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A MULTIVARIABLE LABORATORY PROCESS WITH AN ADJUSTABLE ZERO*

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Abstract

A novel multivariable laboratory process that consists of four interconnected water tanks is presented. The linearized dynamics of the system have a multivariable zero that is possible to move along the real axis by changing a valve. The zero can be placed in both the left and the right half-plane. In this way the quadruple-tank process is ideal for illustrating many concepts in multivariable control, particularly performance limitations due to multivariable right half-plane zeros. Accurate models are derived from both physical and experimental data and multi-loop control is illustrated.

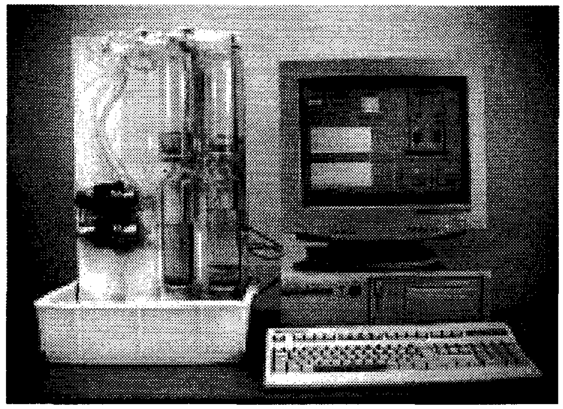

Figure 1 The quadruple-tank laboratory process shown together with a new controller interface running on a Pentium PC.

1. Introduction

There is an increased industrial interest in the use of multivariable control techniques. They are needed to achieve improved performance of complex industrial processes [16]. Therefore, it is important to include multivariable methods in the control curriculum. Of course, true understanding and engineering skills are only obtained if these concepts are illustrated in laboratory exercises. However, few multivariable laboratory processes have been reported in the literature. Mechanical systems such as the helicopter model [11, 1] and the active magnetic bearing process [19] have been developed at ETH in Zürich. Davison has developed a water tank process, where multivariable water level control and temperature–flow control can be investigated [3]. Some multivariable laboratory processes are commercially available, for example from Quanser Consulting in Canada, Educational Control Products in U.S., and Feedback Instruments and TecQuipment in U.K.

This paper describes a new laboratory process that consists of four interconnected water tanks and two pumps. The system is shown in Figure 1. Its inputs are the voltages to the two pumps and the outputs are the water levels in the lower two tanks. This quadruple-tank process is a simple interconnection of two double-tank processes, which are standard processes in many control laboratories [2]. The setup is thus simple, but still the process can illustrate interesting multivariable phenomena. The linearized model of the quadruple-tank process has a multivariable zero, which can be located in either the left or the right half-plane by simply changing a valve. Control performance limitations due to zero locations can be derived from complex analysis [4, 15]. These illustrate fundamental restrictions on the possible choice of closed-loop system. For example, right half-plane zeros impose restrictions on the sensitivity function: if the sensitivity is forced to be small in one frequency band, it has to be large in another, possibly yielding an overall bad performance. The fundamentals for what can be achieved with linear control have also received industrial interest and application [17, 5].

The outline of the paper is as follows. A nonlinear

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model for the quadruple-tank process based on physical data is derived in Section 2. It is linearized and some properties of the linear model is emphasized. In Section 3 linear models are estimated from experimental data and they are compared to the physical model. Simple multi-loop PI control of the quadruple-tank process is performed in Section 4 and some concluding remarks are given in Section 5. See [6] for a thorough description of the quadruple-tank process and its properties.

2. Physical Model

In this section we derive a mathematical model for the quadruple-tank process from physical data. A schematic diagram of the quadruple-tank process is shown in Figure 2. The target is to control the level in the lower two tanks with two pumps. The process inputs are v_1 and v_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltages from level measurement devices). Mass balances and Bernoulli's law yield

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1, \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2, \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2, \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1,
\end{aligned} \tag{1}$$

where A_i is the cross-section of Tank i, a_i the cross-section of the outlet hole, and h_i the water level. The voltage applied to Pump i is v_i and the corresponding flow is $k_i v_i$. The parameters $\gamma_1, \gamma_2 \in [0,1]$ are determined from how the valves are set. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and the flow to Tank 4 is $(1-\gamma_1)k_1 v_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted g. The measured level signals are $k_c h_1$ and $k_c h_2$. The parameter values of the laboratory process are given in the following table:

A_1, A_3	[cm^2]	28
A_2, A_4	[cm^2]	32
a_1, a_3	[cm^2]	0.071
a_2, a_4	[cm^2]	0.057
k_c	[V/cm]	0.50
g	[cm/s^2]	981

The model and control of the quadruple-tank process are studied at two operating points: P_- at which the system will be shown to have minimum-phase characteristics and P_+ at which it will be shown to have nonminimum-phase characteristics. The operating points correspond to the following parameter values:

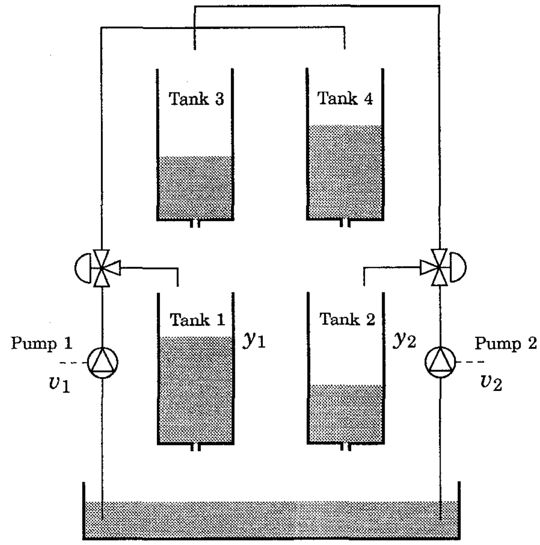

Figure 2 The quadruple-tank laboratory process. The water levels in Tank 1 and Tank 2 are controlled by two pumps. When changing the position of the valves, the location of a multivariable zero for the linearized model is moved.

		P_-	P_+
(h_1^0, h_2^0)	[cm]	(12.4, 12.7)	(12.6, 13.0)
(h_3^0, h_4^0)	[cm]	(1.8, 1.4)	(4.8, 4.9)
(v_1^0, v_2^0)	[V]	(3.00, 3.00)	(3.15, 3.15)
(k_1, k_2)	[cm^3/Vs]	(3.33, 3.35)	(3.14, 3.29)
(γ_1, γ_2)		(0.70, 0.60)	(0.43, 0.34)

Multivariable zeros

Linearization of (1) gives the transfer matrix

$$G(s) = \begin{pmatrix} \frac{\gamma_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{pmatrix},$$

where $T_i = A_i a_i^{-1}\sqrt{2h_i^0/g}$, $c_1 = T_1 k_1 k_c / A_1$, and $c_2 = T_2 k_2 k_c / A_2$. The multivariable zeros are in our case the zeros of the numerator polynomial of the rational function

$$\det G(s) = \frac{c_1 c_2}{\gamma_1\gamma_2 \prod_{i=1}^4 (1+sT_i)} \times \left[(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\right].$$

The transfer matrix G thus has two finite zeros for $\gamma_1, \gamma_2 \in (0, 1]$. A root-locus argument gives that one of them is always in the left half-plane, but the other can be located either in the left or the right half-plane. Introduce the parameter $\eta := (1-\gamma_1)(1-\gamma_2)/\gamma_1\gamma_2$. If η is small, the two zeros are close to $-1/T_3$ and $-1/T_4$, respectively. Furthermore, one zero tends to $-\infty$ and one zero tends to $+\infty$ as $\eta \to \infty$. If $\eta = 1$ one zero is located at the origin. This case corresponds to $\gamma_1 + \gamma_2 = 1$. It follows that the system is nonminimum phase for $0 < \gamma_1 + \gamma_2 \le 1$ and minimum phase for $1 < \gamma_1 + \gamma_2 \le 2$. Recall that $\gamma_1 + \gamma_2 = 1.30 > 1$ for P_- and $\gamma_1 + \gamma_2 = 0.77 < 1$ for P_+.

The multivariable zero being in the left or in right half-plane has a straightforward physical interpretation. For simplicity assume that $q_1 = q_2$. Then the sum of the flows to the upper tanks is $[2 - (\gamma_1 + \gamma_2)]q_1$ and the sum of the flows to the lower tanks is $(\gamma_1 + \gamma_2)q_1$. Hence, the flow to the lower tanks are greater than the flow to the upper tanks if and only if the system is minimum phase. It is intuitively easier to control y_1 with u_1 and y_2 with u_2 if most of the flows goes directly to the lower tanks. This gives an immediate physical interpretation of the control limitations imposed by the multivariable zero.

For the two operating points P_- and P_+ we have the following time constants and zeros:

	P_-	P_+
(T_1, T_2)	$(62, 90)$	$(63, 91)$
(T_3, T_4)	$(23, 30)$	$(39, 56)$
Zeros	$(-0.060, -0.018)$	$(-0.057, 0.013)$

The dominating time constants are thus similar in both operating conditions. The physical modeling gives the two transfer matrices

$$G_-(s) = \begin{pmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.8}{1+90s} \end{pmatrix}, \qquad (2)$$

$$G_+(s) = \begin{pmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{pmatrix}. \qquad (3)$$

Figure 3 shows the response of the minimum-phase model G_- compared to real data obtained from an identification experiment discussed in next section. The inputs are pseudo-random binary sequences (PRBSs) with low amplitudes, so that the dynamics are captured by a linear model. The model outputs agree very well with the responses of the real process. The nonminimum-phase model G_+ shows similar accuracy.

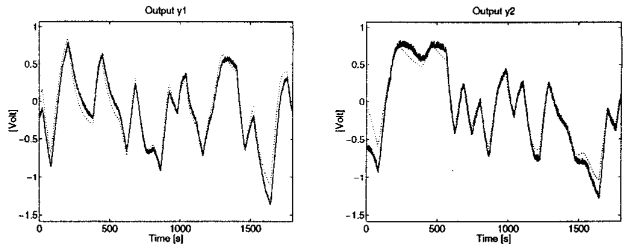

Figure 3 Validation of the linear physical model G_-. The outputs from the model (dashed lines) together with the outputs from the real process (solid lines) are shown in the minimum-phase setting.

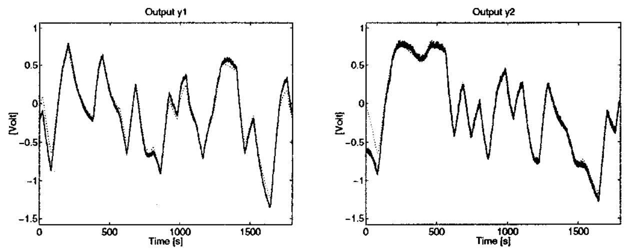

Figure 4 Validation of state-space model for the minimum-phase setting. Outputs from identified model (dashed) together with the outputs from the real process (solid) are shown.

3. System Identification

The physical model derived in previous section is now compared to a model estimated using standard system identification techniques [9, 8].

Both SIMO and MIMO identification experiments were performed with PRBS signals as inputs. The levels of the PRBS signals were chosen so that the process dynamics were approximately linear.

Black-box and gray-box identification methods were tested using Matlab's System Identification Toolbox [10]. Linear SISO, MISO, and MIMO maps were identified in ARX, ARMAX, and state-space forms. All model structures gave similar responses to validation data. Here we only present some examples of the results. We start with a black-box approach. Figure 4 shows validation data for the minimum-phase setting together with a simulation of a state-space model derived with the sub-space algorithm N4SID [18, 10]. The state-space model has three real poles corresponding to time constants 8, 41, and 113. It has one multivariable zero in -0.99. The simulation of the nonminimum-phase model is of similar accuracy. This model is of fourth order and has time constants 11, 31, 140, and 220. Its two zeros are located in -0.288 and 0.019. The validation result in Figures 4 is of similar quality as the result for the physical model shown in Figures 3. Note that the minimum-phase setting gives an identified model with no RHP zero, whereas the nonminimum-phase setting gives a dominating RHP zero (*i.e.*, a RHP zero close to the origin compared to the time scale given by the time constants).

Gray-box models with structure fixed to a linear state-space equation gave similar validation results as the previously shown. Because of the fixed structure, the number of poles and zeros are the same as for the physical model. For the minimum-phase setting we have time constants $(T_1, T_2, T_3, T_4) = (96, 99, 32, 39)$ and zeros at -0.045 and -0.012, whereas for the nonminimum-phase setting we have $(T_1, T_2, T_3, T_4) = (77, 112, 53, 55)$ and zeros 0.014 and -0.051. The zeros agree very well with the ones derived from the physical model.

4. Multi-Loop Control

The control law $u = \text{diag}\{C_1, C_2\}(r - y)$ is in this section applied to the real process as well as to nonlinear and linear process models. PI controllers of the form

$$C_\ell(s) = K_\ell\left(1 + \frac{1}{T_{i\ell}s}\right), \qquad \ell = 1, 2$$

are tuned manually based on the linear physical models (2) and (3).

For the minimum-phase setting P_- the controller parameters $(K_1, T_{i1}) = (3.0, 30)$ and $(K_2, T_{i2}) = (2.7, 40)$ are easily obtained. They give reasonable performances as shown in Figure 5, where the responses are given for a step in the reference signal r_1. The top four plots show control of the simulated nonlinear model in (1) (dashed lines) and control of the identified linear state-space model (solid). The four lower plots show the responses of the real process. The discrepancies between simulations and the true time responses are small.

It is hard to find good controller parameters for the nonminimum-phase setting P_+. The controller parameters $(K_1, T_{i1}) = (1.5, 110)$ and $(K_2, T_{i2}) = (-0.12, 220)$ stabilize the process, but give much slower responses than for the minimum-phase setting, see Figure 6. Note the different time scales compared to Figure 5. The settling time is approximately ten times longer for the nonminimum-phase setting. The control signal u_2 seems to be noiseless. This is due to the low gain K_2. It is no coincidence that K_2 is chosen negative. Because $\det G_+(0) < 0$, there exists no multi-loop PI controller with $K_1 = K_2 > 0$ that stabilizes the system, see Theorem 14.3-1 in [12]. Even if the controller gains are small the closed-loop system will be unstable.

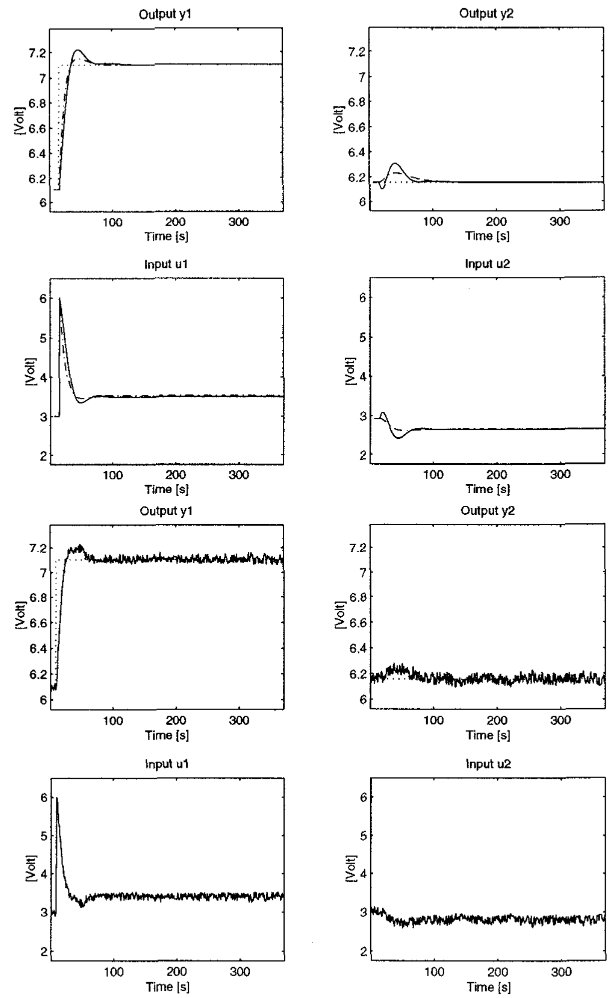

Figure 5 Results of PI control of minimum-phase system. The upper four plots show simulations with the nonlinear physical model (dashed) and the identified linear model (solid). The four lower plots show experimental results.

5. Conclusions

A new multivariable laboratory process that consists of four interconnected water tanks has been described. It was shown that the quadruple-tank process is well suited to illustrate performance limitations in multivariable control design caused by RHP zeros. This followed from that the linearized model of the process has a multivariable zero that in a direct way is connected to the physical position of two valves. Models from physical data and experimental data were derived and they were shown to have responses similar to the real process. Decentralized PI control showed that it was much more difficult to control the process in the nonminimum-phase case than in the minimum-phase case.

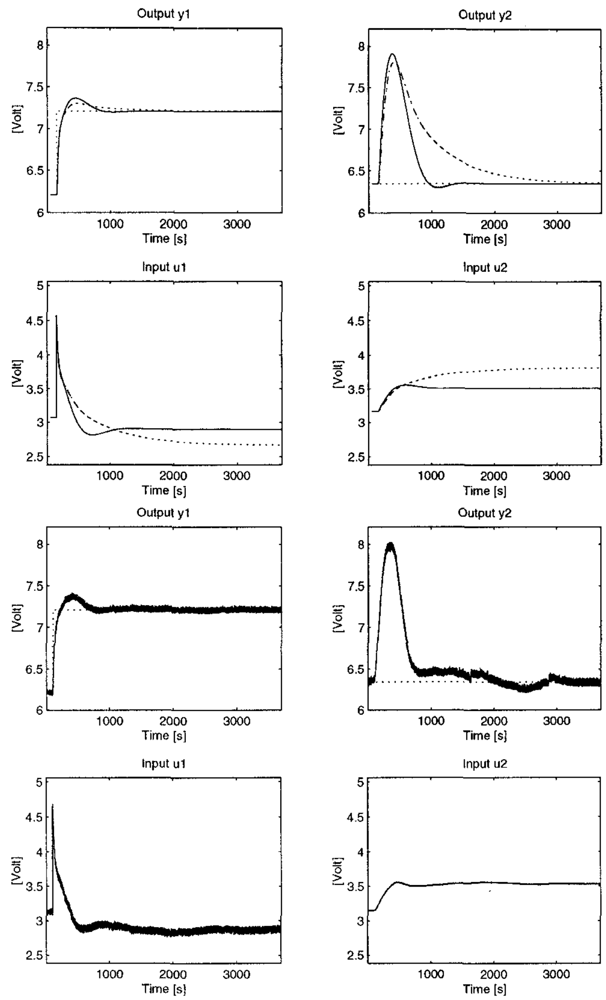

Figure 6 Results of PI control of nonminimum-phase system. Same variables are shown as in Figure 5. Note the ten times longer time scale.

The experiments described in this paper have been performed using a PC interface developed in the man-machine interface generator InTouch from Wonderware Corporation [13]. Ongoing work includes multivariable controller design for the quadruple-tank process. Multivariable controller-tuning method based on relay feedback experiments have been investigated on the process [7, 14].

6. References

[1] M. ÅKESSON, E. GUSTAFSON, and K. H. JOHANSSON. "Control design for a helicopter lab process." In *IFAC'96, Preprints 13th World Congress of IFAC*, San Francisco, CA, July 1996.

[2] K. J. ÅSTRÖM and M. LUNDH. "Lund control program combines theory with hands-on experience." *IEEE Control Systems Magazine*, **12:3**, pp. 22–30, 1992.

[3] E. J. DAVISON. "Description of multivariable apparatus for real time-control studies (MARTS)." Technical Report 8514a, Dept. of Electrical Engineering, Univ. of Toronto, Canada, 1985.

[4] J. FREUDENBERG and D. LOOZE. *Frequency Domain Properties of Scalar and Multivariable Feedback Systems.* Springer-Verlag, Berlin, Germany, 1988.

[5] G. C. GOODWIN. "Defining the performance envelope in industrial control." In *16th American Control Conference*, Albuquerque, NM, 1997. Plenary Session I.

[6] K. H. JOHANSSON. *Relay feedback and multivariable control.* PhD thesis ISRN LUTFD2/TFRT--1048--SE, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, November 1997.

[7] K. H. JOHANSSON, B. JAMES, G. F. BRYANT, and K. J. ÅSTRÖM. "Multivariable controller tuning." In *17th American Control Conference*, Philadelphia, PA, 1998.

[8] R. JOHANSSON. *System Modeling and Identification.* Prentice Hall, Englewood Cliffs, NJ, 1993.

[9] L. LJUNG. *System Identification—Theory for the User.* Prentice Hall, Englewood Cliffs, NJ, 1987.

[10] L. LJUNG. *System Identification Toolbox, Version 4.0.3.* The Mathworks, Inc, 1997.

[11] M. MANSOUR and W. SCHAUFELBERGER. "Software and laboratory experiment using computers in control education." *IEEE Control Systems Magazine*, **9:3**, pp. 19–24, 1989.

[12] M. MORARI and E. ZAFIRIOU. *Robust Process Control.* Prentice-Hall, Englewood Cliffs, NJ, 1989.

[13] J. R. NUNES. "Modeling and control of the quadruple-tank process." Master thesis ISRN LUTFD2/TFRT--5588--SE, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1997.

[14] V. RECICA. "Automatic tuning of multivariable controllers." Master thesis ISRN LUTFD2/TFRT--5592--SE, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1998.

[15] M. M. SERON, J. H. BRASLAVSKY, and G. C. GOODWIN. *Fundamental Limitations in Filtering and Control.* Springer-Verlag, 1997.

[16] F. G. SHINSKEY. *Controlling Multivariable Processes.* Instrument Society of America, Research Triangle Park, NC, 1981.

[17] G. STEIN. "Respect the unstable." In *30th IEEE Conference on Decision and Control*, Honolulu, HI, 1990.

[18] P. VAN OVERSCHEE and B. DE MOOR. "N4SID: Subspace algorithms for the identification of combined deterministic-stochastic systems." *Automatica*, **30:1**, pp. 75–93, 1994.

[19] D. VISCHER and H. BLEULER. "A new approach to sensorless and voltage controlled AMBs based on network theory concepts." In *2nd International Symposium on Magnetic Bearings*, Institute of Industrial Sciencek, Tokyo University, 1990.