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Guaranteeing safety for heavy duty vehicle platooning: Safe set computations and experimental evaluations

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ABSTRACT

In this paper, we consider the problem of finding a safety criteria between neighboring heavy duty vehicles traveling in a platoon. We present a possible framework for analyzing safety aspects of heavy duty vehicle platooning. A nonlinear underlying dynamical model is utilized, where the states of two neighboring vehicles are conveyed through radar information and wireless communication. Numerical safe sets are derived through the framework, under a worst-case scenario, and the minimum safe spacing is studied for heterogenous platoons. Real life experimental results are presented in an attempt to validate the theoretical results in practice. The findings show that a minimum relative distance of 1.2 m at maximum legal velocity on Swedish highways can be maintained for two identical vehicles without endangering a collision. The main conclusion is that the relative distance utilized in commercial applications today can be reduced significantly with a suitable automatic control system.

1. Introduction

The traffic intensity is escalating in many parts of the world, making traffic congestion a growing issue. In parallel, freight transport services has increased dramatically and will continue to do so as economies grow. Current drivers are already faced with several challenging scenarios each time they venture out on the road—challenges that will become harsher with increasing traffic intensity. In addition to more complex traffic situations, an increase in traffic naturally gives a higher fossil fuel usage and inherently a higher emission of harmful exhaust gas. Hence, governments, agencies, the private sector, and individuals around the world are trying to find ways to reduce the emissions and design systems to aid the driver in handling difficult situations. Combating climate change and rooting out its main causes, a problem due to increase in greenhouse gases, are among the top priorities in Europe.

Heavy duty vehicle (HDV) platooning, as illustrated in Fig. 1, serves as a possible partial solution to the mentioned issues. The concept of platooning for congestion and energy reduction is not new. Many experienced HDV drivers know that driving at a short intermediate distance to a vehicle ahead results in a lower required throttle action to propel the vehicle forward. It is due to a lowered air drag when operating in a formation. By packing HDVs close to each other, the total road capacity can be increased and emissions can be reduced. Additionally, when governing vehicle platoons by an automated control strategy, the overall traffic flow is expected to improve. It is fuel efficient to minimize the relative distance between the vehicles to achieve maximum reduction in air drag, but, as traffic intensity grows, the complexity of the coupled traffic dynamics increases. The actions of one vehicle may in turn affect all vehicles in a linked chain. Through improved sensor technology, wireless communication, GPS devices, and digital maps, advanced driver assistance systems are being developed to aid the driver. Key enabling technologies such as vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication have matured. However, they impose constraints in terms of accuracy, reliability, and delays. Therefore, safety constraints with respect to how close we might drive to a vehicle ahead without risking collision are a challenge. A question arises of how close the automated vehicles might operate without endangering a collision.

Commercially available systems, such as the adaptive cruise control (ACC), in a collision avoidance scenario currently uses radar measurements consisting of the relative distance and velocity to a preceding vehicle and adjusts the velocity automatically. A delay arises from measuring the behavior of the preceding vehicle to producing the actual brake torque at the wheels. As an

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Fig. 1. HDVs traveling in a platoon can achieve significant fuel reduction.

alternative to radar measurements, wireless communication of the breaking signal may be utilized. However, delays are still imposed due to data processing, retransmissions, etc. Thus, the impact of the vehicle control on the safety must be established and verified. Rigorous guarantees cannot only be obtained through extensive simulations, but mathematical tools need to be developed and real-world experiments performed.

The main contribution of this paper is to establish empiric results for validation of the analytical framework and numerical safe set computation for collision avoidance in HDV platooning scenarios. We propose an automated and reproducible method to derive empiric results for validation of safe sets. We show how the method has been evaluated experimentally using real HDVs provided by Scania CV AB on a test site near Stockholm. A differential game formulation of the problem enables the safe set derivation by capturing the event when the lead vehicle blunders in the worst possible manner. Based on the theoretic and empiric results, we determine criteria for which collisions can be avoided in a worst-case scenario and thereby establish the minimum possible safe distance in practice between vehicles in a platoon. We show that the minimum relative distance with respect to safety depends on the nonlinear behavior of the brake system and delays in information propagation along with the implemented control actions.

1.1. Problem formulation

We consider an HDV platooning scenario with two vehicles where the follower vehicle receives information regarding the relative position and velocity of the vehicle ahead.[1] The objective is to determine the minimum relative distance between the vehicles that can be maintained without endangering a collision. The aim is to find the largest set of initial states, irrespective of how the lead vehicle behaves, for which there exists a controller that manages to keep all executions inside a safe subset of the state space.

A differential game formulation of the problem enables such a set derivation by capturing the event when the lead vehicle blunders in the worst possible manner. We model the game as the lead vehicle (player u_1) is trying its best to create a collision and the follower vehicle (player u_2) is trying its best to avoid a collision.

Hence, the problem at hand can be set up as a two-vehicle dynamic pursuer-evader game:

$$\max_{u_2 \in \mathcal{E}} \min_{u_1 \in \mathcal{D}} p^T f(x,u), \tag{1}$$

where $f(x,u)$ denotes the system dynamics, x the state vector, $u=(u_1,u_2)$ the control inputs, and p the costates. Details are given in Section 4.

The problem we solve in this paper is the following: compute an unsafe set $\mathcal{U}(\tau)$ from where there is a possibility that a collision occurs within time $\tau > 0$ despite the best control effort from the follower vehicle. Numerical techniques for such computations are utilized and experimental validations are made.

1.2. Outline

The outline of the paper is as follows. First an overview of the related work in this area is presented in Section 2. Then the system model is presented in Section 3. In Section 4 we start by presenting the theoretical premise for computing safe sets and then apply it to platooning. Safe sets are derived for homogeneous and heterogeneous HDV platoons. The experimental setup for evaluating the safe sets is given in Section 5 together with experimental evaluations. Conclusions are given in Section 6. A preliminary version of this paper was presented by Alam, Gattami, Johansson, and Tomlin (2011), in which the analytical model was validated through an advanced simulation model. Some further details of the work is presented in the thesis by Alam (2011).

2. Related work

A multitude of control strategies for vehicle platooning can be found in the literature since 1950s. This section first outlines some of the early theoretical work and more recent implementation-relevant literature on automated vehicle platooning. Then a brief overview on collision avoidance and safety in vehicle applications is given.

2.1. Cooperative vehicle platooning

Vehicle platooning can be described as a chain of vehicles traveling at a given intermediate distance and velocity. The primary objective for each vehicle with respect to safety is to maintain its distance to the preceding vehicle in the platoon. A platoon of N vehicles is often modeled in the literature as a set of moving point masses

$$\ddot{x}_i + k_i \dot{x}_i = u_i, \quad i = 1, \ldots, N, \tag{2}$$

where x_i represents the position of vehicle i, $k_i \geq 0$ denotes a system damping coefficient and u_i is the applied control force. Early theoretical work on control of vehicular platoons was done by Levine and Athans (1966). A centralized LQR control design was considered for high-speed vehicle platoons, implicitly assuming precise models and that computational complexity and V2V communication constraints would not be an issue.

Communication constraints are sometimes a crucial issue in practice, see Gupta, Hassibi, and Murray (2004) and Alam, Gattami, and Johansson (2011). It is not realistic to assume that every vehicle in the platoon would know the state of every other vehicle instantaneously due to physical constraints in the information flow. However, it is reasonable to assume that a vehicle will be able to communicate with some vehicles within a given range. Hence, it is argued that the problem at hand is a distributed control problem with varying information flow patterns.

For vehicle platooning it is essential to use realistic models, as argued by Sahlholm and Johansson (2010) and Guzzella and

[1] The extension to platoons with $N > 2$ vehicles is discussed in the paper. The developed approach generalizes to this case, but the numerical computation are harder.

Sciarretta (2007), and not just identical low-order linear models, which has often been the case in the literature. In HDV platooning, mass and road slope has a significant effect on the system dynamics. Research into implementation aspects of vehicle platooning is only recently emerging. In Shaw and Hedrick (2007) heterogeneous vehicle strings under simple decentralized control laws with a constant spacing control policy were analyzed. Naus, Vugts, Ploeg, van de Molengraft, and Steinbuch (2009) presented a setup for cooperative ACC for which feasibility of the actual implementation was one of the main objectives. Another control design approach for cooperative ACC was presented in Bu, Tan, and Huang (2010). In Alam, Gattami, and Johansson (2010) it has been shown that there is a 4.7–7.7% fuel reduction potential in HDV platooning. A platoon of three automated HDVs was studied in Tsugawa, Kato, and Aoki (2011), and they obtained fuel savings up to 14%.

2.2. Safety in vehicle platooning

String stability for vehicle platoons is an important concept, see Swaroop and Hedrick (1996). It is related to the ability to suppress a disturbance in position, velocity, or acceleration, as it propagates along the platoon, which in turn might lead to a collision. Focusing on preventing collisions, the errors in spacing between the vehicles in the platoon are often considered. In Rajamani and Zhu (2002) practical systems with ACC were considered, where both manually driven and automated cars can coexist. It was shown that the intermediate spacing can be reduced while maintaining string stability through wireless communication. Control difficulties for large (or even infinite) vehicle platoons were studied in Bamieh and Jovanović (2005). In Liang, Alam, and Gattami (2011) it was shown that string stability can be obtained through an ordering strategy with respect to vehicle mass. Ensuring string stability does however not guarantee safety. If a collision occurs due to a harsh braking by any vehicle in the platoon, a collision can still occur downstream.

Collision avoidance has been studied in many areas of engineering such as maritime transportation, unmanned aerial vehicles (UAV), see Ryan, Zennaro, Howell, Sengupta, and Hedrick (2004), mobile robotics, see Siegwart and Nourbakhsh (2004), and automotive engineering. The literature on safety strategies for HDVs is scarce, even though collision avoidance for cars is a vast area. In the car industry, ACC systems with collision avoidance have been studied extensively and are now commercially available. A review can be found in Vahidi and Eskandarian (2003). In Seiler, Song, and Hedrick (1998) longitudinal collision avoidance algorithms by Mazda and Honda were reviewed and human factors were considered. Critical distances for a collision avoidance system is derived as a function of velocity and relative velocity. In this paper, we consider a similar reference framework, but for HDVs. Additionally, we use a dynamical game formulation to capture the worst possible behavior by a preceding vehicle. In Gustafsson (2009), automotive safety systems were reviewed. Recent work on collision avoidance for cars can be found in the thesis by Ali (2012), where reachability analysis tools were utilized for threat assessment and a novel automotive safety function was proposed, based on vehicle state and road preview information. Zonotopes, a special case of polytopes, is an approach for computing reachable sets by abstracting to differential inclusions of simpler dynamics, see Girard (2005). An approach to verify maneuvers of an automated car was presented in Althoff and Dolan (2012). Makhlouf and Kowalewski (2012) considered safety verification of a platoon of vehicles under a varying communication network using zonotopes.

The differential game approach based on optimization (Basar & Olsder, 1995, Chapter 5.3), adopted in this paper has previously been applied to air traffic management, e.g., Bayen, Shanthanam, Mitchell, and Tomlin (2003). The computational challenges for our system are similar, but the implementation aspects are quite different. For example, the delays imposed by inter-vehicle communication and sensor processing are important for platoon safety. Moreover, the braking capability of each vehicle plays a crucial role. Still we are able to use the same mathematical framework, conveniently packaged in the level set Matlab toolbox by Mitchell (2007).

3. Modeling

In this section we present the models that serve as a basis for the analysis and numerical studies. First a brief description is given on the internal and external forces affecting a vehicle in motion. The longitudinal model is then extended to describe the dynamics of an HDV platoon. More details on the vehicle model and its derivation can be found in Alam (2011).

3.1. Vehicle model

The main propelling parts of an HDV consist of engine, clutch, gearbox, propeller shaft, final drive, drive shafts, and wheels. Here, we consider a diesel engine, which in turn is connected to the clutch, producing the desired driving torque. The connection between the gearbox and the clutch is considered to be stiff. The transformation is modeled as a conversion ratio γ_t, which varies according to the specific gearbox transmission characteristics. Typically a slight drop in power transfer occurs in the gear box due to frictional losses. This characteristic of the gear box is modeled as an efficiency η_t. The frictional losses in the propeller shaft are negligible and the connection is considered to be stiff. Like the gearbox, the final drive is characterized by a conversion ratio γ_f and an efficiency η_f. Here, the relation between the propeller shaft and the final drive torque and angular velocity is established by neglecting the inertia. The wheels are assumed to have no slip.

Considering the given assumptions, the longitudinal driving force produced in the powertrain is given by

$$F_{\text{powertrain}} = \frac{\gamma_t \gamma_f \eta_t \eta_f}{r_w} T_e(\omega_e, \delta) - \frac{J_w + \gamma_t^2 \gamma_f^2 \eta_t \eta_f J_e}{r_w^2} \dot{v} - \frac{T_b}{r_w}, \tag{3}$$

where the first term denotes the vehicle propulsion force produced by the engine, the second term is the internal inertial force, and the third term is the force produced by the brakes. v is the vehicle velocity, r_w denotes the wheel radius, J_w, J_e denote the engine and wheel inertia, and T_b denotes the braking torque. T_e is the net engine torque, which is a function of the engine angular velocity, ω_e, and the injected fuel amount, δ.

External forces are imposed on the vehicle in motion. The external forces mainly consists of rolling resistance, F_{roll}, gravitational force, F_{gravity}, and air drag, F_{airdrag}. The rolling resistance occurs due to the resistive frictional force that occurs between the road surface and the wheels. It is given by, $F_{\text{roll}} = c_r mg \cos\alpha$, where c_r denotes the roll resistance coefficient, g the gravitational constant, m the vehicle mass, and α the slope of the road. The gravitational force, $F_{\text{gravity}} = mg \sin\alpha$, can act as a positive or negative longitudinal force depending on the incline of the road. The aerodynamic drag has a strong impact on an HDV and can amount up to 50% of the total resistive forces at full speed. It is given by, $F_{\text{airdrag}} = \frac{1}{2} c^w \Phi(d) A \rho v^2$, where A denotes the maximum cross-sectional area of the vehicle, ρ the air density, c^w the air drag coefficient, d the relative distance between the vehicles, and $0 < \Phi(d) = k_{pw} d + l_{pw} \leq 1$ denotes the empirically derived air drag reduction due to the preceding vehicle. The parameters k_{pw} and l_{pw}

are empirically derived, Alam (2011, Chapter 3). A single HDV in motion experiences an increased air pressure at the front of the vehicle and a pressure drop at the tail. This pressure change produces the aero dynamic drag inflicted upon the vehicle. The pressure is significantly reduced for a follower vehicle, operating at 50 m or less, since the preceding vehicle reduces the air flow inflicted upon its frontal surface, inducing a physical coupling between the vehicles. The preceding vehicle also experiences a small air drag reduction at very short intermediate spacings. However, it can be neglected for the purpose of this study. Hence, we assume that $\Phi(d)=1$ for the lead vehicle in a platoon.

Let $i=1,\ldots,N$ denote the vehicle position in the platoon. Applying Newton's second law of motion along with all the external forces described above, a non-linear vehicle model is derived as

$$\frac{ds_i}{dt}=v_i m_t\frac{dv_i}{dt}=F_i^{\mathrm{e}}-F_i^{\mathrm{b}}-F^{\mathrm{air\ drag}}(v_i,d_{i-1,i})-F^{\mathrm{roll}}(\alpha_i)-F^{\mathrm{gravity}}(\alpha_i)$$
$$=k_i^{\mathrm{e}}T_i^{\mathrm{e}}(\omega_{\mathrm{e}},\delta)-F_i^{\mathrm{b}}-k_i^{\mathrm{d}}(d_{i-1,i})v_i^2-k_i^{\mathrm{fr}}\cos\alpha_i-k_i^{\mathrm{g}}\sin\alpha_i \quad (4)$$

where

$$m_{t_i}=\frac{J_{\mathrm{w}_i}}{r_{\mathrm{w}_i}^2}+m_i+\frac{\gamma_{\mathrm{t}}^2\gamma_{\mathrm{f}}^2\eta_{\mathrm{t}}\eta_{\mathrm{f}}J_{\mathrm{e}}}{r_{\mathrm{w}_i}^2} \quad (5)$$

is the total inertial mass, $F_i^{\mathrm{e}}\geq 0$ denotes the force produced by the engine through fuel injection, $F_i^{\mathrm{b}}\geq 0$ denotes the braking force, s_i is the absolute traveled distance for the ith HDV from a reference point common to all vehicles in the platoon, and k_i^z, $z\in\{\mathrm{e},\mathrm{d},\mathrm{fr},\mathrm{g}\}$, are characteristic coefficients. The control input $u_i=F_i^{\mathrm{e}}-F_i^{\mathrm{b}}$, is assumed to be a continuous function. The forces F_i^{e} and F_i^{b}, are assumed not to be applied simultaneously.

3.2. Platoon model

The aim of this paper is to guarantee safety for HDVs in a platoon. For simplicity of this study, the road is assumed to be flat. For the two-vehicle platoon in Fig. 2, the system can be reduced to

$$\dot{x}=f(x,u_1,u_2)=\begin{bmatrix} c_1^{\mathrm{u}}u_1-c_1^{\mathrm{d}}v_1^2-c_1^{\mathrm{fr}} \\ -v_{21} \\ c_2^{\mathrm{u}}u_2-c_1^{\mathrm{u}}u_1-c_2^{\mathrm{d}}(d)(v_1+v_{21})^2+c_1^{\mathrm{d}}v_1^2-c_2^{\mathrm{fr}}+c_1^{\mathrm{fr}}\end{bmatrix} \quad (6)$$

where $x=[v_1\ dv_{21}]^T$ and $c_i^z=k_i^z/m_{t_i}$. The state variable v_1 is the velocity of the lead vehicle, $d=s_1-s_2$ denotes the relative distance between the vehicles, and $v_{21}=v_2-v_1$ denotes their relative velocity. The model is limited to forward longitudinal direction. The collision scenarios we focus on is when vehicles are traveling closely spaced with a given initial velocity and relative distance.

4. Computing safe sets

In this section, safe sets computations based on pursuit-evasion games and reachability are first briefly described. Then safe sets are derived for the HDV scenarios of interest.

4.1. Safe set computation based on a pursuit-evasion game

A pursuit-evasion game is a family of problems in which one group of members tries to capture another group in a given setting, where the system dynamics are given by

$$\dot{x}=f(x,u_1,u_2),\quad x(0)=x_0, \quad (7)$$

see Isaacs (1965, Chapter 1.5) and Basar and Olsder (1995, Chapter 8). Here $x(t)\in\mathbb{R}^n$, is the state of the system, $u_1(t)\in\mathbb{R}^{m_1}$ are the actions of one group of players, referred to as the *pursuers*, and $u_2(t)\in\mathbb{R}^{m_2}$ are the actions of the second group of players, referred to as the *evaders*. The problem can be described as a finite-time game where one player is trying to minimize and the other player is trying to maximize a cost function depending only on the terminal state and time. By introducing the function $H(x,p,u)=p^Tf(x,u_1,u_2)$, the game can be formulated as

$$\max_{u_2\in\mathcal{E}}\min_{u_1\in\mathcal{D}}H(x,p,u)=H(x,p,u^*)=:H^*(x,p) \quad (8)$$

where $p\in\mathbb{R}^n$ denotes the costates. The sets $\mathcal{D}$ and $\mathcal{E}$ are compact sets representing all possible actions of the pursuers and the evaders, respectively. The function H^* is the Hamiltonian. Necessary conditions for optimality can be derived based on these data, see Basar and Olsder (1995, Chapter 8).

Given actions $u_1\in\mathcal{E}$ and $u_2\in\mathcal{D}$, the ability to reach a defined unsafe set from a set of feasible initial states x_0 is of interest for establishing safety criteria. In Mitchell, Bayen, and Tomlin (2005), it was shown that the unsafe set $\mathcal{U}(\tau)$ in which the pursuer in a two-person dynamic game can create a collision in the next τtime units despite the best effort from the evader, can be computed as $\mathcal{U}(\tau)=\{x\in\mathbb{R}^3|\phi(x,-\tau)\leq 0\}$, where $\phi(\cdot,\cdot)$ is the viscosity solution of the (modified) Hamilton–Jacobi–Isaacs partial differential equation

$$\frac{\partial\phi(x,t)}{\partial t}+\min(0,H(x,p,u^*))=0,\quad t\leq 0, \quad (9)$$

with suitable terminal conditions $\phi(x,0)=\phi_0(x)$. The equation is solved by starting at the boundary of the unsafe set $\partial\mathcal{U}(\tau)$. The set of states under consideration is

$$\chi(\tau)=\{x\in\partial\mathcal{U}(\tau)|p^T(0)f(x,u_1^*,u_2^*)<0\}, \quad (10)$$

which denotes all states heading into the unsafe set. The reachable set is calculated by starting at $\partial\mathcal{U}(0)$ and simultaneously solving the equations corresponding to the optimality conditions. Hence, the trajectories are computed on the boundary of the usable part, from where it is possible to move away from the unsafe set. The procedure gives the surface sets that partitions the safe and unsafe regions.

4.2. Safe set computation for a two-vehicle platoon

In the problem at hand $f(x,u_1,u_2)$ in (7) corresponds to the platoon system (6) and u_1^*,u_2^* are the optimal strategies for the lead vehicle and the follower vehicles, respectively. The unsafe set $\mathcal{U}(\tau)$ corresponds to a vehicle collision at time $\tau>0$. Note that the game formulation leads to a conservative estimate of the unsafe set, since the lead vehicle's optimal control action is based on knowledge of how the follower vehicle will respond, but not vice

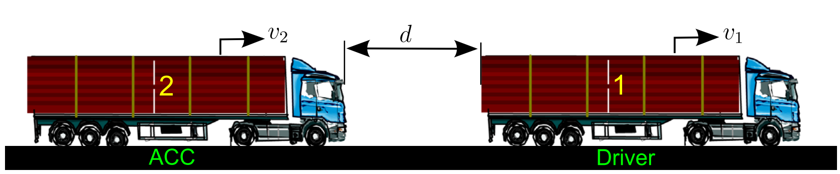

Fig. 2. Two-vehicle platoon on a flat road where vehicle 1 is referred to as the lead vehicle and vehicle 2 is the follower vehicle.

versa (see Mitchell et al., 2005). Such a formulation is preferable to ensure safety in practice.

The Hamiltonian function for the two-vehicle platoon is

$$
\begin{aligned}
H^*(x,p) &:= \max_{u_2 \in \mathcal{E}} \min_{u_1 \in \mathcal{D}} p^T f(x,u_1,u_2) \\
&= \max_{u_2 \in \mathcal{E}} \min_{u_1 \in \mathcal{D}} [p_1 \;\; p_2 \;\; p_3] \begin{bmatrix} \dot{v}_1 \\ \dot{d} \\ \dot{v}_{21} \end{bmatrix} \\
&= -p_2 v_{21} - (p_3 - p_1) c_1^{\mathrm{u}} u_1^* + p_3 c_2^{\mathrm{u}} u_2^* + (p_3 - p_1) c_1^{\mathrm{d}} v_1^2 \\
&\quad - p_3 c_2^{\mathrm{d}}(d)(v_1 + v_{21})^2 + (p_3 - p_1) c_1^{\mathrm{fr}} + p_3 c_2^{\mathrm{fr}}.
\end{aligned} \tag{11}
$$

With the formulation in (11) the lead vehicle determines its optimal control strategy based upon information regarding the follower vehicle's strategy. This is a reasonable assumption as we wish to find a set which guarantees that a collision can be avoided despite the worst possible behavior of the lead vehicle. The costates fulfill

$$
\dot{p} = -\frac{\partial H^*}{\partial x} = \begin{bmatrix} 2p_3 c_2^{\mathrm{d}}(d)(v_1 + v_{12}) - 2(p_3 - p_1) c_1^{\mathrm{d}} v_1 \\ p_3 \frac{1}{2} c_2^{\mathrm{w}} A_2 \rho k_{pw} (v_1 + v_{21})^2 \\ 2p_3 c_2^{\mathrm{d}}(d)(v_1 + v_{21}) \end{bmatrix}. \tag{12}
$$

The optimal strategy can easily be computed as

$$
\begin{aligned}
u_1^* &= \frac{\hat{F}_1^{\mathrm{e}} - \hat{F}_1^{\mathrm{b}}}{2} + \mathrm{sgn}(p_3 - p_1) \frac{\hat{F}_1^{\mathrm{e}} + \hat{F}_1^{\mathrm{b}}}{2}, \\
u_2^* &= \frac{\hat{F}_2^{\mathrm{e}} - \hat{F}_2^{\mathrm{b}}}{2} + \mathrm{sgn}(p_3) \frac{\hat{F}_2^{\mathrm{e}} + \hat{F}_2^{\mathrm{b}}}{2},
\end{aligned} \tag{13}
$$

where $\hat{F}_i^b > 0$ is the maximum brake force and $\hat{F}_i^e > 0$ is the maximum engine force of vehicle $i=1, 2$.

4.3. Safe set computation for an N-vehicle platoon

The safe set computation approach for a two-vehicle derived in the previous section can be generalized to $N > 0$ vehicles. First we consider an $N=3$ HDV platoon. Two states are added to (6) for the three-vehicle platoon system:

$$
\dot{x} = f(x,u_1,u_2,u_3) = \begin{bmatrix} c_1^{\mathrm{u}} u_1 - c_1^{\mathrm{d}} v_1^2 - c_1^{\mathrm{fr}} \\ -v_{21} \\ c_2^{\mathrm{u}} u_2 - c_1^{\mathrm{u}} u_1 - c_2^{\mathrm{d}}(d_{12}) v_2^2 + c_1^{\mathrm{d}}(v_1 + v_{21})^2 - c_2^{\mathrm{fr}} + c_1^{\mathrm{fr}} \\ -v_{32} \\ c_3^{\mathrm{u}} u_3 - c_2^{\mathrm{u}} u_2 - c_3^{\mathrm{d}}(d_{23})(v_1 + v_{21} + v_{32})^2 + c_2^{\mathrm{d}}(d_{12}) v_2^2 - c_3^{\mathrm{fr}} + c_2^{\mathrm{fr}} \end{bmatrix}, \tag{14}
$$

where $x = [v_1 \; d_{12} \; v_{21} \; d_{23} \; v_{32}]^T$ and we have used $v_j = v_1 + \sum_{i=1}^{j-1} v_{i+1,i}$, for $j > 1$, in the expression for the third and last state. The costates satisfy $\dot{p} = -\partial H^*/\partial x$ with the Hamiltonian function

$$
\begin{aligned}
H^*(x,p) &:= \max_{(u_2,u_3) \in \mathcal{E}} \min_{u_1 \in \mathcal{D}} p^T f(x,u_1,u_2,u_3) \\
&= -(p_3 - p_1) c_1^{\mathrm{u}} u_1^* + (p_3 - p_5) c_2^{\mathrm{u}} u_2^* + p_5 c_3^{\mathrm{u}} u_3^* - p_2 v_{\mathrm{r}_{12}} - p_4 v_{\mathrm{r}_{23}} \\
&\quad - p_1 (c_1^{\mathrm{w}} v_1^2 + c_1^{\mathrm{fr}}) - p_3 (c_2^{\mathrm{d}}(d) v_2^2 - c_1^{\mathrm{d}}(v_1 + v_{\mathrm{r}_{12}})^2 + c_2^{\mathrm{fr}} - c_1^{\mathrm{fr}}) \\
&\quad - p_5 (c_3^{\mathrm{d}}(d_{23})(v_1 + v_{\mathrm{r}_{12}} + v_{\mathrm{r}_{23}})^2 - c_2^{\mathrm{d}}(d_{12}) v_2^2 + c_3^{\mathrm{fr}} - c_2^{\mathrm{fr}}),
\end{aligned} \tag{15}
$$

where $\mathcal{E} = \mathcal{E}_2 \times \mathcal{E}_3$ is the compact set of controller actions for the two follower vehicles. The optimal strategies are given by

$$
\begin{aligned}
u_1^* &= \frac{\hat{F}_1^{\mathrm{e}} - \hat{F}_1^{b}}{2} + \mathrm{sgn}(p_3 - p_1) \frac{\hat{F}_1^{\mathrm{e}} + \hat{F}_1^{b}}{2}, \\
u_2^* &= \frac{\hat{F}_2^{\mathrm{e}} - \hat{F}_2^{b}}{2} + \mathrm{sgn}(p_3 - p_5) \frac{\hat{F}_2^{\mathrm{e}} + \hat{F}_2^{b}}{2}, \\
u_3^* &= \frac{\hat{F}_3^{\mathrm{e}} - \hat{F}_3^{b}}{2} + \mathrm{sgn}(p_5) \frac{\hat{F}_3^{\mathrm{e}} + \hat{F}_3^{b}}{2}.
\end{aligned} \tag{16}
$$

Thus, for a three-vehicle platoon the complexity of the pursuit-evasion solution increases. Note that the solution has a physical interpretation. For example, the middle vehicle must consider its safety strategy both with respect to its preceding and following vehicle to avoid a collision with either HDV.

In general, for an N-vehicle platoon the state is given by

$$
x = [v_1 \;\; d_{12} \;\; v_{21} \;\; d_{23} \;\; \ldots \;\; v_{N,N-1}]^T
$$

and its system dynamics corresponding to (15) is readily derived. Note that the controls u_1 and $(u_2, \ldots, u_N)$ enter linearly. Hence, the optimal inputs to the Hamiltonian function, $H^*(x,p)$, can again be given analytically as

$$
u_i^* = \begin{cases} \hat{F}_i^{\mathrm{e}} & \text{if } p^T D_i > 0, \; i = 1, \ldots, N \\ \hat{F}_i^{\mathrm{b}} & \text{otherwise,} \end{cases} \tag{17}
$$

where $D \in \mathbb{R}^{2N-1 \times N}$ is a matrix, with column vectors D_i in the ith column, containing 1 or -1 in the appropriate elements. E.g., $D_2 = [0\ 0\ 1\ 0\ -1]^T$ for the second vehicle in the three-vehicle platoon. The unsafe set $\mathcal{U}(\tau)$ is a $2N-1$ dimensional vector space. Even though analytical expressions for the N-vehicle optimal strategies can be found through the given framework, the computational cost for solving the partial differential equation to derive the safe sets is exponential in the state dimension. An added HDV to the platoon corresponds to the addition of two new states. Thus, it is computationally too expensive for online applications and in general hard to find accurate numerical solutions for the N-vehicle problem when N is large.

4.4. Numerical evaluations

To compute the unsafe set from the solution of (9), the level sets methods toolbox of Mitchell (2007) is utilized. In this section the scenarios are calculated; first we consider a simple setup with two identical platooning HDV's and then we study mass heterogeneity and uncertainties in braking capacity.

4.4.1. Identical HDVs

The collision avoidance scenario is first investigated for two identical HDVs. The vehicles have identical vehicle parameters in (4) and the gross mass of each HDV was chosen to be 40 000 kg, which is a standard weight for European long-haulage HDVs. Their braking capacity is set to create a maximum deceleration of -3 m/s^2, which is considered to be a harsh braking. Commercial HDVs generally have a speed restriction of 90 km/h. The unsafe set is calculated backwards for $\tau \in [0, 8.33 \text{ s}]$, where the upper bound is the time it takes for the follower HDV to come to a full stop under the maximum deceleration constraint.

Fig. 3 shows the boundary $\partial\mathcal{U}(\tau)$, of the unsafe set contained between the plotted level surface and $\mathcal{U}_0$. The safe set $\mathcal{S} = \mathbb{R}^3 \backslash \mathcal{U}$ is indicated as well. As $v_{21} = v_2 - v_1$ increases, the relative distance $d = s_1 - s_2$ must also increase. The fold in the boundary surface area is due to the physical constraint $v_1 \geq 0$. Any trajectory heading behind that surface area would imply that the lead vehicle has reversed to create a collision. If the follower vehicle is within the safe set, it will always be able to avoid a collision regardless of the best effort of the lead vehicle (pursuer) with respect to a compact set of controller actions. Thus a least restrictive controller could be implemented outside the unsafe set without endangering a collision. However, if it is within the unsafe set a collision might occur given that the lead vehicle acts in the worst possible manner.

In platooning applications the vehicles generally travel in what we here refer to as a *normal mode*, where each vehicle is traveling at a constant fixed velocity, $v_{21} = 0$, and a desired relative distance is set by the driver. Fig. 3 reveals that a collision can be avoided for two identical HDVs if the lead vehicle is traveling at a higher

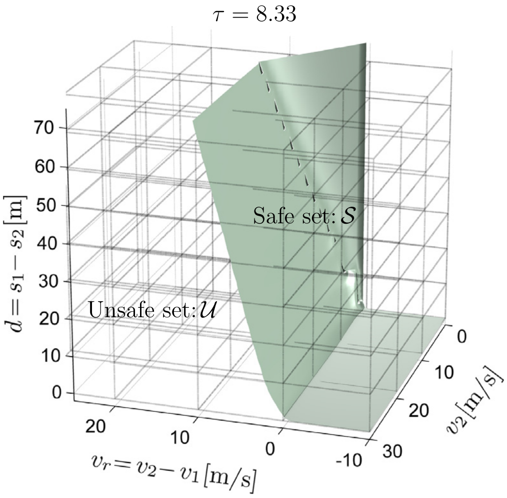

Fig. 3. The backward reachable set obtained under the assumption that no delay is present in the system.

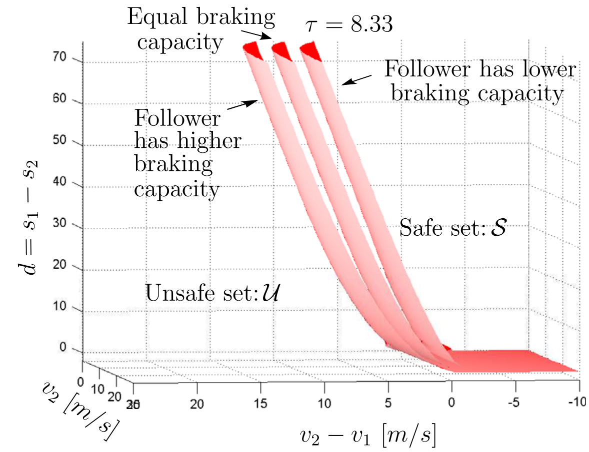

Fig. 4. The case when both vehicles have identical vehicle configuration is given by the level surface in the center. If the braking capacity for the follower vehicle increased by 20%, the nominal level surface shifts in the positive v_{21}-direction (to the left) and the slope of the surface has decreased. Similarly, if the follower vehicle has a 20% higher braking capacity, the surface shifts in the negative v_{21}-direction (to the right) and the slope increases.

velocity than the follower vehicle. However, if the vehicles are operating in normal mode and has a relative distance $d \le \psi$, where $\psi = \partial\mathcal{U}(\tau)|_{v,v_{21}=0}$, a collision could occur. The lead vehicle experiences a greater air drag and is therefore able to obtain a slightly higher braking force. Thus, if the vehicles are both traveling at a velocity $v \le 2.5$ m/s, a collision could occur for $\psi = 0.2$ m. As both vehicles' velocities increase, the air drag and inherently the obtainable brake force become higher for the lead vehicle compared to the follower. Thus, a larger relative distance of $d > \psi = 1.1$ m must be maintained at $v = 25$ m/s to stay out of the unsafe region. Hence, the minimum relative distance that can be obtained for two identical vehicles depends on their initial velocity. Assuming that no delay is present in the system and the vehicles are traveling in normal mode, the vehicles could maintain a relative distance of 1.2 m without endangering safety.

4.4.2. System uncertainties and vehicle parameters

System uncertainties or varying vehicle parameters, such as mass, could cause a difference in braking capabilities between the vehicles. Having different braking capacity changes the shape of the safe sets. If the follower vehicle has a higher braking capacity, the level surface derived for neighboring vehicles with identical braking capacity will shift in the positive v_{21}-direction and the slope of the surface will decrease, as shown in Fig. 4. This means that the follower vehicle will be able to lie closer without endangering a collision. The minimum safe relative distance is therefore shorter compared to the case of two identical vehicles.

However, if the lead vehicle has a greater braking capability a perturbation arises in the level surface at $v_{21} \approx 0$ and the slope becomes steeper. In this case a minimum distance of $d = 13$ m must be maintained to remain outside the reachable set at normal mode. Thus, the relative distance must be increased significantly if the lead vehicle has a stronger braking capability.

Delays for the platoon control system commonly occur due to detection, transmission, computation, and producing the control command. A delay in the system implies that the lead vehicle will be able to act, change the relative velocity and distance, before the follower vehicle is able to react. A delay can be translated into a shift of the reachable set in Fig. 3 by Δd units in the positive direction along the d-axis and by Δv_{21} units in the negative direction along the v_{21}-axis. However, no change occurs in the follower vehicle's velocity v_2, since it does not react. Depending on the radar and the collision detection algorithm, a worst-case delay is approximately 500 ms for the considered vehicles. Hence, the lead vehicle will be able to reduce the relative velocity by 3.25 m/s and the relative distance by 0.8 m if it is driving 25 m/s at normal mode. Thus if the follower vehicle maintains $d \ge 2$ m, a collision can always be avoided for two identical vehicles according to the safe set in Fig. 3.

5. Collaborative braking experiments

In this section we first give the experimental setup for the two-vehicle platoon. To obtain reproducible results, the experiment procedure was automated. We present different braking scenarios that arise based on predefined reference speeds and intermediate distances. Several experiment results are presented subsequently to evaluate the derived safe sets.

5.1. Experimental setup

Two standard Scania HDVs are utilized with additional control and communication hardware. Both HDVs have a 6×3 vehicle configuration and the masses were measured to be 25 t for the lead vehicle and 23.6 t for the follower vehicle. They are equipped with standard radars, which sends the relative distance with a 40 ms interval to the central coordinator ECU and is gated every 100 ms. An external supplier provides the radar with an internal filter of undisclosed characteristics. Both vehicles are equiped with a fully automatic gearboxes. Standard ECUs, utilized in Scania HDVs, are modified to add the automated optimal control logic. As illustrated in Fig. 5, a wireless sensor unit (WSU) carrying the standard wireless communication protocol 802.11p is mounted in each vehicle. The WSU is directly connected to the HDVs internal CAN system and messages are broadcast on demand. Thereby, the internal CAN signals such as velocity, acceleration, and control inputs are available to both vehicles. The data are logged in the lead vehicle.

To evaluate the safe sets the vehicles' cruise controller and internal brake request functionality are utilized. The safe set is divided into three regions, which gives three different scenarios to be evaluated. The first scenario is when the vehicles operate at normal mode, with $v_{21} \approx 0$. The second experiment scenario evaluates the level surface for when the follower vehicle has a higher initial velocity, $v_{21} \ge 0$, and the control action is implemented. The final scenario encompasses the cases when a lead vehicle

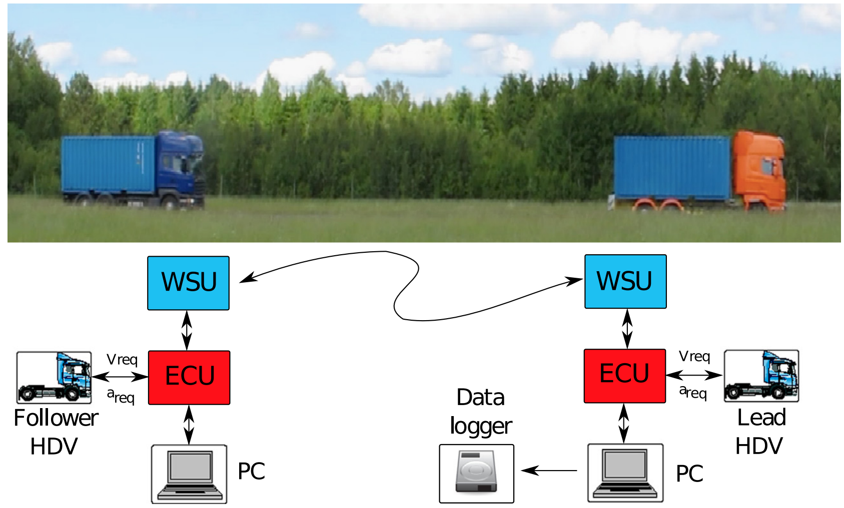

Fig. 5. A schematic overlay of the experimental hardware setup. The top picture shows the HDVs utilized in this experiment. The right HDV is the lead and the left is the follower. The WSU, ECU, and PC communicate through CAN. As soon as new information is obtained through the ECU or the vehicle, it is broadcast through the WSU.

would increase its velocity until $v_{21} \le 0$ and then suddenly initiate emergency braking. An automated procedure is used to guide the vehicles to the desired initial conditions $(v_{21}(t_0), v_2(t_0), d(t_0))$ given by the level surface for each experiment. The experiments are conducted at several different initial velocities and data is logged for evaluation.

The control action, with a maximum reference deceleration of $-3\ \mathrm{m/s^2}$ is implemented at an intermediate distance of more than 30 m due to safety precautions. The brake system is calibrated as an attempt to eliminate any braking discrepancies in the vehicles. Finally, to minimize delays that can occur in the system, all control signals are first sent to the WSU. The WSU then transmits the signal while echoing the same information back through the internal CAN system. Thereby, both vehicles will be able to initiate their control actions nearly simultaneously.

5.2. Braking scenarios

The results from a single run of experiments conducted for each of the three different scenarios are given in Fig. 6. The top plots show the velocity trajectories v_1 and v_2 for the lead vehicle and for the follower vehicle, respectively. The intermediate spacing, d, between the vehicles during the experiments are shown in the middle plot and the acceleration for each vehicle are shown in the bottom two plots, where a_1 and a_2 denote the acceleration for the lead vehicle and for the follower vehicle, respectively. The solid lines show normal mode scenario when the vehicles are traveling with the same velocity. The dashed lines show the results for the scenario when the follower vehicle has a lower initial velocity and the dotted lines show the results for the scenario when the follower vehicle has a higher initial velocity. In the normal mode scenario, the vehicles accelerate to a given reference speed and then maintain it. The lead vehicle initially maintains a 5 km/h lower speed compared to the follower vehicle, as shown in the top plot. As the follower vehicle approaches and an intermediate distance of less than 50 m is reached, the lead vehicle changes its speed to match a relative distance of 30 m $\pm$ 0.1 m. When it is reached, both vehicles initiate their optimal control inputs with a maximum braking capacity of $-3\ \mathrm{m/s^2}$. For this experiment the braking is observed at the 47 s time marker, when both the vehicles have reached 70 km/h. As given by the solid line in the middle plot of Fig. 6, the relative distance remains nearly unchanged during the implementation of the braking strategies. Having almost identical vehicle configuration, the braking is initiated at an intermediate distance of 30 m and both vehicles come to rest at approximately 37 m. The bottom two plots show that the follower vehicle obtains a slightly higher deceleration in the beginning and the intermediate spacing is therefore increased. The relative distance changes more dramatically when the follower vehicle has a higher initial velocity. This is displayed by the dotted lines, which is the scenario when the follower vehicle approaches with a higher velocity of 60 km/h and the lead vehicle is traveling at 20 km/h. The collision test in this case is initiated at the 57 s time marker, where the initial intermediate spacing is 65 m and both vehicles come to full stop at 23 m. If the lead vehicle starts to accelerate and reaches a higher

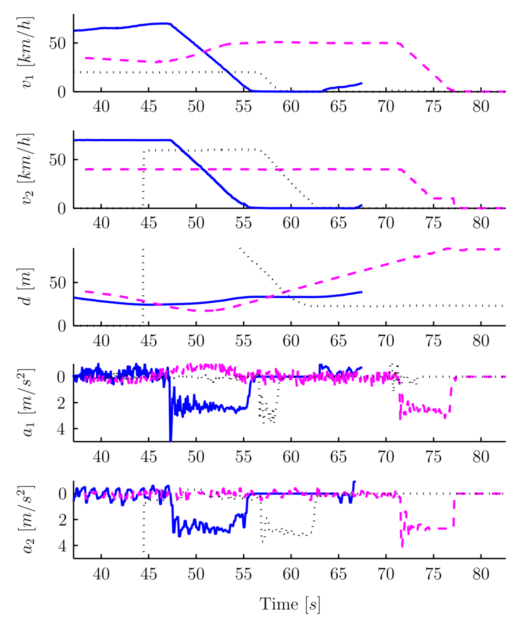

Fig. 6. Experiment results for three different braking scenarios. The top plot shows the velocity trajectories, v_1, for the lead vehicle and the second to top plot shows the velocity trajectories, v_2, for the follower vehicle. The corresponding intermediate distance trajectories are shown in the middle plot. The bottom two plots show the acceleration a_1 and a_2 for the lead vehicle and for the follower vehicle, respectively. Solid line shows the results for an experiment conducted when the vehicles are traveling at the same initial velocity. Results for when follower vehicle approaches the lead vehicle with a higher velocity are given by the dotted lines. Experimental results for when the lead vehicle initiates an acceleration and then emergency brakes are given by the dashed lines.

relative speed when initiating an emergency brake, the intermediate spacing between the vehicles with similar braking capacity seems to increase. This is given by the dashed trajectories in Fig. 6. Here, the collision test is initiated around the 71 s time marker. The action is initiated when the vehicles have an intermediate spacing of 76 m and come to a full stop at 87 m.

It should be noted that as the vehicles decelerate, their gearboxes automatically change to lower gears. At lower speed, the difference in gearbox has a stronger effect. Hence, the dynamic behavior of the vehicles are different during deceleration at lower velocities. Furthermore, the bottom two plots in Fig. 6 show a variation in the deceleration trajectories for the HDVs. Braking is carried out by executing a precomputed pressure to the brake disks in relation to the requested deceleration. The braking dynamics is nonlinear and changes with time.

5.3. Safe set evaluation

Several collision experiments were conducted at various reference velocities to evaluate various points on the safe set, as illustrated by the trajectories in Fig. 7. The $\times$ denotes the starting point for each trajectory and each trajectory ends when both vehicles have come to a full rest, with $v_2 = v_{21} = 0$. The control action in the experiments was implemented at an intermediate distance of more than 30 m due to safety precautions. However, the initial points of all the trajectories have been shifted to the minimum safe relative distance given by the safe set, based on the relative velocity and follower vehicle velocity at initial time of implementing the optimal control inputs. A delay of 200–300 ms has occasionally occurred and hence the minimum safety distance have been adjusted accordingly. Fig. 7 shows the empirically obtained deceleration trajectories in comparison with the safe set for vehicles with identical braking power. None of the trajectories starting with $v_{21} \leq 0$ intersect the level surface for any initial v_2. However, some of the trajectories for starting points at $v_{21} \geq 0$ intersects the level surface and then comes back out again. This is due to the varying deceleration that occurs because of the nonlinearities in the braking system, which was seen in the bottom two plots of Fig. 6. The trajectories would not intersect a safe set derived for a maximum braking capability of $\hat{F}^b/m_t = 4.8\ \text{m/s}^2$, which is the deceleration that the lead vehicle initially obtained in the cases when the surface is breached. The braking capabilities were noted to vary significantly during some of the braking scenarios. This is not captured by the level set surface that divides the safe and unsafe sets. The vehicle control action momentarily exceeds the upper boundary of the assumed available control action. However, since the safe set is conservative, the vehicles come to full rest without causing a collision.

Owing to the fact that the braking capability can change during an emergency braking, several experiments were conducted with varying braking capability for both vehicles. These experiments were focused on normal mode platooning, since it is the most common mode of operation. Fig. 8 shows a two-dimensional projection of the deceleration trajectories, omitting the minor variation in relative velocity during the optimal control implementation. The top plot shows the collision tests for varying initial velocities and similar vehicle braking capacity. It can be seen that the intermediate spacing remains fairly constant throughout the collision tests. However, below a velocity of 5 m/s the vehicles start changing gears, which have a clear impact on the vehicle dynamics. The model for deriving the safe sets does not take gear change logic into consideration. Nevertheless, the safe sets are conservative and a collision is hence still avoided. The middle plot, in Fig. 8, shows the deceleration trajectories for when the follower vehicle has a 30–40% lower braking capacity. It can be seen that the intermediate spacing is constantly reducing. Most of

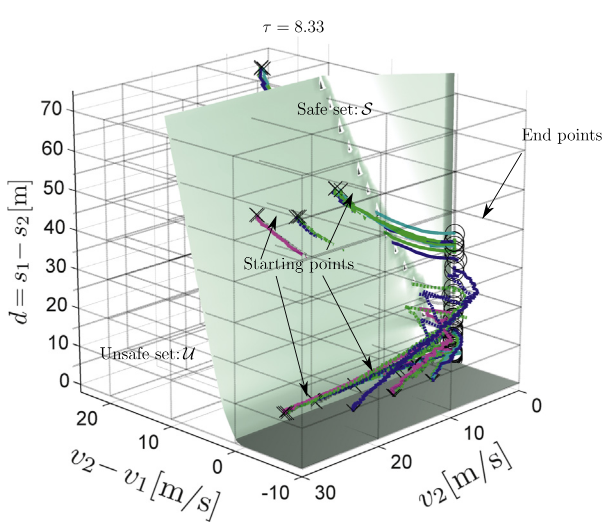

Fig. 7. Three-dimensional plot of the empirical deceleration trajectories for varying initial velocities. The $\times$ denotes the starting point for each trajectory and $\circ$ the end points (at $v_1 = v_2 = 0$). Each color indicates an experiment obtained for a given initial vehicle velocity at time of emergency braking. The starting point of the trajectories is shifted to the minimum safe relative distance in the safe set based on the relative velocity and current follower vehicle velocity at the time of initiating the optimal control input.

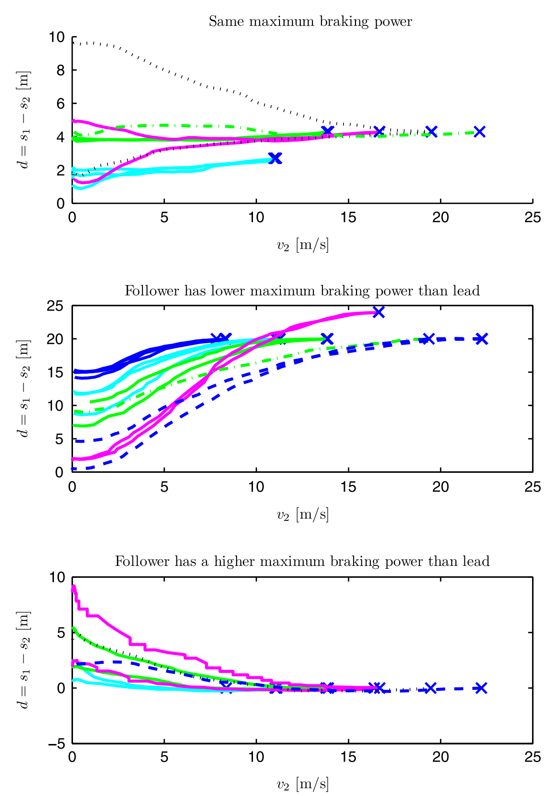

Fig. 8. Two-dimensional plot of the empirical braking trajectories for varying initial velocities, where the $\times$ denotes the starting point for each trajectory. The trajectories are presented in the (d, v_2)–plane, omitting the slight variation in the relative velocity. Each color indicates a set of experiments obtained for a given initial follower vehicle velocity at time of emergency braking. The starting point of the trajectories is shifted to the minimum safe relative distance in the safe set with respect to initial relative velocity and current follower vehicle velocity. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

the trajectories end with both vehicles at rest and still a few meters to spare. Finally, the bottom plot shows the deceleration trajectories for when the follower vehicle has a 20–30% higher braking capacity. It can be seen that the intermediate spacing remains the same or increases, which is congruent with the results obtained from the corresponding safe set. Hence, if the follower vehicle has a higher braking capacity, an intermediate spacing within decimeters could have been maintained with the presented system and still no collision would have occurred.

6. Conclusions

A minimum distance between two HDVs can be deduced with respect to a compact set of controller actions without endangering a collision despite the worst possible action by the lead vehicle. Our results show that during normal operation a minimum distance of 1.2 m should be maintained to ensure that a collision can be avoided for two identical vehicles and 2 m when a worst-case delay of 500 ms delay is present in the system, which is lower compared to what is utilized in commercial applications today without endangering safety.

A stronger overall braking capability in the follower vehicle creates the possibility of reducing the relative distance further. Thus, in platooning applications the results suggest that HDVs with stronger braking capabilities should always be placed behind to enable the shortest possible relative distance without endangering a collision. However, this is contrary to fuel-efficient control criteria, where the follower HDV should have less stringent control actions.

Even though the model and procedure utilized for deriving the safe sets do not encompass all nonlinear features of an HDV in motion, the sets serve as a reliable reference to ensure that a collision can be avoided. Both theoretical and empirical results show that it is suitable to order HDVs according to increasing braking capacity. Thereby, the intermediate spacing can be reduced significantly without compromising safety. Naturally, it only holds under the assumption that reliable V2V communication is available.

Although the experimental results confirm the numerical evaluations, the results reveal that the influence of the unmodeled nonlinear braking system and gear changes are not negligible. Furthermore, extending the experimentally evaluated and validated model to both longitudinal and lateral motion is of high relevance. Finally, we believe that finding a novel computational method for computing safe sets for N HDVs is interesting, since establishing what minimum distance that can be maintained between HDVs in a heterogenous platoon is not evident. Investigating these factors requires a more advanced vehicle model and computation method and is therefore left as future work.

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References

Alam, A. (2011). *Fuel-efficient distributed control for heavy duty vehicle platooning* (Licentiate thesis). SE-100 44 Stockholm, Sweden: Royal Institute of Technology.

Alam, A., Gattami, A., & Johansson, K. H. (September 2010). An experimental study on the fuel reduction potential of heavy duty vehicle platooning. In *13th international IEEE conference on intelligent transportation systems*. Madeira, Portugal.

Alam, A., Gattami, A., & Johansson, K. H., (December 2011). Suboptimal decentralized controller design for chain structures: Applications to vehicle formations. In *50th IEEE conference on decision and control and European control conference*. Orlando, FL, USA.

Alam, A., Gattami, A., Johansson, K. H., & Tomlin, C. J. (August 2011). Establishing safety for heavy duty vehicle platooning: A game theoretical approach. In *18th world congress of the international federation of automatic control (IFAC)*. Milan, Italy.

Ali, M. (2012). *Decision making and control for automotive safety* (Ph.D. thesis). Göteborg, Sweden: Department of Signals and Systems, Chalmers University of Technology.

Althoff, M., & Dolan, J. M. (June 2012). Reachability computation of low-order models for the safety verification of high-order road vehicle models. In: *American control conference*. Montreal, Canada.

Bamieh, B., & Jovanović, M. R. (2005). On the ill-posedness of certain vehicular platoon control problem. *IEEE Transactions on Automatic Control*, *50*(September (9)), 1307–1321.

Basar, T., & Olsder, G. J. (1995). *Dynamic noncooperative game theory* (2nd ed.). New York: Academic Press ISBN: 978-0-89871-429-6.

Bayen, A. M., Shanthanam, S., Mitchell, I., & Tomlin, C. J. (August 2003). A differential game formulation of alert levels in ETMS data for high altitude traffic. In *Guidance navigation and control* (*GNC*) *conference* (Vol. AIAA, pp. 2003-5341). Austin, Texas.

Bu, F., Tan, H. -S., & Huang, J. (July 2010). Design and field testing of a cooperative adaptive cruise control system. In *American control conference* (pp. 4616–4621), developed under the California PATH research project.

Girard, A., 2005. Reachability of uncertain linear systems using zonotopes. In: *Proceedings of the 8th international conference on hybrid systems: Computation and control* (HSCC'05) (pp. 291–305).

Gupta, V., Hassibi, B., & Murray, R. (July 2004). On the synthesis of control laws for a network of autonomous agents. In *Proceedings of the American control conference* (Vol. 6, pp. 4927–4932). Boston, MA, USA.

Gustafsson, F. (2009). Automotive safety systems. *IEEE Signal Processing Magazine*, *26*(July (4)), 32–47.

Guzzella, L., & Sciarretta, A. (2007). *Vehicle propulsion systems* (2nd ed.). Berlin/Heidelberg/New York: Springer ISBN: 978-3-540-74691-1.

Isaacs, R. (1965). *Differential games*. John Wiley: Dover Publications, Inc. (Reprinted (1999)).

Levine, W., & Athans, M. (1966). On the optimal error regulation of a string of moving vehicles. *IEEE Transactions on Automatic Control*, *11*(July (3)), 355–361.

Liang, K., Alam, A., & Gattami, A. (November 2011). The impact of heterogeneity and order in heavy duty vehicle platooning networks. In *3rd IEEE vehicular networking conference*. Amsterdam, Netherlands.

Makhlouf, I. B., & Kowalewski, H. D. S. (June 2012). Safety verification of a controlled cooperative platoon under loss of communication using zonotopes. In *4th IFAC conference on analysis and design of hybrid systems*. TU Eindhoven, Netherlands.

Mitchell, I. (June 2007). *A toolbox of level set methods* (*version 1.1*) (Technical Report TR-2007-11). BC, Canada: Vancouver. [Online]. Available at: ⟨http://www.cs.ubc.ca/~mitchell/ToolboxLS/toolboxLS-1.1.pdf⟩.

Mitchell, I., Bayen, A., & Tomlin, C. (2005). A time-dependent Hamilton–Jacobi formulation of reachable sets for continuous dynamic games. *IEEE Transactions on Automatic Control*, *50*(July (7)), 947–957.

Naus, G., Vugts, R., Ploeg, J., van de Molengraft, R., & Steinbuch, M. (September 2009). Towards on-the-road implementation of cooperative adaptive cruise control. In *16th world congress on intelligent transport systems and services*. Stockholm, Sweden.

Rajamani, R., & Zhu, C. (2002). Semi-autonomous adaptive cruise control systems. *IEEE Transactions on Vehicular Technology*, *51*(September (5)), 1186–1192.

Ryan, A., Zennaro, M., Howell, A., Sengupta, R., & Hedrick, J. (December 2004). An overview of emerging results in cooperative UAV control. In *43rd IEEE conference on decision and control* (Vol. 1, pp. 602–607).

Sahlholm, P., & Johansson, K. H. (2010). Road grade estimation for look-ahead vehicle control using multiple measurement runs. *Control Engineering Practice*, *18*(11), 1328–1341.

Seiler, P., Song, B., & Hedrick, J. (February 1998). Development of a collision avoidance system. In *SAE 1998 world congress*. Detroit, MI, USA. SAE paper 980853.

Shaw, E., & Hedrick, J. (July 2007). String stability analysis for heterogeneous vehicle strings. In *American control conference* (pp. 3118–3125). New York, USA.

Siegwart, R., & Nourbakhsh, I. R. (2004). *Introduction to autonomous mobile robots*. MIT Press.

Swaroop, D., & Hedrick, J. (1996). String stability of interconnected systems. *IEEE Transactions on Automatic Control*, *41*(March (3)), 349–357.

Tsugawa, S., Kato, S., & Aoki, K. (September 2011). An automated truck platoon for energy saving. In *IEEE/RSJ international conference on intelligent robots and systems* (pp. 4109–4114).

Vahidi, A., & Eskandarian, A. (2003). Research advances in intelligent collision avoidance and adaptive cruise control. *IEEE Transactions on Intelligent Transportation Systems*, *4*(September (3)), 143–153.