{++}^n$.

Proof : Using the same arguments as in [Chen and Francis, 1995, Mirkin, 2007], the L_2-induced norm of the operator Δ_{sh} is equal to $\frac{2}{\pi}\mu$, where μ is the largest interval of integration for the operator Δ_{sh} given in (6). Therefore, for all $r \in L_2^n$, the inequality

$$\|\Delta_{sh}r\|_{L_2}^2 \leq \frac{4}{\pi^2}\mu^2\|r\|_{L_2}^2$$

holds or equivalently there exists $S_1 \in \mathbb{S}_{++}^n$ such that

$$\int_0^{+\infty} \varphi_{S_1}(\Delta_{sh}[r](t))dt \leq \frac{4}{\pi^2}\mu^2 \int_0^{+\infty} \varphi_{S_1}(r(t))dt$$

where $\varphi_X(\alpha) = \alpha^T X\alpha$ for any matrices $X = X^T$ and vectors α of appropriate dimensions. Considering now $x(t) \in L_{2e}^n$, $x_T(t) \in L_2^n$, we have

$$\int_0^{+\infty} \left\{ -\varphi_{S_1}(\Delta_{sh}[x_T](t)) + \frac{4}{\pi^2}\mu^2\varphi_{S_1}(x_T(t)) \right\} dt \geq 0. \quad (13)$$

Moreover, the passivity of the operator Δ_{sh} has been proved in [Fujioka, 2009]. So, for any $S_2 \in \mathbb{S}_{++}^n$ we have

$$\int_0^\infty r^T(t)S_2\Delta_{sh}[r](t)dt \geq 0.$$

for all $r \in L_2$. As previously, for all $x(t) \in L_{2e}^n$,

$$\int_0^\infty x_T^T(t)S_2\Delta_{sh}[x_T](t)dt \geq 0. \quad (14)$$

Finally, the sum of the inequalities (13) and (14) can be easily arranged as the result of the lemma. $\diamondsuit$

Lemma 4. The operator Δ_γ, which has an L_2-induced norm equal to γ^{-1}, is characterized by the IQC:

$$\langle \begin{bmatrix} 1_r \\ \Delta_\gamma \end{bmatrix} x_T, \begin{bmatrix} -\eta 1_q & 0 \\ 0 & 1_r \end{bmatrix} \begin{bmatrix} 1_q \\ \Delta_\gamma \end{bmatrix} x_T \rangle \leq 0,$$

for all $x \in L_{2e}^q$ where $\eta = \gamma^{-2}$.

The role of the operator Δ_γ is to close a virtual loop between the performance output and the exogenous input of the system to be analyzed. The virtual loop block has L_2-gain γ^{-1}. When the gain γ^{-1} is too large, stability is lost. Hence, the goal is to find the maximal γ^{-1} for which stability is preserved. The inverse of this value coincides with the actual L_2-gain of the system.

3.3 Main result:

Theorem 5. The system (8) is asymptotically stable for all $(m,\tau,T) \in \mathscr{S}_\mu$ if there exist matrices $P, S_1, S_2 \in \mathbb{S}_{++}^n$ and a scalar $\eta > 0$ such that the LMI

$$[\mathcal{E} \; -\mathcal{A}]_{\perp}^{T} \Theta [\mathcal{E} \; -\mathcal{A}]_{\perp} \prec 0 \tag{15}$$

holds where $\mathcal{E}, \mathcal{A}$ are defined in (12) and

$$\Theta = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & -P & 0 & 0 \\ 0 & -\frac{4}{\pi^2}\mu^2 S_1 & 0 & 0 & -S_2 & 0 \\ 0 & 0 & -\eta \mathbf{1}_q & 0 & 0 & 0 \\ \hline & & & 0 & 0 & 0 \\ & * & & 0 & S_1 & 0 \\ & & & 0 & 0 & \mathbf{1}_r \end{array}\right]. \tag{16}$$

Moreover, the closed-loop system satisfies $||y||_{L_2} \leq \sqrt{1/\eta}||v||_{L_2}$.

Proof : The result is based on Theorem 1 applied to the system (11)–(12) using Lemmas 2–4. It is easy to see that by inserting (16) into (1) and (2) and using Lemmas 2–4, then the system is well-posed if (15) is satisfied. The stability of the system follows from well-posedness according to Lemma 4, see e.g. Iwasaki and Hara [1998]. Finally, the $\mathcal{L}_2$-gain is guaranteed from Lemma 4. The proof is complete. ◇

Remark 2. Extensions to robust analysis with respect to parametric uncertainties is possible [Peaucelle et al., 2007].

3.4 Numerical examples:

Example 1: Let us consider the system

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.8 & -0.01 \\ 1 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} Fx(t_k) \\ F &= \begin{bmatrix} -2.0348 & -1.8108 \end{bmatrix}. \end{aligned} \tag{17}$$

It is determined by eigenvalues analysis that the maximal constant sampling period that preserves stability is equal to 2.0142 and provides an upper bound on the maximal varying sampling period. The results obtained using Theorem 5 are compared to [Tang et al., 2008] and those reported in [Hespanha et al., 2007], namely [Yu et al., 2004, Yue et al., 2004]. Fig. 3 shows the maximal allowable sampling period T_{max} for a given maximal value of the delay τ_{max} (when no dropout occurs). The determined stability region $\mathscr{S}_\mu$ for (17) assessed by the different approaches is the surface below the corresponding line . This shows that our approach gives tighter bounds than the previous results.

Assume now the delay $\tau(t) \in [0, 0.43]$ and dropouts may occur. The maximal number of consecutive dropouts is depicted in Fig. 4 and, as expected, the smaller the maximal sampling period is, the larger is the number of admissible consecutive dropouts.

Example 2: The following system is considered:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1.006 & -1.006 \end{bmatrix} x(t_k). \tag{18}$$

whose maximal admissible constant sampling period is 5.8117.

The results are compared to [Yu et al., 2004, Yue et al., 2004, Tang et al., 2008, Naghshtabrizi and Hespanha, 2006]. In Table 1, the corresponding MATIs are given and we can see that the proposed technique leads to better

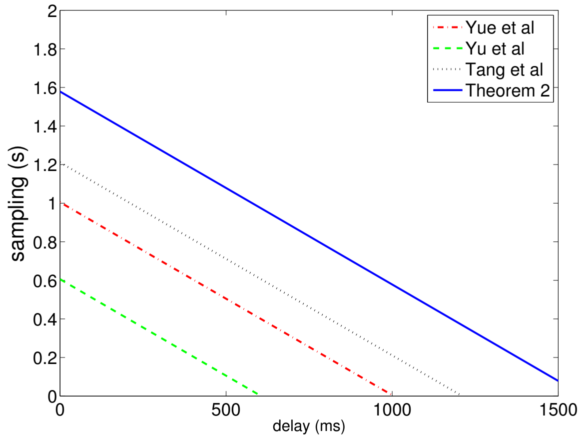

Fig. 3. Stability regions of (17) using different approaches (when no dropout occurs).

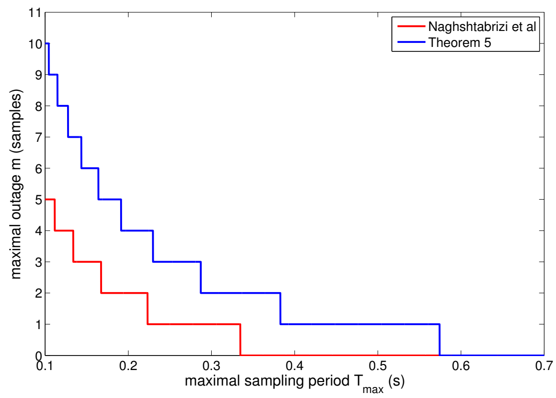

Fig. 4. Stability regions in term of number of consecutive dropouts for system (17) with a maximal delay $\tau_{max} = 430ms$.

results both in terms of efficiency and computational complexity.

4. SYNTHESIS

This section is devoted to the stabilization of NCS via state-feedback. A dilated LMI is devised from (15)-(16) in order to make the stabilization problem tractable. This is stated in the following theorem:

Theorem 6. There exists a matrix $K \in \mathbb{R}^{m\times n}$ such that the closed-loop system (8) is asymptotically stable for all $(m, \tau, T) \in \mathscr{S}_\mu$ if there exist matrices $P, S_1 \in \mathbb{S}^n_{++}$, $X \in \mathbb{R}^{n\times n}$, $U \in \mathbb{R}^{m\times n}$ and a scalar $\gamma > 0$ such that the LMI

$$\begin{bmatrix} -X^{\mathcal{S}} & P + A_{cl}' & -BU & E & 0 & X & \alpha S_1 \\ \star & -P & 0 & 0 & {C_{cl}'}^T & 0 & 0 \\ \star & \star & -S_1 & 0 & -(DU)^T & 0 & 0 \\ \star & \star & \star & -\gamma I & F^T & 0 & 0 \\ \star & \star & \star & \star & -\gamma I & 0 & 0 \\ \star & \star & \star & \star & \star & -P & -\alpha S_1 \\ \star & \star & \star & \star & \star & \star & -S_1 \end{bmatrix} \prec 0 \tag{19}$$

Table 1.

	μ	no. of var.	for $n=2$
[Yu et al., 2004]	unfeasible	$4\frac{n(n+1)}{2}$	12
[Yue et al., 2004]	0.970	$2\frac{n(n+1)}{2}+6n^2$	30
[Tang et al., 2008]	0.995	$4\frac{n(n+1)}{2}+16n^2$	76
[Naghshtabrizi and Hespanha, 2006] (without delay)	1.272	$7\frac{n(n+1)}{2}+16n^2$	85
Theorem 5	1.561	$3\frac{n(n+1)}{2}+1$	10

holds with $A'_{cl} = AX + BU$, $C'_{cl} = CX + DU$ and $\alpha = \mu\frac{\pi}{2}$. Furthermore, the closed-loop system controlled with gain $K = UX^{-1}$ satisfies $||y||_{L_2} \leq \gamma||v||_{L_2}$.

Proof : The proof is based on the projection lemma [Gahinet and Apkarian, 1994] similarly as in [Tuan et al., 2003, Briat, 2008, Briat et al., 2009]. Since it relies on standard but tedious algebraic manipulations, the proof is only sketched. Denote the matrix (19) by Ω. A congruence transformation with respect to[2] $\mathcal{C} := \mathrm{diag}(I_3 \otimes Y, I, I, I_2 \otimes Y)$, $Y = X^{-1}$ yields

$$\begin{aligned} \Omega' &= \mathcal{C}^T \Omega \mathcal{C} \\ &= \mathcal{C}^T \left[\Omega|_{Y=0} + U^T Y V + (U^T Y V)^T \right] \mathcal{C} \end{aligned} \tag{20}$$

where

$$\begin{aligned} U &= [1\ 0\ 0\ 0\ 0\ 0\ 0], \\ V &= [-1\ A_{cl}\ -BK\ E\ 0\ 1\ 0], \end{aligned}$$

with $A_{cl} = A + BK$ and $K = UY^{-1}$. The projection lemma implies

$$U_\perp^T \left[\Omega'|_{Y=0}\right] U_\perp \prec 0, \tag{21}$$

$$V_\perp^T \left[\Omega'|_{Y=0}\right] V_\perp \prec 0. \tag{22}$$

So, assuming that (19) holds then so do (21) and (22). After some manipulations, it can be shown that (22) is equivalent to (15) with $S_2 = 0$ (using Schur complements) and thus, stability follows together with the $\mathcal{L}_2$-gain property $||y||_{L_2} \leq \gamma||v||_{L_2}$. $\diamondsuit$

Example 1. Let us consider the unstable open-loop system

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.8 & -0.01 \\ 1 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v(t) \\ y(t) &= x_1(t) \end{aligned} \tag{23}$$

Since Theorem 6 provides an LMI condition for controller design, only a comparison with other results involving LMIs can be considered. Techniques based on LMIs with pre-tuning terms cannot be compared immediately since their complexity is higher due to the necessity of finding optimal pre-tuning terms, making the overall problem nonlinear.

In Yu et al. [2004], it is shown that the system is stabilizable provided that $\mu \leq 0.6011$. Using Theorem 6, it is shown that the system is still stabilizable for $\mu \leq 3.64826$ with a controller gain $K = [-0.3482\ -0.3097]$. When the closed-loop system stability is checked using Theorem 5, we find that the system remains stable if $\mu \leq 9.19286$ showing that, as usual, stabilization results are more conservative than stability ones. In Fig. 5, the L_2 norm of the closed-loop system is plotted with respect to the MATI μ.

[2] where $\otimes$ denotes the Kronecker product

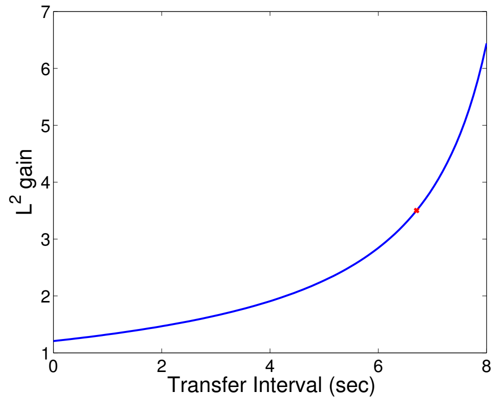

Fig. 5. L_2-gain of the controlled system (23) with respect to the MATI

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