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Innovations-based Priority Assignment for Control over CAN-like Networks

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Abstract— **We present an innovations-based prioritization mechanism to efficiently use network resources for data gathering, without compromising the real-time decision making capability of the control systems. In the envisioned protocol, each sensor assigns the Value of Information (VoI) contained in its current observations for the network as the priority. Tournaments are used to compare priorities and assign transmission slots, like in the CAN bus protocol. By using a rollout strategy, we derive feasible algorithms for computing the VoI-based priorities for the case of coupled and decoupled systems. In the case of decoupled systems, performance guarantees with regard to the control cost of the VoI-based strategy are identified. We illustrate the efficiency of the proposed approach on a platooning example in which the vehicles receive measurements from multiple sensors.**

I. INTRODUCTION

We consider a scenario where multiple sensors communicate over a shared network to perform estimation and control tasks. The shared network prevents simultaneous transmissions from sensors. Prioritising data packets based on their content can help to ensure delivery of important packets and provide performance guarantees for control and estimation. We identify a prioritisation scheme that offers a significant improvement in performance over other typically used solutions.

The networked control scenario described above is quite typical in automotive systems. An automobile contains many different electronic control units (ECU) for various subsystems, such as antilock braking, cruise control, etc. Each subsystem communicates with other subsystems, receives measurements from sensors and transmits control signals to actuators. The CAN bus standard specifies the protocol for communication between various components within a vehicle [1]. Data transmission on the CAN uses a lossless bit-wise arbitration method to compare the device IDs, which serve as static priorities, and resolve contention between components that attempt simultaneous transmissions. Autonomous vehicles and automated driving solutions are the next technological leap for automotive systems. These solutions offer higher reliability and faster reaction time, while simplifying the task of driving.

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An autonomous vehicle must be capable of sensing its environment and navigating without human input. In addition to onboard sensing solutions using Radar, Lidar, GPS, and computer vision, autonomous vehicles must be able to receive information from infrastructure nodes and other vehicles in the vicinity. An important challenge is to incorporate the profusion of data sources in to the CAN bus. Dynamic or state-based priorities are likely to play an important role in ensuring efficient use of the CAN bus.

The idea of using the state or measurement of a physical system to determine channel access has been prevalent for some time now [2]–[4]. The deviation in the state from the nominal value was used to determine a priority in Try-Once-Discard (TOD) [2]. Maximum error first is the prioritization principle used in TOD, to guarantee input-to-state stability for deterministic systems with disturbances. The implementation of the original idea was centralized, and required a network coordinator to collect and compare errors from the various physical processes in the network. Distributed implementations of this algorithm and the effect of packet losses have been studied in [5] and [6]. A similar approach has been used in [7] to identify a dynamic utilization policy for the Time Division Multiple Access (TDMA) slots of the IEEE 802.15.4 protocol. An alternate approach for stochastic systems over a network was presented and analyzed in [8] for decoupled systems. Other works study the stochastic stability of controlling decoupled systems with error-dependent randomized priorities [9], [10]. While the priority mechanism has been heuristically chosen in [8]–[10], our formulation of the priority assignment is based on the framework of VoI, which allows for a systematic and analytic approach to compare with centralized decision policies.

Our main contribution is the systematic development of an innovations-based priority scheme for scheduling multiple sensor data over a CAN-like networked control system. We introduce a specific variant of VoI for calculating priorities. The concept of VoI is well-known in information analysis and optimal decision making, and is defined as the price a decision maker is willing to pay to utilize certain information [11]. It is extensively applied in the area of information economics [12], [13] and in the field of health care for determining diagnostic value [14]. Closer to our work, VoI is also used in the problem of sensor selection for data fusion [15]–[17]. While the typical VoI formulation presumes knowledge of only the statistics of the information, the novelty of our approach is that each sensor takes the current measurement into account when computing the VoI. Inspired by [18], we use a rollout strategy, which assumes

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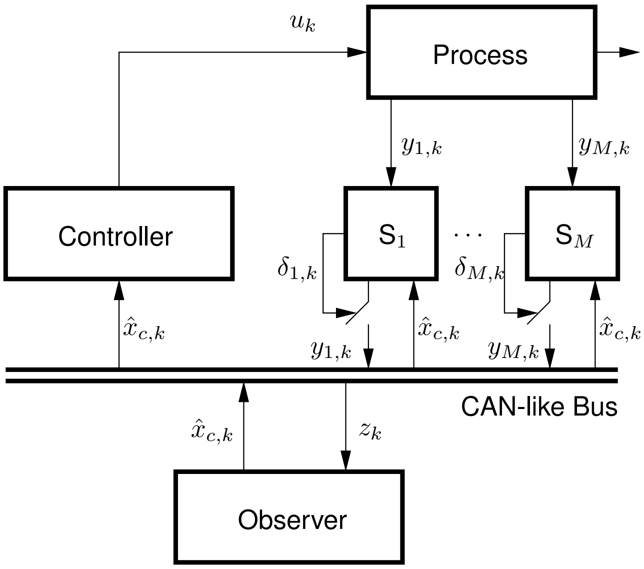

Fig. 1. Multi-sensor networked control system over CAN-like bus. Sensor S_j, $1 \leq j \leq M$ transmit data according to the triggering variable $\delta_{j,k}$. The augmented measurement vector z_k is used for updating the state estimate $\hat{x}_{c,k}$ at the observer.

that the future transmission schedule is predetermined by a baseline heuristic [19], in order to keep the determination of the VoI feasible. For decoupled systems, we show that the VoI-based priority takes the form of a weighted squared innovation at each sensor, where the weighting matrix can be determined recursively. Moreover, we show that the proposed scheme is identical to the optimal centralized scheduling rule based on the rollout strategy. Thus, we have a performance guarantee for the prioritization scheme: the control cost is upper bounded by the cost obtained by the baseline schedule used in the rollout strategy. We also highlight how these results can be extended to the coupled case of a first-order system through a suitable approximation of the VoI.

The rest of this paper is organized as follows. In Section II, we formulate the problem and introduce the priority assignment scheme. The properties of the priority scheme are analyzed in Section III for decoupled systems, and in Section IV for coupled systems. In Section V, we illustrate the efficiency of the proposed innovations-based strategy on a cruise control problem in platoons.

II. PROBLEM FORMULATION

Our problem formulation is divided into four parts in Sections II-A–II-D, which describe the model of the physical process and the sensors, the priority-based transmission scheme over a CAN-like network, the controller and observer, and our novel approach to synthesize priorities, respectively. An overview of the networked control system is given in Fig. 1.

A. Multiple sensor LQG framework

We consider a set of M sensors, indexed by j, $1 \leq j \leq M$ that generate measurements $y_{j,k}$. The plant state evolves as per the law

$$x_{k+1} = Ax_k + Bu_k + w_k$$

where the state $x_k \in \mathbb{R}^n$ and the control $u_k \in \mathbb{R}^p$. The initial value of the state x_0 is assumed to be zero mean Gaussian with covariance R_0. The process noise w_k is assumed to be an independent and identically distributed (i.i.d.) zero-mean Gaussian with covariance R_w. The measurements $y_{j,k}$ are given by

$$y_{j,k} = C_j x_k + v_{j,k}$$

with $y_{j,k} \in \mathbb{R}^{m_j}$ and $C_j \in \mathbb{R}^{m_j \times n}$. The measurement noise $v_{j,k}$ is also assumed to be an i.i.d. zero-mean Gaussian sequence with covariance $R_{v,j}$. We assume that all the primitive random variables, i.e., x_0, w_k and $v_{j,k}$, are mutually independent.

We aim to minimize the finite horizon cost

$$J^{\mathcal{C}} = \mathbf{E}\left[x_N^{\mathsf{T}} Q_N x_N + \sum_{k=0}^{N-1} x_k^{\mathsf{T}} Q_1 x_k + u_k^{\mathsf{T}} Q_2 u_k\right] \quad (1)$$

with $Q_1 \in \mathbb{R}^{n \times n}$, $Q_2 \in \mathbb{R}^{p \times p}$, and $Q_N \in \mathbb{R}^{n \times n}$ being positive definite matrices.

B. Priority-based transmission

Each sensor assigns a priority to its available data. Tournaments are then performed to determine which sensors get to transmit in the N_T available tournament slots, and in which order, based on the assigned priorities.

Let $\delta_{j,k} \in \{0, 1\}$ denote the channel access outcome of the jth sensor. It is defined as

$$\delta_{j,k} = \begin{cases} 1 & \text{node } j \text{ wins any of the } N_T \text{ tournaments} \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, let us define the last time sensor j transmitted data to the observer as

$$\tau_{j,k} = \max\{\ell \,|\, \delta_{j,\ell} = 1 \wedge \ell < k\} \quad (2)$$

with $\tau_{j,k} = -1$ when no transmissions of sensor j have occurred yet. Let the vector $s_k \in \{1, \ldots, M\}^{N_T}$ denote the indices of the sensors that obtain a transmission slot and z_k denote the data received by the central observer at time k. The data to be transmitted by each sensor will be defined in the following sections.

C. Control and filtering structure

The controller uses the certainty equivalent law

$$u_k = -L_k \hat{x}_{c,k} \quad (3)$$

where $\hat{x}_{c,k}$ is an estimate of x_k, $L_k = \left(Q_2 + B^{\mathsf{T}} \Xi_{k+1} B\right)^{-1} B^{\mathsf{T}} \Xi_{k+1} A$ and Ξ_k is the standard solution to the finite horizon Riccati equation. The observer is based on a Kalman filter that uses the received information z_k and s_k. Its exact form will be introduced in the following sections.

The cost function in (1) can be rewritten as

$$\begin{aligned} J^{\mathcal{C}} =& \mathbf{E}[x_0^{\mathsf{T}}\Xi_0 x_0] + \mathbf{E}[\sum_{k=0}^{N-1} w_k^{\mathsf{T}}\Xi_{k+1}w_k] \\ &+ \mathbf{E}[\sum_{k=0}^{N-1}(u_k + L_k x_k)^{\mathsf{T}}\hat{\Gamma}_k(u_k+L_k x_k)]. \end{aligned} \tag{4}$$

with $\hat{\Gamma}_k = B^{\mathsf{T}}\Xi_{k+1}B + Q_2$, $k \in \{0,\dots,N-1\}$ (see lemma 6.1 of chapter 8 in [20]). By using the certainty equivalent law (3) and by noting that the first and second terms in (4) are constant, we can restrict our attention to the minimization of the following weighted mean square error cost

$$J = \mathbf{E}\left[\sum_{k=0}^{N-1}(x_k - \hat{x}_{c,k})^{\mathsf{T}}\Gamma_k(x_k - \hat{x}_{c,k})\right] \tag{5}$$

with $\Gamma_k = L_k^{\mathsf{T}}(B^{\mathsf{T}}\Xi_{k+1}B + Q_2)L_k$, $k \in \{0,\dots,N-1\}$.

D. Synthesis of priorities

Our priority assignment scheme aims to minimize the cost J in (5), through the use of two complementary concepts: VoI and rollout algorithms. The priorities are determined based on the VoI associated with each sensor's information. In our scenario, the VoI is defined as the improvement in the cost J due to transmitting a sensor's data as against not transmitting it. A distinct feature in our approach is that each sensor computes the VoI based on its own information structure, which varies based on the scenario considered. In the decoupled case, it is sufficient to keep track of its own measurements and the transmission sequence, whereas in the coupled case, previously transmitted data from other sensors is also required.

A full computation of the VoI in this problem setting is a difficult task, in general, because scheduling choices must be made for the entire horizon, and not just for the current time instant. Furthermore, scheduling choices made using sensors' local information lead to complex cost-to-go functions and impede tractability. Thus, we use a rollout strategy, which assumes that future scheduling decisions are predetermined by a baseline heuristic, to simplify computation of the VoI [19].

In particular for decoupled subsystems, the advantages of using this suboptimal strategy are: (i) it gives us a means to circumvent the curse of dimensionality as the VoI can be computed explicitly, (ii) it allows us to obtain performance guarantees with respect to the baseline schedule, (iii) it enables us to determine distributed prioritization schemes, which are generally difficult to design in the dynamic programming framework.

The VoI of sensor j is defined as the difference between the cost-to-go when no sensors transmit at time k and when sensor j transmits at time k. In both cases, we assume that a baseline schedule $\{\bar{s}_{k+1},\dots,\bar{s}_{N-1}\}$ is used in future steps. The cost-to-go functions are computed based on the available information structure $I_{j,k}$ at the sensor j to be defined in the subsequent sections. Therefore, the VoI Δ_j at time k is defined as

$$\begin{aligned} \Delta_j =& \mathbf{E}[\sum_{\ell=k}^{N-1}(x_\ell - \hat{x}_{c,\ell})^{\mathsf{T}}\Gamma_\ell(x_\ell - \hat{x}_{c,\ell})|I_{j,k}, s_k = \varnothing] \\ &- \mathbf{E}[\sum_{\ell=k}^{N-1}(x_\ell - \hat{x}_{c,\ell})^{\mathsf{T}}\Gamma_\ell(x_\ell - \hat{x}_{c,\ell})|I_{j,k}, s_k = j]. \end{aligned} \tag{6}$$

III. ANALYSIS OF DECOUPLED SYSTEMS

For M decoupled systems, we establish a link between the VoI-based priority assignment and the optimal scheduling policy minimizing cost J in (5). This enables us to specify a performance guarantee of the VoI strategy obtained at the end of this section.

A. Model assumptions

We consider M isolated dynamical subsystems with dedicated sensor and controller

$$\begin{aligned} x_{k+1}^{(j)} &= A^{(j)}x_k^{(j)} + B^{(j)}u_k^{(j)} + w_k^{(j)} \\ y_{j,k} &= C_j x_k + v_{j,k} = C^{(j)}x_k^{(j)} + v_{j,k} \end{aligned} \tag{7}$$

with $x_k^{(j)} \in \mathbb{R}^{n^{(j)}}$ and $u_k^{(j)} \in \mathbb{R}^{p^{(j)}}$. The system matrix A can therefore be written as $A = \mathrm{diag}[A^{(1)},\dots,A^{(M)}]$. The weighting matrices in the control cost (1), the input matrix B, and the covariance matrix of the process noise, R_w can also be similarly written.

Because of the decoupled system structure, the cost function J defined in (5) can be written as follows

$$J = \mathbf{E}\left[\sum_{j=1}^{M}\sum_{k=0}^{N-1}(x_k^{(j)} - \hat{x}_k^{(j)})^{\mathsf{T}}\Gamma_k^{(j)}(x_k^{(j)} - \hat{x}_k^{(j)})\right] \tag{8}$$

where $\Gamma_k^{(j)}$ is positive semidefinite and $\hat{x}_k^{(j)}$ is the Kalman estimate of $x_k^{(j)}$ at time step k attained at the remote observer.

A local Kalman filter at each sensor node keeps track of $x_k^{(j)}$ based on the complete observation history at the sensor, denoted $Y_{j,k} = \{y_{j,0},\dots,y_{j,k}\}$. It computes the estimate

$$\begin{aligned} \tilde{x}_{k|k}^{(j)} &= \tilde{x}_{k|k-1}^{(j)} + K_k^{(j)}(y_{j,k} - C^{(j)}\tilde{x}_{k|k-1}^{(j)}) \\ P_{k|k}^{(j)} &= (I_{n^{(j)}} - K_k^{(j)}C^{(j)})P_{k|k-1}^{(j)} \\ \tilde{x}_{k+1|k}^{(j)} &= A^{(j)}\tilde{x}_{k|k-1}^{(j)} + B^{(j)}u_k^{(j)} \\ P_{k+1|k}^{(j)} &= A^{(j)}P_{k|k}^{(j)}(A^{(j)})^{\mathsf{T}} + R_w^{(j)} \end{aligned}$$

where $K_k^{(j)} = P_{k|k-1}^{(j)}(C^{(j)})^{\mathsf{T}}(C^{(j)}P_{k|k-1}^{(j)}(C^{(j)})^{\mathsf{T}} + R_{v,j})^{-1}$ and $\tilde{x}_{0|-1} = 0$, $P_{0|-1}^{(j)} = R_0^{(j)}$. When possible, the local estimate $\tilde{x}_{k|k}^{(j)}$ is transmitted to the observer. This estimate summarizes the new information $\{y_{j,\tau_{j,k}},\dots,y_{j,k}\}$. The received signal at time k is then defined as

$$z_k = \left[\left(\tilde{x}_{k|k}^{(s_{k,1})}\right)^{\mathsf{T}} \cdots \left(\tilde{x}_{k|k}^{(s_{k,M})}\right)^{\mathsf{T}}\right]^{\mathsf{T}}. \tag{9}$$

The observer for subsystem j can be described as

$$\hat{x}_{c,k}^{(j)} = \begin{cases} \tilde{x}_{k|k}^{(j)} & \delta_{j,k} = 1 \\ \hat{x}_{c,k|k-1}^{(j)} & \delta_{j,k} = 0 \end{cases} \tag{10}$$

where $\hat{x}^{(j)}_{c,k|k-1} = (A^{(j)} - B^{(j)} L^{(j)}_k)\hat{x}^{(j)}_{c,k-1}$ is the linear prediction of the state, $\hat{x}^{(j)}_{c,0} = 0$ for $\delta_{j,0} = 0$ and $L^{(j)}_k$ is the control gain of subsystem j.

B. Computation of priorities

Our measure for the priority of a sensor is based on the VoI introduced in Section II-D. The decoupled structure of the system enables us to calculate its value exactly. The information structure for computing the VoI is $I_{j,k} = \{Y_{j,k}, \tau_{j,k}\}$. Roughly speaking, the last transmission time $\tau_{j,k}$ from (2) summarizes all required information on the triggering of transmissions for sensor j. The information structure can also be recursively expressed as $I_{j,k+1} = \{I_{j,k}, y_{j,k+1}, \delta_{j,k}\}$.

Because of the fact that data from sensor j is not beneficial for state estimation in the other subsystems $i \neq j$, we obtain the following simplified expression of the VoI based on (6).

$$\begin{aligned}\Delta_j &= \\ &\mathbf{E}[\sum_{\ell=k}^{N-1}(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})^\mathsf{T}\Gamma^{(j)}_\ell(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})|I_{j,k}, \delta_{j,k} = 0] \\ &- \mathbf{E}[\sum_{\ell=k}^{N-1}(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})^\mathsf{T}\Gamma^{(j)}_\ell(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})|I_{j,k}, \delta_{j,k} = 1]\end{aligned} \tag{11}$$

We implicitly assume that the baseline schedule $\{\bar{s}_{k+1}, \dots, \bar{s}_{N-1}\}$ applies in the future, and implies a transmission outcome of $\{\bar{\delta}_{j,k+1}, \dots, \bar{\delta}_{j,N-1}\}$ for sensor j. The first term of the running cost can be written as

$$\begin{aligned}&\mathbf{E}[(x^{(j)}_k - \hat{x}^{(j)}_{c,k})^\mathsf{T}\Gamma^{(j)}_k(x^{(j)}_k - \hat{x}^{(j)}_{c,k})|I_{j,k}, \delta_{j,k} = 0] \\ &= \mathrm{tr}\left[\Gamma^{(j)}_k \, \mathbf{E}[(x^{(j)}_k - \hat{x}^{(j)}_{c,k|k-1})(x^{(j)}_k - \hat{x}^{(j)}_{c,k|k-1})^\mathsf{T}|I_{j,k}]\right]\end{aligned}$$

Define, $e_{j,k} = x^{(j)}_k - \tilde{x}^{(j)}_{k|k}$ and $\tilde{e}_{j,k} = \tilde{x}^{(j)}_{k|k} - \hat{x}^{(j)}_{c,k|k-1}$. Then, we have

$$\begin{aligned}&\mathbf{E}[(x^{(j)}_k - \hat{x}^{(j)}_{c,k|k-1})(x^{(j)}_k - \hat{x}^{(j)}_{c,k|k-1})^\mathsf{T}|I_{j,k}] \\ &= \mathbf{E}[(e_{j,k} + \tilde{e}_{j,k})(e_{j,k} + \tilde{e}_{j,k})^\mathsf{T}|I_{j,k}] \\ &= \mathbf{E}[e_{j,k}e^\mathsf{T}_{j,k}|I_{j,k}] + \tilde{e}_{j,k}\tilde{e}^\mathsf{T}_{j,k}.\end{aligned} \tag{12}$$

The last equality holds because $\mathbf{E}[e^\mathsf{T}_{j,k}|I_{j,k}] = 0$ and because $\tilde{e}_{j,k}$ is computable for a given $I_{j,k}$ as per

$$\tilde{e}_{j,k} = \sum_{n=\tau_{j,k}+1}^{k} (A^{(j)})^{k-n} K_{j,n} \tilde{y}_{j,n} \tag{13}$$

where $\tilde{y}_{j,n} = y_{j,n} - C_j \tilde{x}^{(j)}_{n|n-1}$ is the innovations process of the local Kalman filter at sensor j. As the first term in the last line of (12) cancels out the second term in (11), the VoI with respect to the running cost can be computed by $\mathrm{tr}[\Gamma^{(j)}_k \tilde{e}_{j,k}\tilde{e}^\mathsf{T}_{j,k}]$.

For the running cost of the second term in (11), we obtain

$$\begin{aligned}&\mathbf{E}[(x^{(j)}_k - \hat{x}^{(j)}_{c,k})^\mathsf{T}\Gamma^{(j)}_k(x^{(j)}_k - \hat{x}^{(j)}_{c,k})|I_{j,k}, \delta_{j,k} = 1] \\ &= \mathrm{tr}\left[\Gamma^{(j)}_k P^{(j)}_{k|k}\right]\end{aligned}$$

Let us now look at the future terms of the cost-to-go function in (11). The observer in (10) implies that the evolution of the estimate $\hat{x}^{(j)}_\ell$ is independent of previous scheduling choices, following a transmission at time ℓ. Define the first transmission time after k of the baseline schedule as

$$\bar{\tau}_{j,k} = \min\{\ell \,|\, \bar{\delta}_{j,\ell} = 1 \wedge k < \ell \leq N-1\},$$

where we define $\bar{\tau}_{j,k} = N-1$ if no transmission is to occur in the future based on the baseline schedule. Then, we only need to consider cost terms until $\bar{\tau}_{j,k}$ in our VoI calculations, as $\hat{x}^{(j)}_\ell$, $\bar{\tau}_{j,k} \leq \ell \leq N-1$, will be the same for $\delta_{j,k} = 0$ and for $\delta_{j,k} = 1$. Hence, the VoI can be computed as

$$\Delta_j = \mathrm{tr}[\sum_{\ell=k}^{\bar{\tau}_{j,k}-1} \Gamma^{(j)}_\ell \tilde{P}_\ell] \tag{14}$$

where $\tilde{P}_\ell$ is recursively given by

$$\begin{aligned}\tilde{P}_k &= \tilde{e}_{j,k}\tilde{e}^\mathsf{T}_{j,k} \\ \tilde{P}_{\ell+1} &= A^{(j)}\tilde{P}_\ell(A^{(j)})^\mathsf{T}\end{aligned} \tag{15}$$

for $k \leq \ell < \bar{\tau}_{j,k}$ with $\tilde{e}_{j,k}$ given by (13).

Remark 1: It follows from (14)–(15) that the VoI Δ_j at time k takes the form of a quadratic function of $\tilde{e}_{j,k}$, which can be defined as the discrepancy between the estimates of $x^{(j)}_k$ at the sensor j and the remote observer. The discrepancy $\tilde{e}_{j,k}$ is a linear combination of the innovations $\tilde{y}_{j,n}$ with $n \in \{\tau_{j,k}+1, \dots, k\}$ given by (13). The VoI need only be computed $k - \bar{\tau}_{j,k}$ steps in the future and therefore allows for the consideration of large horizons N.

C. Performance guarantee

We provide a performance guarantee for the cost using the VoI strategy, in comparison to the cost using the baseline schedule. For this purpose, we introduce the centralized decision rule that chooses N_T sensors using the complete information $I_k = \{I_{1,k}, \dots, I_{M,k}\}$. Though this scheme violates the imposed restrictions in the information structure to compute priorities, it is only used to derive a performance guarantee for the VoI-based prioritization scheme.

A centralized scheduler that aims to minimise cost J in (8) assuming a rollout strategy with a deterministic baseline schedule $\{\bar{s}_1, \dots, \bar{s}_{N-1}\}$, must solve the following problem:

$$\min_{s_k} \mathbf{E}[\sum_{j=1}^{M}\sum_{\ell=k}^{N-1}(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})^\mathsf{T}\Gamma^{(j)}_\ell(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})|I_k] \tag{16}$$

where s_k is assumed to be a function of y_k and the side information $\hat{x}_{k|k-1}, P_{k|k-1}$. As the systems are decoupled and we are using a deterministic baseline strategy, measurements of a sensor $i \neq j$ are independent of variables appearing in subsystem j. Therefore, the cost in (16) decomposes into

$$\sum_{j=1}^{M}\mathbf{E}[\sum_{\ell=k}^{N-1}(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})^\mathsf{T}\Gamma^{(j)}_\ell(x^{(j)}_\ell - \hat{x}^{(j)}_{c,\ell})|I_{j,k}]. \tag{17}$$

This implies that each subsystem can evaluate its costs independently. By selecting the N_T measurements that yield the greatest benefit reflected by the difference of these cost terms, we obtain the optimal decision rule minimizing the cost (8). Hence, this rule coincides with the VoI-based priority assignment defined in (11). It should be noted that we implicitly excluded cases in which the VoI is identical for different subsystems as this occurs with probability zero. Hence, we have the following intermediate result.

Lemma 1: Let the system be defined as in (7). Then, the VoI-based priority assignment is an optimal solution to the minimization problem posed in (16).

Using this result, we provide a performance guarantee for the VoI-based strategy as stated in the subsequent theorem.

Theorem 1: Let the system be defined as in (7) and let $\{\bar{s}_1, \ldots, \bar{s}_{N-1}\}$ be a baseline scheduler with cost $\bar{J}$. Then, $\bar{J}$ is an upper bound for the cost resulting from the priority assignment based on the VoI Δ_j defined in (14) using the rollout strategy with baseline schedule $\{\bar{s}_1, \ldots, \bar{s}_{N-1}\}$.

Proof: Let $y_k = \{y_{1,k+1}, \ldots, y_{M,k+1}\}$. then, the centralized information structure follows the recursion

$$I_{k+1} = \{I_k, y_k, s_k\}.$$

Let $s_k = \pi_k^{\text{RO}}(I_k)$ be the rollout strategy based on the heuristic $\{\bar{s}_1, \ldots, \bar{s}_{N-1}\}$, $0 \leq k \leq N-1$. Let the running cost at time k be defined as

$$c_k(I_k, s_k) = \mathbf{E}[\sum_{j=1}^{M}(x_k^{(j)} - \hat{x}_{c,k}^{(j)})^\mathsf{T}\Gamma_k^{(j)}(x_k^{(j)} - \hat{x}_{c,k}^{(j)})|I_k, s_k].$$

Define the cost-to-go of the rollout strategy as $J_k^{\text{RO}}(I_k)$ and the cost-to-go of the heuristic as $\bar{J}_k(I_k)$ at time k, respectively. Similar to the result for rollout algorithms with full-state information in [19], we prove inductively that there is a cost improvement of the rollout strategy at each time k, i.e., $J_k^{\text{RO}}(I_k) \leq \bar{J}_k(I_k)$.

For $k = N$, we have $J_N^{\text{RO}}(I_N) = \bar{J}_N(I_N) = 0$ as there is no terminal cost in J. Assume that $J_{k+1}^{\text{RO}}(I_{k+1}) \leq \bar{J}_{k+1}(I_{k+1})$ for all I_{k+1}. Then, we have from (16)

$$\begin{aligned}
&J_k^{\text{RO}}(I_k)\\
&= \mathbf{E}[c_k(I_k, \pi_k^{\text{RO}}(I_k)) + J_{k+1}^{\text{RO}}(\{I_k, y_{k+1}, \pi_k^{\text{RO}}(I_k)\})|I_k]\\
&\leq \mathbf{E}[c_k(I_k, \pi_k^{\text{RO}}(I_k)) + \bar{J}_{k+1}(\{I_k, y_{k+1}, \pi_k^{\text{RO}}(I_k)\})|I_k]\\
&\leq \mathbf{E}[c_k(I_k, \bar{s}_k) + \bar{J}_{k+1}(\{I_k, y_{k+1}, \bar{s}_k\})|I_k]\\
&= \bar{J}_k(I_k)
\end{aligned}$$

The first inequality is due to the induction hypothesis, while the second inequality arises from the fact that $\pi_k^{\text{RO}}(I_k)$ solves (16). This completes the induction.

As shown in Lemma 1, the centralized decision rule is identical to using tournaments with the VoI-based priority assignment in (11). Hence, we conclude the proof. ∎

IV. Analysis of Coupled Systems

We now examine the case of coupled subsystems, and present an extension of our VoI-based prioritization scheme for this case.

A. Model assumptions

We consider M sensors, measuring the state of a first-order system, as described by

$$\begin{aligned}
x_{k+1} &= ax_k + bu_k + w_k \ ,\\
y_{j,k} &= c_j x_k + v_{j,k} \ , \quad \text{for } 1 \leq j \leq M \ .
\end{aligned} \tag{18}$$

The initial value x_0, process noise w_k and measurement noises $v_{j,k}$ are all i.i.d. zero mean Gaussian noise processes with variances $\sigma_{x_0}^2$, σ_w^2 and $\sigma_{v,j}^2$, respectively.

B. Observer design

We present a design for the observer and filters at each sensor node for a generic transmission scheme that selects N_T out of M sensors for transmission at any time $k \geq 0$. As before, let s_k denote the indices of the sensors that transmit their data to the observer at time k. Each sensor transmits a local unbiased estimate $\hat{x}_{j,k}$ and thus, the observer receives $z_k = \begin{bmatrix}\hat{x}_{s_{1,k},k}^\mathsf{T} & \cdots & \hat{x}_{s_{N_T,k},k}^\mathsf{T}\end{bmatrix}^\mathsf{T}$. It generates the Best Linear Unbiased Estimate (BLUE) [22], $\hat{x}_{c,k}$, using

$$\hat{x}_{c,k} = \sum_{j \in s_k} \alpha_{j,k}\hat{x}_{j,k} \ , \tag{19}$$

where the weights $\alpha_{j,k}$ must satisfy $\sum_{j \in s_k} \alpha_{j,k} = 1$ to ensure an unbiased estimate. The variance of the estimation error is given by

$$\sigma_{c,k}^2 = \boldsymbol{\alpha}_k^\mathsf{T} P_{s,k} \boldsymbol{\alpha}_k \ , \tag{20}$$

where $\boldsymbol{\alpha}_k = \begin{bmatrix}\alpha_{1,k} & \ldots & \alpha_{N_T,k}\end{bmatrix}^\mathsf{T}$ and $P_{s,k}$ is the error covariance matrix corresponding to the transmitted estimates, with the $(i,j)^{\text{th}}$ element given by $(P_{s,k})_{i,j} = \mathbf{E}[(x_k - \hat{x}_{s_{i,k},k})(x_k - \hat{x}_{s_{j,k},k})]$. The weights are chosen to minimize the estimation error variance and we obtain

$$\boldsymbol{\alpha}_k = P_{s,k}^{-1}U/(U^\mathsf{T}P_{s,k}^{-1}U) \ , \tag{21}$$

where U is a vector of ones. The observer transmits its estimate $\hat{x}_{c,k}$ to all the nodes in the network.

Each sensor runs a local filter to generate an estimate of the state, using its measurement history $\{y_{j,l}\}_{l=0}^{k}$ along with the information available to the observer. The local filter runs three updates: a prediction update, an initial filtering update and a final filtering update. The prediction update results in the estimate $\hat{x}_{j,k|k-1}$, as given by

$$\begin{aligned}
&\hat{x}_{j,k|k-1} = a\hat{x}_{j,k-1|k-1} + bu_{k-1} \ , \quad \text{and } \hat{x}_{j,0|-1} = 0 \ ,\\
&\sigma_{j,k|k-1}^2 = a^2\sigma_{j,k-1|k-1}^2 + \sigma_w^2 \ .
\end{aligned} \tag{22}$$

Here, $\hat{x}_{j,k-1|k-1}$ denotes the final filtered estimate from the previous time step. Next, the sensor node uses its own measurement $y_{j,k}$ to generate the initial filtered estimate

$$\begin{aligned}
\hat{x}_{j,k} &= \hat{x}_{j,k|k-1} + \kappa_{j,k}\tilde{y}_{j,k} \ ,\\
\sigma_{j,k}^2 &= \sigma_{j,k|k-1}^2 - \kappa_{j,k}^2\sigma_{\tilde{y},j,k}^2 \ ,
\end{aligned} \tag{23}$$

where $\tilde{y}_{j,k} = y_{j,k} - c_j\hat{x}_{j,k|k-1}$, $\sigma_{\tilde{y},j,k}^2 = c_j^2\sigma_{j,k|k-1}^2 + \sigma_{v,j}^2$ and $\kappa_{j,k} = c_j\sigma_{j,k|k-1}^2/\sigma_{\tilde{y},j,k}^2$. This is the estimate that each sensor node tries to transmit to the rest of the network. Some succeed and each node receives $\hat{x}_{c,k}$ from the observer. The nodes combine it with their own estimates to generate the BLUE final filtered estimate

$$\hat{x}_{j,k|k} = \beta_{j,k}\hat{x}_{j,k} + (1-\beta_{j,k})\hat{x}_{c,k} \ ,$$
$$\sigma^2_{j,k|k} = \begin{bmatrix}\beta_{j,k} & 1-\beta_{j,k}\end{bmatrix}\begin{bmatrix}\sigma^2_{j,k} & \rho_{jc,k}\\ \rho_{jc,k} & \sigma^2_{c,k}\end{bmatrix}\begin{bmatrix}\beta_{j,k}\\ 1-\beta_{j,k}\end{bmatrix} \ , \tag{24}$$

where $\rho_{jc,k} = \mathbf{E}[(x_k - \hat{x}_{j,k})(x_k - \hat{x}_{c,k})] = \sum_{i\in s_k}\alpha_{i,k}\rho_{ij,k}$ and $\rho_{ij,k} = \mathbf{E}[(x_k - \hat{x}_{i,k})(x_k - \hat{x}_{j,k})]$. The weight $\beta_{j,k}$ is chosen to minimize $\sigma^2_{j,k|k}$ and we obtain

$$\beta_{j,k} = \frac{\sigma^2_{c,k} - \rho_{jc,k}}{\sigma^2_{c,k} + \sigma^2_{j,k} - 2\rho_{jc,k}} \ .$$

An important term used in the above calculations is $\rho_{ij,k} = \mathbf{E}[(x_k - \hat{x}_{i,k})(x_k - \hat{x}_{j,k})]$, and this can be computed as follows:

$$\begin{aligned}\rho_{ij,k} = {} & a^2(1-\kappa_{i,k}c_i)(1-\kappa_{j,k}c_j)\big(\beta_{i,k-1}\beta_{j,k-1}\rho_{ij,k-1}\\ & + (1-\beta_{i,k-1})(1-\beta_{j,k-1})\sigma^2_{c,k-1}\\ & + \beta_{i,k-1}(1-\beta_{j,k-1})\rho_{ic,k-1} + (1-\beta_{i,k-1})\beta_{j,k-1}\rho_{jc,k-1}\big)\\ & + (1-\kappa_{i,k}c_i)(1-\kappa_{j,k}c_j)\sigma^2_w \ ,\end{aligned}$$

where $\rho_{ij,0} = (1-\kappa_{i,0}c_i)(1-\kappa_{j,0}c_j)\sigma^2_{x_0}$.

To reduce the computational burden on the sensor nodes, the observer could pre-compute $\beta_{j,k}$ for all $j \in \{1,\dots,M\}$ and transmit this information along with its estimate $\hat{x}_{c,k}$ to the nodes in the network. Then, the nodes can simply combine its estimate with the observer's estimate using (24). Note that this would require the observer to replicate the filtering constants $\sigma^2_{\tilde{y},j,k}$, $\kappa_{j,k}$ and $\sigma^2_{j,k}$ of each sensor node, and maintain a full cross-covariance matrix of the elements $\rho_{ij,k}$, for every $i,j \in \{1,\dots,M\}$.

C. Computation of priorities

We now return to the definition of VoI in (6), and apply it to a network of coupled subsystems. In this network, the lack of a transmission can also convey information to other sensor nodes and the observer, due to the nature of the scheduling policy. Using this information would render the innovations process at each sensor node non-Gaussian, and make this formulation analytically intractable. We overcome such difficulties by simplifying the information patterns at the sensor nodes.

There are three important aspects to our VoI formulation for such a network. Firstly, the information available to each sensor $I_{j,k}$ is limited to its local information and the transmitted information from the rest of the network. Any side information that can be gleamed from the scheduling policy is not included. Secondly, at any transmission instant, the scheduling sequences considered are simply $s_k = j$ or $s_k = \varnothing$, as in (6). Finally, the future transmission sequences are obtained from a baseline scheduler. Note that this VoI formulation requires each sensor node to replicate the parameters used by the filter at the observer.

V. AUTOMOTIVE EXAMPLES

In order to illustrate the efficiency of our prioritisation scheme, we consider two automotive examples.

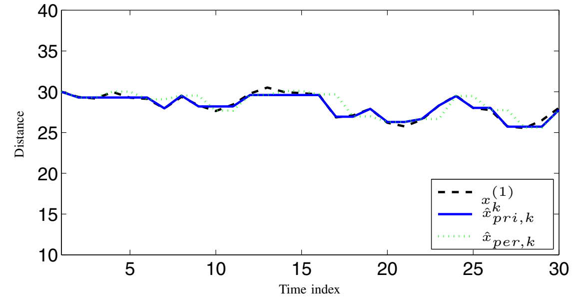

Fig. 2. Typical sample path of the state $x_k^{(1)}$ and the state estimates $\hat{x}_{pri,k}$, $\hat{x}_{per,k}$ with priority-based scheduling and with periodic scheduling, respectively, for subsystem 1 with $M = 2$, $N_T = 1$.

We present a distance tracking example for a platoon of vehicles. We wish to estimate the distances of M vehicles in the platoon to our own vehicle. We model each vehicle's distance as an independent random walk ($A^{(j)} = 1$) subject to a zero-mean Gaussian disturbance with variance 1. A noisy distance measurement with $C^{(j)} = 1$ and $v_{j,k} \sim \mathcal{N}(0, 0.01)$ from each vehicle's radar is to be transmitted to us. Our aims to minimize the cost J in (8) with $\Gamma^{(j)}_k = 1$ for this decoupled scenario using the observer in (10). The performance is evaluated in Table I for the priority-based scheduling and a periodic scheduling by conducting a Monte Carlo simulation with a horizon of $N = 100,000$ each. The priority-based scheme assumes a periodic baseline strategy. As stated in Theorem 1, the performance J_{priority} is upper bounded by the performance of the periodic scheme denoted as J_{periodic}. The cost is almost halved by the priority scheme. In Fig. 2, a typical sample path is drawn for the state evolution and its corresponding estimates that use either periodic or priority-based scheduling. Here, we observe that the priority-based scheme adapts faster to critical changes in the distance compared to the periodic schedule.

M	2	3	4	6	8
J_{priority}	0.5	1.5	3.1	8.5	17
J_{periodic}	1.0	3.0	6.0	15.0	28

TABLE I

PERFORMANCE COMPARISON FOR STATE TRACKING

Next, we present a cruise control example to illustrate the prioritization method for coupled systems. We use a simple mathematical model for the motion of the car, as given by $m\dot{v} = F - F_d$, where m is the total mass of the vehicle, v is the speed of the car, F is the force generated by the engine torque and F_d are the disturbance forces due to gravity, friction and air drag. The expressions for each of these forces and the parameters for our model are taken from [23]. By linearizing around $v_0 = 27$ m/s and discretizing the resulting differential equation, we arrive at a first-order state space representation, such as the one in (18), where the state $x_k = v_k - v_0$ and the control signal $u_k = v_k - v_0$ denote the difference from the respective equilibrium values. The control signal at equilibrium can be computed as $v_0 = b^{-1}(1-a)v_0$. The system parameters are computed to be $a = 0.9766$ and $b = 1781.9$, and the variances are chosen to be $\sigma_{x_0}^2 = \sigma_w^2 = 0.01$. The car contains a number of sensors to measure or estimate its own velocity, such as a radar, an accelerometer, a GPS, etc. We assume that there are four sensors with $c_j = 1$ for $1 \le j \le 4$ and the measurement noise variances set to 0.01, 0.02, 0.03 and 0.04, respectively. Typically, the variance values are chosen to best fit a training sequence of measurements while parameterizing the plant model and calibrating the sensors.

For the purpose of reference tracking, we introduce the integral state $i_{k+1} = i_k + r_k - \hat{x}_{c,k|k}$, where $\hat{x}_{c,k|k}$ is the estimate at the observer. Using the augmented state $\chi_k = \begin{bmatrix} \hat{x}_{c,k|k}^\top & i_k^\top \end{bmatrix}^\top$, we design and implement a standard LQG reference tracking controller. We use the VoI computed at every sensor with respect to the observer's estimate to synthesize the priority of the sensor. We compare the results of a reference change experiment against a static periodic scheduler in Fig. 3, and find that the VoI prioritization improves tracking. A Monte carlo simulation also shows that the VoI scheme results in a smaller cost, as shown in Table II.

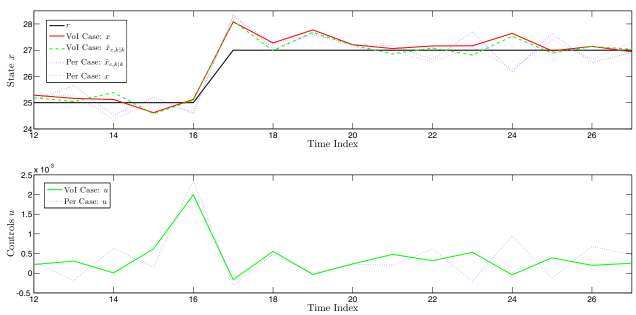

Fig. 3. This plot compares the performance of a periodic scheduler against the VoI prioritization scheme in a reference tracking experiment on the cruise control system. We can see that the VoI scheme results in a better tracking performance.

Scheduler	Cost J
VoI Prioritization Scheme	0.1902
Periodic Scheduler	0.5022

TABLE II

A TABLE COMPARING THE ESTIMATION COSTS J FROM A MONTE CARLO SIMULATION

VI. CONCLUSIONS

We have shown that innovations-based priority assignment strategies for sensor scheduling can improve the control performance over a CAN-like bus. We demonstrated how the concept of VoI can be applied to systematically synthesize dynamic priorities, while obtaining a performance guarantee for the resulting cost using rollout algorithms with a static baseline schedule. In addition, the priorities were found to be computed efficiently by a quadratic form of the innovations process at each sensor. We also presented an extension of the innovations-based prioritization scheme to the case of coupled systems by suitable approximating the VoI. The numerical simulations indicate a significant performance gain when using the proposed innovations-based approach compared with periodic sensor scheduling.

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