Wireless Networked Control System Co-Design

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Abstract—**A framework for the joint design of wireless network and controllers is proposed. Multiple control systems are considered where the sensor measurements are transmitted to the controller over the IEEE 802.15.4 protocol. The essential issues of wireless networked control systems (NCSs) are investigated to provide an abstraction of the wireless network for a co-design approach. We first present an analytical model of the packet loss probability and delay of a IEEE 802.15.4 network. Through optimal control techniques we derive the control cost as a function of the packet loss probability and delay. Simulation results show the feasible control performance. It is shown that the optimal traffic load is similar when the communication throughput or control cost are optimized. The co-design approach is based on a constrained optimization problem, for which the objective function is the energy consumption of the network and the constraints are the packet loss probability and delay, which are derived from the desired control cost. The co-design is illustrated through a numerical example.**

Keywords: Co-design, Linear Quadratic Gaussian controller, Medium Access Control, IEEE 802.15.4, Optimization.

I. Introduction

There are major advantages in terms of increased flexibility, reduced installation, and maintenance costs in the use of wireless communication technology in industrial control systems [1], [2]. The IEEE 802.15.4 standard has received considerable attention as a low data rate and low power protocol for wireless sensor network (WSN) applications in industrial control, home automation, and smart grids [1]–[3].

Although WSNs provide a great advantage for the process and manufacturing industries, they are not yet efficiently deployed. One of the most significant reasons is the lack of proper modeling of the network behavior. Any wireless network introduces random packet losses and delays due to the harsh nature of the wireless channel, limited bandwidth, and interference generated by other wireless devices. The tradeoff between tractability and accuracy of the analytical model of a wireless network is important in order to hide the system complexity through a suitable abstraction without losing critical aspects of the network. Furthermore, WSNs require energy-efficient operation due to the limited battery power of each sensor node. Design methods on how to achieve high performance of control systems through a communication network have been recently proposed. The approaches can be grouped in two categories: design of the control algorithm and design of the communication protocol. Some research has been done in designing robust

controllers and estimators that are adaptive and robust to the communication faults: packet dropout as a Bernoulli random process [4] or with deterministic rate [5], packet delay [6], and data rate limitation [7]. On the other side, communication protocols and their parameters are designed in order to achieve a given control performance. In [8], the authors present a scheduling policy to minimize a linear quadratic (LQ) cost under computational delays. In [9], the authors proposed an adaptive tuning scheme of the parameters of the link layer, medium access control (MAC) layer and sampling period through numerical results in order to minimize the LQ cost. However, these approaches often consider only one aspect of the network faults: packet dropout [4],[5], packet delay [6], [8], or data rate limitation [7]. In [9], although the authors consider the simulation results of the wireless network, the framework has not been designed out of an analytical consideration of control performance. Even though many communication protocols are available in [10] and [11], these protocols are designed mainly to achieve high reliability and high energy efficiency for various applications of WSNs and not specifically for control applications.

In this paper there are two original contributions:
1) We investigate the essential issues of wireless networked control systems (NCSs) by considering the effects of wireless network on control performance.
2) We propose a co-design approach to meet the desired control cost while minimizing the energy consumption of the network.

In particular, we show the feasible control performance by considering the wireless network effects. This paper explicitly considers both the control cost of control applications and the network performance with respect to energy consumption, which is the most important requirement of communication protocol design for WSNs. The key issue addressed here is how to derive the explicit relation between the performance of the control systems and the characteristics of a wireless network. Furthermore, the well-defined design procedure is investigated to achieve high performances in wireless NCSs.

The outline of the paper is as follows. Section II defines the considered problem of control over a wireless network. In Section III, we describe the IEEE 802.15.4 standard and its network model. The design of the estimator and the controllers is presented in Section IV. In Section V, we discuss the essential issues of wireless NCSs based on simulation results and propose a co-design approach. In Section VI, we illustrate it through numerical examples. Section VII concludes the paper.

II. Problem Formulation

The problem considered is depicted in Fig. 1, where multiple plants are controlled over a WSN using the IEEE 802.15.4 protocol. M plants contend to transmit sensor measurements
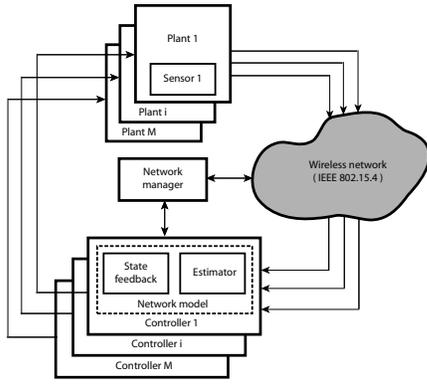
Fig. 1. Overview of the networked control system setup. M plants need to be controlled by M controllers. The wireless network closes the loop from the sensor nodes to the controllers.

to the controller over a wireless network that induces packet losses and varying delays. We assume that a sensor node is attached to each plant. A contention-based IEEE 802.15.4 protocol is used to determine which sensor node accesses the wireless channel. Throughout this paper we consider control applications where nodes asynchronously generate packets when a timer expires. When a node sends a packet successfully or discards a packet, it stays in an idle period for $\overline{h}$ seconds without generating packets. The data packet transmission is successful if an acknowledgement (ACK) packet is received. We assume that the controller commands are always successfully received by the actuator. Many practical NCSs have several sensing channels, whereas the controllers are collocated with the actuators, as in heat, ventilation and air-conditioning control systems [12].

We consider a plant i, for $i = 1, \dots, M$, given by a linear stochastic differential equation

$$ dx(t) = Ax(t)dt + Bu(t)dt + dw(t) \quad (1) $$

where $x(t) \in \mathbb{R}^n$ is the plant state and $u(t) \in \mathbb{R}^m$ is the control signal. The process disturbance $w(t) \in \mathbb{R}^n$ has a mean value of zero and uncorrelated increments, with incremental covariance $R_w dt$. We neglect the plant index i to simplify notation. Let us consider the sampling of the plant with time-varying sampling period $h_k = t_{k+1} - t_k$ and delay d_k [13]. The sampling period is $h_k = \overline{h} + d_k$ where the idle period $\overline{h}$ is constant and the random delay is d_k, which is bounded $d_k \leq d_{\text{req}}$. We assume that the random sequences $\{d_k\}$ and $\{h_k\}$ are bounded, $0 < d_k < h_k$ and $0 < h_{\min} \leq h_k \leq h_{\max}$. In addition, they are independent and have known distributions. Notice that the networked induced delay d_k is less than h_k and allows the packets to arrive at the controller in the correct order. By considering zero-order-hold, a time-varying discrete-time system is obtained

$$ x_{k+1} = \Phi_k x_k + \Gamma_0^k u_k + \Gamma_1^k u_{k-1} + w_k $$
$$ y_k = C x_k + v_k \quad (2) $$

where $\Phi_k = e^{Ah_k}$, $\Gamma_0^k = \left[\int_0^{h_k - d_k} e^{As} ds \right] B$, $\Gamma_1^k = \left[\int_{h_k - d_k}^{h_k} e^{As} ds \right] B$, and v_k is a discrete-time white Gaussian noise with zero mean and variance R_v. The parameter k is the discrete time index. The initial state x_0 is white Gaussian with mean $\overline{x}_0$ and covariance P_0.

Packet loss is first modelled as a random process whose parameters are related to the behavior of the network. The measurement at the controller side is given by

$$ \hat{y}_k = \begin{cases} C x_k + v_k, & \gamma_k = 1, \\ 0, & \gamma_k = 0, \end{cases} \quad (3) $$

where γ_k is a Bernoulli random variable with $\Pr(\gamma_k = 1) = 1 - p$, where p is the packet loss probability which models the packet loss between the sensor and the controller.

By considering both the packet loss and delay induced by a wireless network, we introduce an augmented discrete-time state variable $z_k = \begin{pmatrix} x_k & u_{k-1} \end{pmatrix}^T$ to analyze the system. The augmented state space is

$$ z_{k+1} = \Phi_d z_k + \Gamma_d u_k + w_k $$
$$ \hat{y}_k = \gamma_k y_k \quad (4) $$

where $\Phi_d = \begin{pmatrix} \Phi & \Gamma_1 \\ 0 & 0 \end{pmatrix}, \Gamma_d = \begin{pmatrix} \Gamma_0 \\ \mathbf{I} \end{pmatrix}$ and $C_d = (C \quad \mathbf{0})$.

In Fig. 1, a network manager block is introduced to achieve an efficient control system over a wireless network. Particularly, the network manager requires an analytical model of the packet loss and delay (i.e., between the sensors and controller). Then, this model is used to design the estimator and controller that compensate for the packet loss and delay induced by the network. The network manager is based on a constrained optimization problem where the objective function, denoted by E_{tot}, is the total energy consumption of the wireless network and the constraint is the desired control cost. Hence, the constrained optimization problem of the control system is

$$ \min_{h, \mathbf{V}} \quad E_{\text{tot}}(h, \mathbf{V}, \delta) \quad (5a) $$
$$ \text{s.t.} \quad J(h, p(h, \mathbf{V}, \delta), d(h, \mathbf{V}, \delta)) \leq J_{\text{req}}. \quad (5b) $$

The decision variables are h, which is the sampling period, and $\mathbf{V}$, which are the protocol parameters of the network. δ includes the parameters of the network setup such as a network topology, length of packet, and number of nodes. $J(h, p(h, \mathbf{V}, \delta), d(h, \mathbf{V}, \delta))$ is the control cost, which is a function of the sampling period h, packet loss probability p, and delay d of the network, and J_{req} is the desired maximum control cost. We remark that the packet loss probability and delay of the network is also a function of the sampling period h, protocol parameters $\mathbf{V}$ and parameters of the network setup δ. Thus, the sampling period h affects the performance of both wireless network and control system. In (5b), the decision variables are feasible if they satisfy a given control cost J_{req}. Note that it is possible to pose different optimization problems under the same framework.

III. WIRELESS MEDIUM ACCESS CONTROL PROTOCOL

In this section, we introduce the effective analytical model of packet loss probability and delay of the wireless network imposed by the IEEE 802.15.4 protocol which was originally derived in [14]. The contention-based MAC protocol of the IEEE 802.15.4 standard is used for control systems in this paper. We first present the overview of the carrier sense multiple access with collision avoidance (CSMA/CA) mechanism of the IEEE 802.15.4 protocol and provide an analytical model of the wireless network.

Consider a node trying to transmit. In the slotted CSMA/CA algorithm, first the MAC sub-layer of the node initializes four variables, i.e., the number of backoffs (NB=0), contention window (CW=2), backoff exponent (BE=*macMinBE*), and retransmission times (RT=0). Then the MAC sub-layer delays for a random number of complete backoff periods in the range $[0, 2^{\text{BE}} - 1]$ units. When the backoff period is zero, the node performs the first Clear Channel Assessment (CCA). If two consecutive CCAs are idle, then the node commences the packet transmission. If either of the CCA fails due to a busy channel, the MAC sublayer will increase the value of both NB and BE by one up to a maximum value *macMaxCSMABackoffs* and *macMaxBE*, respectively. Hence, the value of NB and BE depend on the number of CCA failures of a packet. Once the BE reaches *macMaxBE*, it remains at the value of *macMaxBE* until it is reset. If NB exceeds *macMaxCSMABackoffs*, then the packet is discarded due to the channel access failure. Otherwise, the CSMA/CA algorithm generates a random number of complete backoff periods and repeats the process. Here, the variable *macMaxCSMABackoffs* represents the maximum number of times the CSMA/CA algorithm is required to backoff. If channel access is successful, the node transmits the frame and waits for ACK. The reception of the corresponding ACK is interpreted as successful packet transmission. If the node fails to receive ACK due to collision or ACK timeout, the variable RT is increased by one unit up to *macMaxFrameRetries* units. If RT is less than *macMaxFrameRetries*, the MAC sublayer initializes two variables CW=0, BE=*macMinBE* and follows the CSMA/CA mechanism to re-access the channel. Otherwise the packet is discarded due to the retry limits.

In such a scenario, a precise and effective analytical model of the slotted CSMA/CA of the IEEE 802.15.4 standard was proposed in [14]. It is modelled through a Markov chain taking into account retry limits, ACKs, unsaturated traffic load, and the parameters of the network setup such as a length of packet and number of nodes. Let $s(t), c(t)$ and $r(t)$ be the stochastic process representing the backoff stage, the state of the backoff counter, and the state of retransmission counter at time t, respectively, experienced by a node to transmit a packet. By assuming that nodes start sensing independently, the stationary probability μ that the node attempts the first carrier sensing in a randomly chosen slot time is constant and independent of the other nodes. It follows that (s, c, r) results in a three dimensional Markov chain with the time unit *aUnitBackoffPeriod* (corresponding to 0.32 ms). The channel accessing probability μ that a node attempts the first CCA, the first busy channel probability α for the first CCA, and the second busy channel probability β for the second CCA are derived by solving the state transition probabilities associated with the Markov chain model. Note that the expressions of μ, α, and β are computed by solving a system of non-linear equations. The precise model gives us the objective function, energy consumption (5a), and the packet loss probability and delay in a numerical form. Note that the protocol parameters $\mathbf{V}$ of the decision variables are the MAC parameters (*macMinBE*, *macMaxCSMABackoffs*, *macMaxFrameRetries*).

IV. DESIGN OF ESTIMATOR AND CONTROLLER

In this section, we investigate how the packet loss probability and delay of the network affect the control performance.

We discuss the design of an optimal feedback controller and present a control cost to analyze the NCSs described in Section II. We first introduce our performance indicator as a control cost function, which is an explicit function of the sampling period h, packet loss probability p, and delay d of the network. Then, we design the estimator and controller under packet losses and delays in Section IV-A and IV-B, respectively. This is achieved by extending the results on optimal stochastic estimation and control under packet losses in [4] with delays in [6].

Let us first define the information set under the packet loss and network induced delay as follows

$$\mathcal{I}_k = \{\mathbf{y}^k, \boldsymbol{\gamma}^k\} \tag{6}$$

where $\mathbf{y}^k = (y_k, y_{k-1}, \ldots, y_1)$ and $\boldsymbol{\gamma}^k = (\gamma_k, \gamma_{k-1}, \ldots, \gamma_1)$.

Consider the control cost function

$$J_N(\boldsymbol{u}^{N-1}, \bar{z}_0, P_0) = \mathbf{E}[z_N^T W_N z_N \\ + \sum_{k=0}^{N-1} (z_k^T W_k z_k + 2 z_k^T N_k u_k + u_k^T U_k u_k)], \tag{7}$$

where $\bar{z}_0 = (\bar{x}_0 \quad 0)^T$, P_0 is the covariance of the initial condition, and the matrices W_k, N_k and U_k are time-invariant, symmetric and positive definite. In the following section, we introduce the estimator design.

A. Estimator Design

The estimator design is based on arguments similar to the standard Kalman filtering. Let us define the following variables

$$\hat{z}_{k|k} = (\mathbf{E}[x_k|\mathcal{I}_k] \quad u_{k-1})^T \\ P_{k|k} = \mathbf{E}[(z_k - \hat{z}_{k|k})(z_k - \hat{z}_{k|k})^T|\mathcal{I}_k].$$

The innovation step is given by

$$\hat{z}_{k+1|k} = \Phi_d \mathbf{E}[z_k|\mathcal{I}_k] + \Gamma_d u_k = \Phi_d \hat{z}_{k|k} + \Gamma_d u_k \tag{8}$$
$$P_{k+1|k} = \Phi_d P_{k|k} \Phi_d^T + R_w \tag{9}$$

where w_k and $\mathcal{I}_k$ are independent and u_k is a deterministic function of $\mathcal{I}_k$. The correction step is given by

$$\hat{z}_{k+1|k+1} = \hat{z}_{k+1|k} + \gamma_{k+1} K_{k+1}(y_{k+1} - C_d \hat{z}_{k+1|k}) \tag{10}$$
$$K_{k+1} = P_{k+1|k} C_d^T (C_d P_{k+1|k} C_d^T + R_v)^{-1}$$
$$P_{k+1|k+1} = P_{k+1|k} - \gamma_{k+1} K_{k+1} C_d P_{k+1|k} \tag{11}$$

where we apply the standard derivation for the Kalman filter.

B. Controller Design

We introduce the feedback control law and present the finite and infinite horizon control cost functions. The cost function given by Eq. (7) can be expressed as

$$J_N^* = V_0(x_0) = \bar{z}_0^T S_0 \bar{z}_0 + \text{Tr}(S_0 P_0) + \sum_{k=0}^{N-1} (\text{Tr}((\Phi_d^T S_{k+1} \\ \times \Phi_d + W_k - S_k) \mathbf{E}_\gamma[P_{k|k}]) + \text{Tr}(S_{k+1} R_w)) \tag{12}$$

where S_k is the solution of the Riccati equation as defined in [4] and Tr denotes the trace of a square matrix. $\mathbf{E}_\gamma[\cdot]$ is the expectation with respect to the arrival sequence $\{\gamma_k\}$.
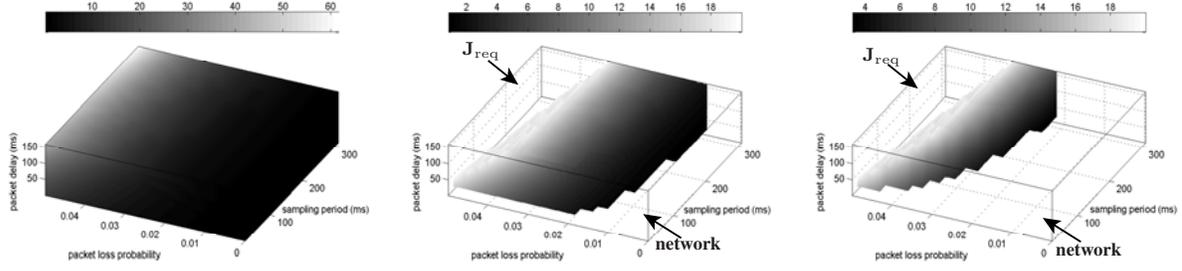
(a) Feasible control cost for the simplified case.

(b) Feasible control cost with $M = 10$.

(c) Feasible control cost with $M = 20$.

Fig. 2. Feasible control cost over different sampling periods, packet loss probabilities, and packet delays. The colors show the control cost. Note that the scales of color bar are different in the figures.

The control input that minimizes the cost function of Eq (7) is

$$u_k = -(\Gamma_d^T S_{k+1} \Gamma_d + U_k)^{-1} \Gamma_d^T S_{k+1} \Phi_d \hat{z}_{k|k} = -L_k \hat{z}_{k|k}. \tag{13}$$

The expected value $\mathbf{E}_\gamma[P_{k|k}]$ is bounded by

$$\tilde{P}_{k|k} \leq \mathbf{E}_\gamma[P_{k|k}] \leq \hat{P}_{k|k}, \qquad \forall k \geq 0$$

where the matrices $\tilde{P}_{k|k}$ and $\hat{P}_{k|k}$ can be found in [4]. Then, it is possible to derive the bound of control cost given in Eq. (12). In the next section, we use two deterministic sequences $J_N^{\min}$ and $J_N^{\max}$, which bound the expected minimum cost as follows

$$\frac{1}{N} J_N^{\min} \leq \frac{1}{N} J_N^* \leq \frac{1}{N} J_N^{\max}, \tag{14}$$

and the two sequences converge to the following values:

$$J_\infty^{\max} = \text{Tr}((\Phi_d^T S_\infty \Phi_d + W_k - S_\infty)(\overline{P}_\infty - (1-p)\overline{P}_\infty C_d^T \\ \times (C_d \overline{P}_\infty C_d^T + R_v)^{-1} C_d \overline{P}_\infty)) + \text{Tr}(S_\infty R_w) \tag{15}$$

$$J_\infty^{\min} = p\text{Tr}((\Phi_d^T S_\infty \Phi_d + W_k - S_\infty)\underline{P}_\infty) + \text{Tr}(S_\infty R_w) \tag{16}$$

where,

$$\overline{P}_\infty = \Phi_d \overline{P}_\infty \Phi_d^T + R_w - (1-p)\Phi_d \overline{P}_\infty C_d^T \\ \times (C_d \overline{P}_\infty C_d^T + R_v)^{-1} C_d \overline{P}_\infty \Phi_d^T \\ \underline{P}_\infty = p\Phi_d \underline{P}_\infty \Phi_d^T + R_w.$$

We remark that Eqs. (15) and (16) are explicit functions of the sampling period h, packet loss probability p, and delay d. The finite horizon cost and the cost bounds of the infinite horizon case will be used as the performance indicators in Section V-A.

V. CO-DESIGN FRAMEWORK

In this section, we first show the feasible control performance by taking into account realistic simulation results. Then, we study the co-design of the wireless NCS.

A. Effects of Wireless Network

In this section, we discuss the fundamental issues of co-design of communication network and controller for wireless NCSs. The control cost (15) is considered as a performance indicator of the control system as described in Section IV. As an example we consider an unstable second-order plant in the form of (1) with

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ P_0 = 0.01I$$

$$W = I, \ N = 0, \ U = 0.01, \ R_w = I, \ R_v = 0.01I,$$

where W, N, U are assumed to be time-invariant in Eq. (7).

Fig. 2 shows the feasible control cost with respect to different sampling periods, packet loss probabilities, packet delays with the simplified case and the realistic wireless networks for the different number of nodes $M = 10, 20$. Note that the simplified case does not explicitly consider the realistic network behavior i.e., independent relationship between sampling period, packet loss probability, and packet delay. In the figures, the colors show the feasible control cost. Fig. 2(a) depicts the simplified case where longer sampling periods increase the control cost. Furthermore, we observe that packet losses at a higher sampling period are more critical than packet losses at a lower sampling period, indicating that we are sampling in a conservative way. Similarly, we derive the effects of packet delay on the control cost. Figs 2(b) and 2(c) depict the feasible region for $M = 10$ and 20 nodes, respectively. Note that we set the desired control cost $J_{\text{req}} = 20$. A point is feasible if it satisfies a given required cost, packet loss probability and delay for each sampling period. The feasible region is the set of all feasible points. In the figure, the transparent region denotes that the desired control cost is not feasible. It is natural that as the control requirement becomes strict, the infeasible region increases, since it also requires lower packet loss probability and delay of the network for lower sampling periods. Observe in Figs. 2(b) and 2(c) that the packet loss probability $p \leq 0.01$ is not feasible when the sampling period is short $h \leq 0.03$ s. Since short sampling periods increase the traffic load of the network, the packet loss probability is closer to the critical packet loss probability, above which the system is unstable. Hence, it is difficult to achieve a low packet loss probability when the sampling period is short. Furthermore, by comparing Figs. 2(b) and 2(c), we see that the infeasible region increases as the number of nodes increases. We remark that the infeasible region due to the
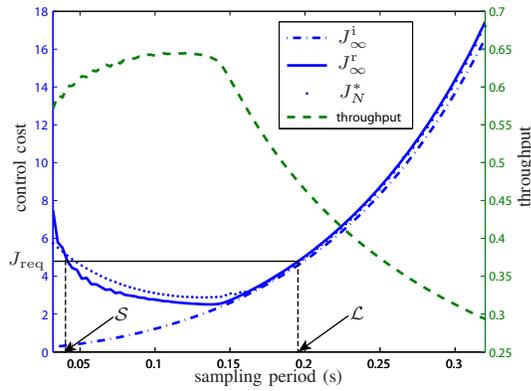
Fig. 3. Control cost and throughput of the wireless network over different sampling periods. J_∞^i and J_∞^r refer to the cost bound $J_\infty^{\max}$ of the infinite horizon control cost given in (15) with ideal case and realistic model in [14], respectively. J_N^* denotes the finite horizon control cost given in (12).

Fig. 4. Flow diagram of co-design framework.

wireless network starts from the origin where the sampling period $h = 0$, no packet loss $p = 0$, and no packet delay $d = 0$. No matter what communication protocol is used, the origin belongs to the infeasible region. The area and shape of the infeasible region depends on the communication protocol.

Fig. 3 shows the control cost and communication throughput over different sampling periods. The throughput is the average rate of successful data transmission over a communication channel, which is the common objective for a communication designer. In the figure, J_∞^i and J_∞^r refers to the cost bound $J_\infty^{\max}$ given by Eq. (15) for the ideal (no packet loss and no delay) and realistic model in [14], respectively. Recall that J_N^* is the finite horizon control cost given by Eq. (12). The cost J_N^* follows the infinite horizon cost J_∞^r based on the realistic model. Due to the absence of packet losses and delays, the control performance when using an ideal network increases monotonically as the sampling period increases. However, when using a real network, a shorter sampling period does not minimize the control cost of the control systems, because of the higher packet loss probability when the traffic load is high. In addition, the two curves of the cost J_∞^i and J_∞^r coincide for longer sampling periods, meaning that when the sampling period is larger, the sampling period is the dominant factor in the control cost compared to the packet loss probability and delay.

Now, let us discuss the throughput of the communication network and control cost of control systems. When we flip the throughput curve on the Y-axis, we observe a similar trend of behavior with the curve of control cost. Note that the closer the throughput is to 1, the better the utility of the wireless network. As the sampling period $h \in [0, 0.13]$ s increases, the control cost decreases and the throughput increases due to mainly high packet loss. For a longer sampling period $h > 0.15$ s, the performance of both the communication and control system degrades as the sampling period increases. The throughput decreases since the network is underutilized. We remark that the objective of both communication design and control design has a very similar trend. Hence, the optimal traffic load of the network is similar when the communication throughput or control cost are optimized. Even though the dynamic interactions between these two objectives, throughput of the communication and control cost of control system, are critical factors for wireless NCSs, these issues are not well investigated in the previous literatures.
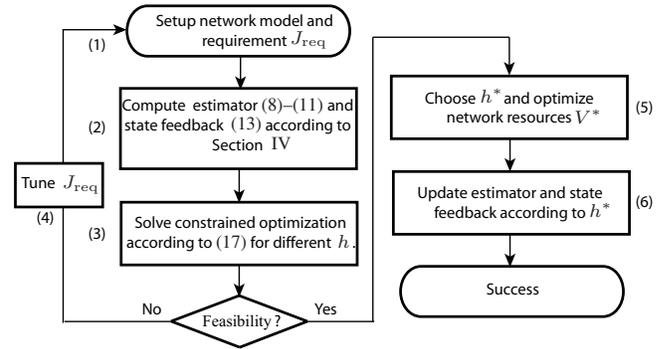
Let us consider a desired maximum control cost J_{req} greater than the minimum value of the control cost. Then, we have two feasible sampling periods $\mathcal{S}$ and $\mathcal{L}$ in Fig. 3. However, the performance of the wireless network is still heavily affected by the choice of the sampling period of $\mathcal{S}$ and $\mathcal{L}$, as we discussed earlier. By choosing $\mathcal{L}$, the throughput of the network is stabilized (see details in [15]), hence, the control cost is also stabilized with respect to small perturbations of the network. Therefore, the wireless NCSs achieve good robustness for both communication and control perspective by choosing $\mathcal{L}$. Furthermore, a longer sampling period $\mathcal{L}$ leads to lower network energy consumption than the shorter sampling period $\mathcal{S}$ in [14]. Recall that the energy efficiency is one of the most critical issues for sensor nodes due to their limited battery power. This motivates our co-design approach of NCSs running over WSNs.

B. Design Procedure

We remind that the problem we consider in this paper is how to determine the optimal sampling period h^* of control systems and the protocol parameters $\mathbf{V}^*$ of the communication protocol of an optimization problem given by Eq. (5). Fig. 4 shows the proposed design flow that each control loop of the network follows. The application designer provides the parameters of network setup δ and the desired maximum control cost J_{req}. δ includes the important factors for modeling the wireless network such as a network topology, length of the packets, and the number of nodes (step 1). It is also possible that each control loop has a different desired maximum control cost J_{req}. The control designer then computes, off-line, an estimator (8)–(11) and a state feedback (13) according to Section IV for different sampling periods, packet loss probabilities, and delays (step 2). The network manager formulates and solves a constrained optimization problem, whereby the objective function is the energy consumption of the network and the constraints are the packet loss probability and delay, which are derived from J_{req} for different sampling periods (step 3). More precisely, the constrained optimization problem is formulated from (5) for a given sampling period h as follows

$$\min_{\mathbf{V}} \quad E_{\text{tot}}(h, \mathbf{V}, \delta) \tag{17a}$$

$$\text{s.t.} \quad p(h, \mathbf{V}, \delta) \le p_{\text{req}}, \tag{17b}$$

$$d(h, \mathbf{V}, \delta) \le d_{\text{req}}. \tag{17c}$$

The decision variables are the communication protocol parameters $\mathbf{V}$ depending on the network designer. The adaptive IEEE 802.15.4 protocol [16] is applied to meet the

| (a) control cost. | (b) power consumption of a node. | (c) interval of packet generated time. | (d) packet loss probability. |
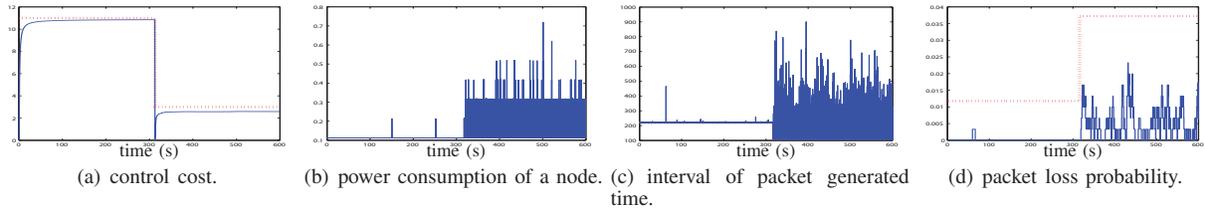
Fig. 5. Optimized control cost, power consumption of the network, interval of packet generated time, and packet loss probability of the proposed co-design approach with $M = 20$ nodes when the control requirement changes from $J_{\text{req}} = 11$ to $J_{\text{req}} = 3$ at 315 s. The particular realization is shown out of $M = 20$ nodes. The dotted line shows the requirement change of each figures.

requirements for packet loss probability and packet delay for a given sampling period. One can find a sub-optimal solution using the steps described in [16]. The network manager finds the local optimal MAC parameters $\mathbf{V}^*(h, p_{\text{req}}, d_{\text{req}})$ of a sub-optimization problem for a given $h, p_{\text{req}}, d_{\text{req}}$. Then, the optimal solution $h^*, \mathbf{V}^*$ is given by the pair $h, \mathbf{V}$ that minimizes the cost function if there are feasible solutions (step 5). Otherwise, the control designer needs to tune J_{req} since the desired control cost is not realistic (step 4). The network manager adapts the optimal sampling period h^* and the optimal protocol parameters V^* of the network (step 5). The control designer updates the estimator and the state feedback according to the optimized $h^*, p(h^*, \mathbf{V}^*, \delta), d(h^*, \mathbf{V}^*, \delta)$ (step 6).

VI. ILLUSTRATIVE EXAMPLE

In this section, we illustrate the proposed co-design procedure described in Section V-B through numerical examples. Fig. 5 shows the adaptation of the requirements in terms of the sampling period, and packet loss probability of the network when the control requirement changes from $J_{\text{req}} = 11$ to $J_{\text{req}} = 3$ at 315 s. The optimal parameters $h^*, p_{\text{req}}, d_{\text{req}}$ are 214.4 ms, 0.012, 74.9 ms before control requirement changes, respectively. Figs. 5(c), and 5(d) show that the adaptive communication protocol satisfies the requirements of h and p_{req}, respectively. Note that the proposed protocol also meets the requirement of the packet delay. The high jitter of Fig. 5(c) is mainly due to the packet loss of Fig. 5(d). After the control requirement changes at time 315 s, the optimal parameters $h, p_{\text{req}}, d_{\text{req}}$ adapt to 102.4 ms, 0.037, 97.4 ms, respectively. We remark that although the requirements of packet loss probability and packet delay are less strict after the requirement changes, the sampling period decreases to meet the requirement $J_{\text{req}} = 3$. Recall that as the sampling period decreases, the packet loss probability and packet delay increase. Observe that the control cost is satisfied and the convergence of the algorithm is very fast. By comparing Figs. 5(a) and 5(b), the tradeoff between the control cost and power consumption of the network is clearly observed.

VII. CONCLUSIONS

The dynamic interactions between communication network and control system are critical factors to guarantee the stability of wireless NCS. In this paper, it is shown how the design framework of the WSNs is applicable to control applications. We first present how the wireless network affects the performance of NCSs by showing the feasible region of the control performance. Furthermore, the optimal traffic load of the network is similar when the communication throughput or control cost are optimized. By considering these results,

we conclude that the sampling period significantly influences not only the control performance, and throughput and energy consumption of the network, but also the robustness of the wireless NCS. A co-design between communication and control application layers is proposed for multiple control systems over the IEEE 802.15.4 wireless network. In particular, a constrained optimization problem is studied, where the objective function is the energy consumption of the network and the constraints are the packet loss probability and delay, which are derived from the desired control cost. Numerical results illustrate the efficiency of the proposed co-design approach.

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