Proceedings of the 18th World Congress
The International Federation of Automatic Control
Milano (Italy) August 28 - September 2, 2011

Establishing Safety for Heavy Duty Vehicle Platooning: A Game Theoretical Approach

Assad Al Alam * **Ather Gattami, Karl H. Johansson** ** **Claire J. Tomlin** ***

* *Scania CV AB, 151 87 Södertälje, Sweden (e-mail: assad.alam@scania.se)*
** *ACCESS Linnaeus Centre, Royal Institute of Technology, 100 44 Stockholm, Sweden (email: gattami@kth.se, kallej@ee.kth.se)*
*** *Electrical Engineering and Computer Sciences, UC Berkeley, Berkeley CA, 94720-1770, USA (email: tomlin@eecs.berkeley.edu)*

Abstract: It is fuel efficient to minimize the relative distance between vehicles to achieve a maximum reduction in air drag. However, the relative distance can only be reduced to a certain extent without endangering a collision. Factors such as the vehicle velocity, the relative velocity, and the characteristics of the vehicle ahead has a strong impact on what minimum relative distance can be obtained. In this paper, we utilize optimal control and game theory to establish safety criteria for heavy duty vehicle platooning applications. The derived results show that a minimum relative distance of 1.2 m can be obtained for two identical vehicles without endangering a collision, assuming that there is no delay present in the feedback system. If a worst case delay is present in the system, a minimum relative distance is deduced based upon the vehicle's maximum deceleration ability. The relative distance can be reduced if the follower vehicle has a greater overall braking capability, which suggests that vehicle heterogeneity and order has substantial impact. The findings are verified by simulations and the main conclusion is that the relative distance utilized in commercial applications today can be reduced significantly with a suitable advanced cruise control system.

Keywords: Game theory, Reachability, Optimal Control, Safety analysis, Heavy Duty Vehicle, Platooning.

1. INTRODUCTION

As traffic intensity is growing, the complexity of the coupled traffic dynamics is increasing. The actions of one vehicle may in turn affect all vehicles in a linked chain. Thus, through rapidly increasing technology in sensors, wireless communication, GPS-devices, and digital maps, advanced driver assistance systems are being developed to aid the driver. However, these information sources impose constraints in terms of accuracy, reliability and delays amongst others. The adaptive cruise control (ACC) is a commercially available system, which utilizes information regarding the relative position and velocity of the vehicle ahead to maintain a safe distance. It has also been considered as a means to enable vehicle platooning in Hedrick et al. (1991) and Rajamani and Zhu (1999). In Alam et al. (2010), it was showed that a fuel reduction of $4.7-7.7\%$ can be obtained through heavy duty vehicle (HDV) platooning by driving vehicles closely spaced to each other. Hence, it is fuel efficient to minimize the relative distance between the vehicles to achieve a maximum reduction in air drag. However, issues such as feedback delay and communication delay for safety and driver comfort arise. Therefore, the question is how close we might drive to a vehicle ahead without risking collision scenarios.

As an example, consider a collision avoidance scenario for a vehicle in a platoon equipped with an ACC-system. The signal from the radar must first generally be received and filtered through the corresponding electronic control unit (ECU). It is then processed by calculating the relative distance and velocity to the vehicle ahead. The ACC receives the information as an input and determines the proper control action. If the control action is braking, a slight delay arises from sending the brake request to producing the actual brake torque at the wheels. Alternatively, wireless communication may be utilized. However, delays are still imposed due to package drops, retransmission time, etc. Thus, the impact of the vehicle control on the safety criterion must be established and verified. Rigorous guarantees cannot be obtained through extensive simulations, but mathematical tools need to be developed.

Collision avoidance for cars has been investigated in Seiler et al. (1998), amongst others. However, in this paper we propose a novel approach by computing so called reachable sets to develop safety criteria for HDV platooning. A variety of reachable sets have been studied in Tomlin et al. (2000) and Mitchell (2002). A collision can occur if the

⋆ This paper was supported by Scania CV AB, VINNOVA - FFI, and the Swedish Research Council.

reachable set is entered, hence the set is refereed to as unsafe for vehicle platooning. Computing safe sets is an efficient method to capture the behavior of entire trajectories simultaneously. In Bayen et al. (2003), reachable sets were computed with a differential game formulation of an alerting logic for conflicts in high altitude air traffic.

The main contribution of this paper is to primarily establish safe sets, which can serve as a reference for HDV platooning in collision avoidance. We determine criteria for which collisions can be avoided in a worst case scenario and thereby establish the minimum possible safe distance to a vehicle ahead. A numerical study is performed to derive safe sets for a two-vehicle platoon. We show that the minimum relative distance with respect to safety depends on the overall braking capabilities of the HDVs within the platoon.

The outline of the paper is as follows. First we revisit some key concepts in Section 2 and state the theoretical premise for calculating reachable sets. Then the problem is formulated in Section 3. In Section 4 we state the solution and investigate the derived results for different HDV configurations. To validate the main results a simulation study is conducted in Section 5. Finally, in Section 6 we present a brief summary of the results in this paper and conclusions.

2. PRELIMINARIES

In this section, fundamental concepts for the problem formulation, such as game theory and reachability, are briefly presented.

A pursuit-evasion game, Basar and Olsder (1995) p.423, is a family of problems in which one group of members try to capture another group in a given setting. The system dynamics is given by

$$\dot{x} = f(t, x, u_1, u_2), \quad x(0) = x_0, \quad x(t) \in R^n,$$

where $f(t, x, u)$ is globally Lipschitz in x and continuous in u. Generally u_1 is the strategy of the player referred to as the *pursuer* and u_2 is the strategy of the player referred to as the *evader*. The problem can be described, through the principle of optimality, as a game where the players are trying to minimize respectively maximize a cost function. The game can then generally be formulated as

$$\max_{u_2 \in \mathcal{U}} \min_{u_1 \in \mathcal{D}} H(x, p, u) \tag{1}$$

where $H(x, p, u) = p^T f(x, u_1, u_2)$ is the Hamiltonian and p denotes the costates, which must satisfy $\dot{p} = -\frac{\partial H^T}{\partial x}$.

For reachability, the classical notion could be described as the ability to reach one state from another. Hence, given a compact set of controller actions $u_1 \in \mathcal{U}$ and $u_2 \in \mathcal{D}$, the ability to reach a defined unsafe set from a set of feasible initial states $x(0)$ is of interest for establishing safety criterion. In Mitchell et al. (2002), it was proved that the unsafe set $\mathcal{G}(\tau)$ in which the pursuer in a two-person dynamic game, Isaacs (1999 (1965)), can create a collision in the next τ time units despite the best effort from the evader, is given by $\mathcal{G}(\tau) = \{x \in \mathbb{R}^3 | \Phi(x, -\tau) \leq 0\}$, where

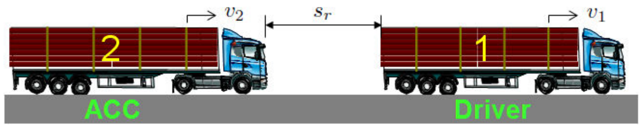


Fig. 1. Two trucks traveling on a flat road where vehicle 1 is referred to as the lead vehicle (the pursuer) and vehicle 2 is the follower vehicle (the evader).

$\Phi(\cdot, \cdot)$ is the viscosity solution of the following modified Hamilton-Jacobi-Isaacs partial differential equation (HJI PDE)

$$\frac{\partial \Phi(x, t)}{\partial t} + \min(0, H(x, p, u^*)) = 0, \quad t \in \mathbb{R}^-, \tag{2}$$

with terminal conditions $\Phi(x, 0) = \Phi_0(x)$.

The procedure for solving the HJI PDE can intuitively be described as starting at any point on the boundary of the unsafe set $\partial\mathcal{G}$. The usable part is then defined as:

$$UP = \{x \in \partial\mathcal{G} \mid p^T(0) f(x, u^*) < 0\}, \tag{3}$$

which denotes all the state trajectories heading in towards the unsafe set. Thus, the reachable set is calculated by starting on $\partial\mathcal{G}$ and under optimality constraints simultaneously solving the ODE:s, $\dot{x}$ and $\dot{p}$, *backwards* by calculating terminal conditions of $x(t_f)$ for when $p^T(t_f)x(t_f) \geq 0$. Hence, all the trajectories are calculated on the boundary of the usable part, for which it is possible to move away from the unsafe set and thus the surface sets which partitions the safe and unsafe regions are derived.

3. SYSTEM AND PROBLEM DESCRIPTION

In this section a mathematical model is derived for the system and then the problem formulation is given.

3.1 System Model

The state equation of a single HDV can be formulated as

$$\begin{aligned} \frac{ds_i}{dt} &= v_i \\ m_t \frac{dv_i}{dt} &= F_{engine} - F_{brake} - F_{airdrag}(v_i) \\ &\quad - F_{roll}(\alpha_i) - F_{gravity}(\alpha_i) \\ &= F_{engine} - k_i^b F_{brake} - k_i^d v_i^2 \\ &\quad - k_i^{fr} \cos\alpha - k_i^g \sin\alpha \end{aligned} \tag{4}$$

where m_t denotes the accelerated mass and $i = 1, 2$ denote the vehicle index. k_i^b, k_i^d, k_i^{fr}, and k_i^g denote the characteristic vehicle and environment coefficients for the brake, air drag, road friction, and gravitation respectively (for a more detailed description see Sahlholm and Johansson (2010)). The presented model is accurate for high velocities. At low velocities the internal dynamics of the vehicle has a larger impact on the longitudinal dynamics.

For simplicity of this study, the road is assumed to be flat and the control input is $u = F_{brake}$. The system can be simplified to three dimensions as partially relative variables with respect to the follower vehicle:

$$\dot{x} = f(x, u_1, u_2) = \begin{cases} v_1 - v_2 = -v_r \\ \dot{v}_2 - \dot{v}_1 = c_2^u u_2 - c_1^u u_1 - c_2^w(s_r)v_2^2 \\ \qquad +c_1^d(v_2 - v_r)^2 - c_2^{fr} + c_1^{fr} \\ c_2^u u_2 - c_2^w(s_r)v_2^2 - c_2^{fr} \end{cases} \tag{5}$$

where $x = [s_r \ v_r \ v_2]^T$ and $c_i^z = \frac{k_i^z}{m_t}$, $z = (b, d, fr, g)$. The state variable s_r denotes the relative distance between the vehicles, v_r is the relative velocity between the vehicles, and v_2 is the velocity of the follower vehicle (Fig. 1). The variable

$$c_2^w(s_r) = c_2^d(1 - \frac{-0.414s_r + 41.3}{100}) \tag{6}$$

denotes the linearized air drag reduction, Wolf-Heinrich and Ahmed (1998), which is a result of driving close to a vehicle ahead.

Both vehicles are only allowed to move in a forward longitudinal direction. Thus, there is a inherent constraint that $v_1 = v_2 - v_r \geq 0$, in (5).

3.2 Problem Formulation

We consider a HDV platooning scenario where each vehicle only receives information regarding the relative position and velocity of the immediate vehicle ahead. The objective is to determine the minimum relative distance between a lead vehicle and a follower vehicle that can be maintained without endangering a collision.

The aim is to find the largest set of initial states, irrespective of how the lead vehicle behaves, for which there exists a controller that manages to keep all executions inside a set of states $F \subseteq \mathbb{R}^3$; a subset in which the system is defined to be safe inside the boundary.

A differential game formulation of the problem enables such a set derivation by capturing the event when the lead vehicle blunders in the worst possible manner. We model the game as the follower vehicle (player u_2) is trying its best to avoid a collision and the lead vehicle (player u_1) is trying its best to create a collision.

Hence, the problem at hand can be set up as a two-vehicle dynamic pursuer-evader game as described in Sec. 2:

$$\max_{u_2 \in \mathcal{U}} \min_{u_1 \in \mathcal{D}} p^T f(x, u_1, u_2) = H(x, p, u^*), \tag{7}$$

where $f(x, u_1, u_2)$ is the system (5) and u_1^*, u_2^* are the optimal strategies. The derived unsafe set $\mathcal{G}(\tau)$ under this condition then becomes conservative, which is preferable to ensure safety.

Thus, the problem we solve in this paper is the following: Compute the unsafe set $\mathcal{G}(\tau)$ for which it is guaranteed that given all the control inputs of the leader and follower vehicle, there is a possibility that a collision occurs despite the best effort from the follower vehicle.

4. SAFETY SETS

In the following sections we first derive the expression for the analytical Hamiltonian and define the unsafe set $\mathcal{G}_0$ for a two-vehicle collision scenario. Then we use the method described in Sec. 2 to derive the boundary conditions of the unsafe and inherently safe set.

The analytical Hamiltonian can be derived from (5) and (7) as:

$$\begin{aligned} H^*(x,p) &:= H(x, p, u^*) \\ &= \max_{u_2 \in \mathcal{U}} \min_{u_1 \in \mathcal{D}} p^T f(x, u_1, u_2) \\ &= [p_1 \ p_2 \ p_3] \begin{bmatrix} \dot{s}_r \\ \dot{v}_r \\ \dot{v}_2 \end{bmatrix} \\ &= -p_1 v_r - p_2 c_1^u u_1^* + (p_2 + p_3)c_2^u u_2^* \\ &\quad - (p_2 + p_3)c_2^w(s_r)v_2^2 + p_2 c_1^d(v_2 - v_r)^2 \\ &\quad - (p_2 + p_3)c_2^{fr} + p_2 c_1^{fr} \end{aligned} \tag{8}$$

The lead vehicle in (8) is given the advantage of determining its optimal control strategy based upon information regarding the following vehicles strategy. This is generally not the case. However we wish to find a set which guarantees that a collision can be avoided despite the best effort of the pursuer with respect to a compact set of controller actions, $u_1 \in \mathcal{D}$ and $u_2 \in \mathcal{U}$.

The costates can be derived as

$$\dot{p} = -\frac{\partial H^*}{\partial x} = \begin{cases} (p_2 + p_3)c_2^d \dfrac{0.414}{100} v_2^2 \\ p_1 + 2p_2 c_1^d(v_2 - v_r) \\ 2(p_2 + p_3)c_2^w(s_r)v_2 - 2p_2 c_1^d(v_2 - v_r) \end{cases}$$

Due to the linear dependency of the optimal control inputs in Eq. (8), they can easily be computed as:

$$\begin{aligned} u_1^* &= \frac{\bar{T}_1 + \underline{T}_1}{2} + \text{sgn}(s_1)\frac{\bar{T}_1 - \underline{T}_1}{2} \\ u_2^* &= \frac{\bar{T}_2 + \underline{T}_2}{2} + \text{sgn}(s_2)\frac{\bar{T}_2 - \underline{T}_2}{2} \end{aligned} \tag{9}$$

where $T_i \in [\underline{T}_i, \bar{T}_i], i = 1, 2$, is the brake torque available to vehicle i and $s_1 = p_1 c_1^u$, $s_2 = (p_1 + p_2)c_2^u$. Since the sign function is undefined for $s_1 = s_2 = 0$, the sign of $\dot{s}_1$ and $\dot{s}_2$ is checked to determine the value in those situations. The control input for each vehicle can switch instantaneously between $\underline{T}$ and $\bar{T}$ in (9). In practical applications this would imply that the brake- or acceleration request can be computed and implemented without any delay. However, the scenarios for when delay is present in the system is not excluded by this assumption, which is discussed further in Sec. 4.3.

We define the unsafe set $\mathcal{G}_0$, as a region when the two vehicles are within d units from each other:

$$\mathcal{G}_0 = \{x \in \mathbb{R}^3 | s_r - d < 0\}. \tag{10}$$

The HDVs modeled here are described as traveling in a longitudinal direction and a collision has therefore occurred if $s_r \leq d = 0$. The maximum braking torque in a HDV depends on the vehicle configuration but can be approximated as $\sim 60000\,\text{Nm/axle}$. A maximum deceleration on a flat and dry road have been measured up to $6.5\,\text{m/s}^2$. Commercial HDVs generally have a speed restriction of $90\,\text{km/h}$. Therefore, the unsafe set is calculated backwards for $\tau = -t = -4\,\text{s}$, which implies that a vehicle is able to reduce its velocity by $93.6\,\text{km/h}$. Hence, the chosen simulation time enables the vehicles to come to a full stop within the calculated safety sets. It is considered as safe if the vehicles have stopped and no collision has occurred.

To compute the unsafe set from the solution of Eq. (2) a state-of-the-art toolbox of Level Sets Methods, Mitchell (2007), is utilized.

4.1 Identical HDVs

The collision avoidance scenario is first investigated for two identical HDVs. Both vehicles have identical vehicle parameters in Eq. (4) and a 4×2 vehicle configuration. The gross mass of each HDV was chosen to be $40000\,\text{kg}$, which is considered to be a standard weight for European Long-haulage HDVs.

Fig. 2 shows the boundary $\partial\mathcal{G}(\tau)$, of the unsafe set contained between the plotted level surface and $\mathcal{G}_0$. As $v_r = v_2 - v_1$ increases, the relative distance $s_r = s_1 - s_2$ must also increase. The fold in the boundary surface area is due to the physical constraint $v_1 \geq 0$. Any trajectory heading behind that surface area would imply that the lead vehicle has reversed to create a collision. If the follower vehicle is within the safe set, it will always be able to avoid a collision regardless of the best effort of the lead vehicle (pursuer) with respect to a compact set of controller actions. Thus a least restrictive controller could be implemented outside the unsafe set without endangering a collision. However, if it is within the unsafe set a collision might occur given that the pursuer acts in the worst possible manner.

In platooning applications the vehicles generally travel in what we here refer to as a *normal mode*, where each vehicle is traveling at a constant fixed velocity, $v_r = 0$, and a desired relative distance is set by the driver. Fig. 2 reveals that a collision can be avoided for two identical HDVs if the lead vehicle is traveling at a higher velocity than the follower vehicle. However, if the vehicles are operating in normal mode and has a relative distance $s_r \leq \gamma$, where $\gamma = \partial\mathcal{G}(\tau)|_{v,v_r=0}$, a collision could occur. The lead vehicle experiences a greater air drag and is therefore able to obtain a slightly higher braking force. Thus, if the vehicles are both traveling at a velocity, $v \leq 2.5\,\text{m/s}$, a collision could occur for $\gamma = 0.2\,\text{m}$. As both vehicles velocity increases the air drag and inherently the obtainable brake force becomes higher for the lead vehicle. Thus, a larger relative distance of $s_r > \gamma = 1.1\,\text{m}$ must be maintained at $v = 25\,\text{m/s}$ to stay out of the unsafe region. Hence, the minimum relative distance that can be obtained for two identical vehicles depends on their current velocity.

4.2 Varying Vehicle Parameters

Here we investigate the unsafe set for a lower or higher mass of the follower vehicle compared to the lead vehicle, keeping all other parameters constant. A difference in mass affects the deceleration stretch compared to when both the vehicles are identical.

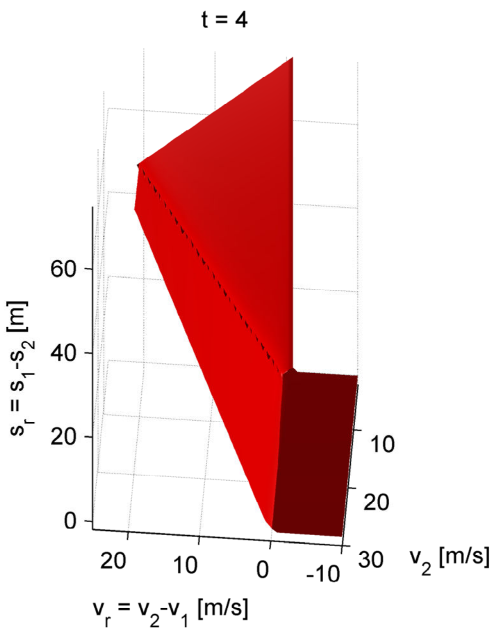


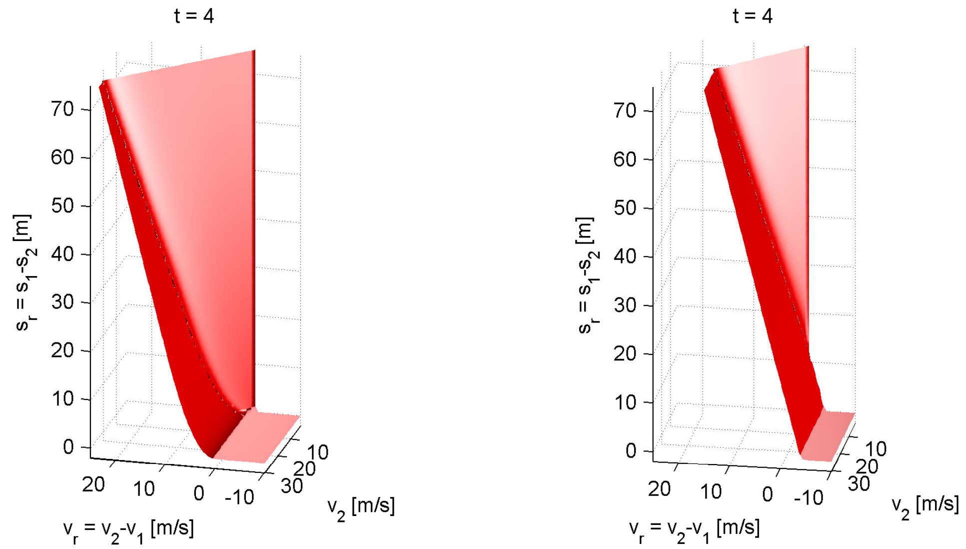
Fig. 2. The backward reachable set obtained under the assumption that no delay is present in the system.

The unsafe set in Fig. 3(a) is derived for the case where the follower vehicle is 10 t lighter than the lead vehicle. A similar unsafe set is formed when the available brake force is solely increased for the follower vehicle. The follower vehicle being lighter implies that it has a lower mass to decelerate. Therefore, the deceleration stretch becomes shorter compared to the lead vehicle. The shape of the reachable set might seem unchanged. However, a shift has occurred in the level surface in the positive v_r-direction, hence the follower vehicle will be able to lie closer without endangering a collision. If no delay is present in the system the vehicles would now in theory be able to lie attached to each other and the lead vehicle could have a slightly lower velocity at the time when a the lead vehicle strives to collide and no collision would occur. The minimum relative distance of $s_r > 1.1\,\text{m}$ is no longer valid since the higher overall braking force induced by a lower mass for the follower vehicle is greater than the larger air drag for the lead vehicle.

However, in Fig. 3(b) the follower vehicle is 10 t heavier than the lead vehicle. Hence, the lead vehicle has a greater braking capability and thus a perturbation arises in the level surface at $v_r \approx 0$. In this case a minimum distance of $s_r = 13\,\text{m}$ must be maintained to remain outside the reachable set at normal mode. Thus, the relative distance must be increased significantly if the lead vehicle has a stronger braking capability.

4.3 Delay and Model Uncertainties

The results presented so far have been derived by assuming that there is no delay present in the system. However, in real life this is rarely the case. In this section we describe



(a) Follower vehicle has a 10 t lower mass.

(b) Follower vehicle has a 10 t higher mass.

Fig. 3. The backward reachable set obtained under the assumption that no delay is present in the system and the vehicles have different configuration.

how delays can be translated to the previously derived results.

Delays commonly occur due to transmission, computation, and producing the control command. Consider the case were a delay of τ_1 [s] occurs due to transmission delay from receiving information regarding the relative distance and velocity. An additional delay of τ_2 [s] occurs in detection and confirmation that a vehicle in front is braking rapidly. The actuation delay is measured to be τ_3 [s]. Thus, the total delay can be $\tau = \sum_{i=1}^{3} \tau_i$ in a worst case scenario. Hence, the lead vehicle will be able to act before the follower vehicle is able to react. This implies that the lead vehicle will be able to reduce the relative velocity and distance, which can easily be computed in the worst case scenario.

A delay can be translated into a shift of the reachable in Fig. 2-3 by Δs_r units in the positive direction along the s_r-axis and by Δv_r units in the negative direction along the v_r-axis. However, no change occurs in the follower vehicles velocity, v_2, since it does not react. Depending on the radar and the collision detection algorithm, the worst case delay can be approximated to be $\tau_1 + \tau_2 \approx 500$ ms. Hence, the lead vehicle will be able to reduce the relative velocity by 3.25 m/s and the relative distance by 0.8 m if it is driving 25 m/s at normal mode. Thus if the follower vehicle maintains $s_r \geq 2$ m, a collision can always be avoided for two identical vehicles according to Fig. 2.

5. SIMULATION & VALIDATION

To investigate the validity of the derived safety regions in Fig. 2, a simulation study is conducted within this section for a finite number of scenarios.

Two identical vehicles were simulated in an advanced simulated model consisting of 3313 variables, 1058 equations, and 626 states for each vehicle, Sandberg (2001). The model is verified to mimic real life behavior. Hence, two different simulation scenarios are presented in Fig. 4. The velocity trajectory for the lead vehicle, the follower vehicle and the relative distance is displayed in the top plot, the middle plot, and the bottom plot respectively.

The first scenario was chosen as a point on the level surface in Fig. 2. It can be described as a HDV traveling on a highway and another HDV appears in front, e.g. from a shoulder at a lower velocity. Upon entering, the vehicles suddenly brake due to an incident ahead on the road. The follower vehicle has a velocity of 20.23 m/s and approaches the lead vehicle entering with a 13 m/s lower velocity. When a relative distance of 45.14 m is reached, both vehicles implement the optimal control input derived in Eq. (9). We can see that a collision is avoided since both vehicles have come to rest at $s_r = 5.32$ m. Hence, collision could be avoided for a higher relative velocity or a lower relative distance, which is a point below the level surface. This is due to the problem formulation, which produces a preferred overapproximated level surface in safety applications.

In the second scenario a point below the level surface in Fig. 2 was chosen by lowering the relative distance and increasing the relative velocity by 9.7 %. The follower vehicle has a velocity of 20.23 m/s and now approaches the lead vehicle with a 14.3 m/s higher velocity. Both vehicles applies their optimal control input at a relative distance of 40.76 m. The lead vehicle has come to a rest while $v_2 > 0$ in Fig. 4 and the relative distance is zero. Thus a collision has occurred.

Several scenarios were investigated for different points on the level surface in Fig. 2 and within the reachable set. The results are summarized in Table 1, where the initial conditions for each collision scenario is given in the middle columns and the final relative distance is given in the fourth column.

As can be deduced from Table 1 and Fig. 4, the safe set ensures that collision is avoided. The level surface is more accurate for simulation no. 4-6, where the HDVs are

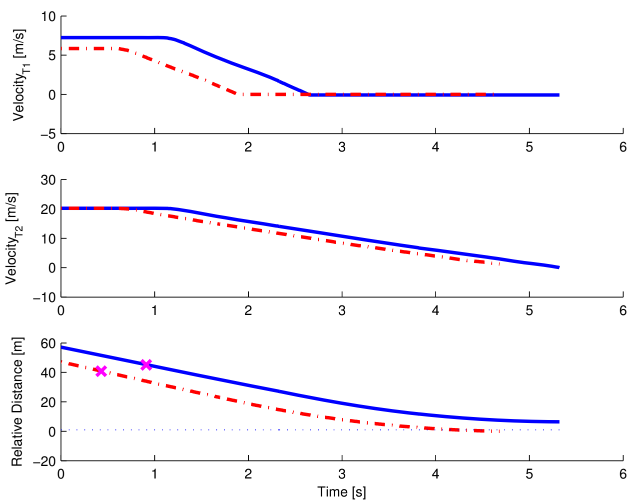


Fig. 4. Simulated emergency brake scenario. Solid line illustrates the scenario with initial conditions selected on the level surface for two identical vehicles and the dashed line illustrates initial conditions behind the level surface. The $\times$:s in the bottom plot indicates the point where both vehicles initiates their optimal control inputs.

operating close to normal mode. Both the HDVs have come to a total halt at an $s_r \leq 0.3$ m. For the scenarios when the relative velocities differ more, simulation no. 1-3, the vehicles come to rest at a larger relative distance due to the increased model uncertainty at lower velocities. Hence, even though Eq. (4) is a fairly simple model of an HDV, it seems to be a sufficient for this application.

6. SUMMARY & CONCLUSION

It is fuel efficient to drive vehicles closely spaced to each other due to the inherent minimized air drag reduction. A minimum distance for two HDVs can be deduced with respect to a compact set of controller actions without endangering a collision despite the worst possible action by the vehicle ahead. Thus, if the minimum distance is determined for each vehicle pair, a collision can be avoided throughout the platoon. The level set toolbox utilized in this paper provides a the means to visualize the reference frame and boundary of the safety set. During normal mode operation a minimum distance of 1.2 m should be maintained to ensure that a collision can be avoided for two identical vehicles and no delay is present in the system. Delays can be measured and implemented in the reachable set formulation through a simple translation. The derived minimum possible relative distance with a measured worst case delay of 500 ms in this paper for identical vehicles is 2 m, which is lower compared to what is utilized in commercial applications today without endangering safety.

A stronger overall braking capability in the follower vehicle creates the possibility of reducing the relative distance further. Thus, in platooning applications the results suggest that vehicles with stronger braking capabilities should always be placed behind to enable the shortest possible relative distance without endangering a collision. Wireless communication could further improve the system by enabling a reduction of the relative distance between each

Table 1. Table of several initial conditions for collision scenarios.

Simulation no.:	$v_r(t_0)$	$s_r(t_0)$	$v_2(t_0)$	$s_r(t_f)$
1)	18.3	71.9	25.1	10.9
2)	11.7	45.7	20.2	10.3
3)	3.6	13.7	10.4	6.7
4)	0.24	1.14	10.4	0.3
5)	0.2	1.14	15.3	0.5
6)	0.5	2.3	25.1	0.05

vehicle from the 2:nd to the N:th vehicle. Producing the maximum brake torque after a request could take up to 0.4 s depending on the vehicle configuration, which will be propagated downstream. However, if the first vehicle transmits that it is emergency braking to all the following vehicles within the platoon, they will be able to commence the control action almost simultaneously and thereby cancel a propagation in actuation delay.

In real life scenarios, additional factors may shape the safety boundary which is not captured by the current model. One of the vehicles might be traveling on a wet patch or the tires might be in a worse condition, reducing the frictional force from the road. Also, nonlinearities might arise in the applied brake force due to temperature variation. Investigating these factors would require a more advanced vehicle model and is therefore left as future work.

REFERENCES

Alam, A.A., Gattami, A., and Johansson, K.H. (2010). An experimental study on the fuel reduction potential of heavy duty vehicle platooning. *13th Intl. IEEE Conf. on Intelligent Transp. Systems*.

Basar, T. and Olsder, G.J. (1995). *Dynamic Noncooperative Game Theory*. Academic Press, New York, 2nd edition. ISBN: 978-0-89871-429-6.

Bayen, A.M., Shanthanam, S., Mitchell, I., and Tomlin, C.J. (2003). A differential game formulation of alert levels in etms data for high altitude traffic. *GNC Conference*, AIAA, 2003-5341.

Hedrick, J.K., McMahon, D., Narendran, V., and Swaroop, D. (1991). Longitudinal vehicle controller design for ivhs systems. In *Proceedings of American Control Conference, 1991*, 3107 –3112.

Isaacs, R. (1999 (1965)). *Differential Games*. Dover Publications, Inc. (John Wiley).

Mitchell, I. (2002). *Application of Level Set Methods to Control and Reachability Problems in Continuous and Hybrid Systems*. Ph.D. thesis, Stanford University.

Mitchell, I. (2007). A toolbox of level set methods (version 1.1). Vancouver, BC, Canada. Tech. Rep. TR-2007-11, [Online]. Available: http://www.cs.ubc.ca/ mitchell/ToolboxLS/toolboxLS.pdf.

Mitchell, I., Bayen, A.M., and Tomlin, C.J. (2002). Computing reachable sets for continuous dynamic games using level set methods. *IEEE Transactions on Automatic Control*.

Rajamani, R. and Zhu, C. (1999). Semi-autonomous adaptive cruise control systems. In *Proceedings of the 1999 American Control Conference*, volume 2, 1491 –1495.

Sahlholm, P. and Johansson, K.H. (2010). Road grade estimation for look-ahead vehicle control using multiple measurement runs. *Control Engineering Practice*, 18(11), 1328 – 1341.

Sandberg, T. (2001). *Heavy Truck Modeling for Fuel Consumption, Simulations, and Measurements*. Licentiate thesis LiU-TEK-LIC-2001:61, Linköpings universitet, S-581 83 Linköping, Sweden.

Seiler, P., Song, B., and Hedrick, J. (1998). Development of a collision avoidance system. *SAE Conference, pp. 97-103*.

Tomlin, C.J., Lygeros, J., and Sastry, S.S. (2000). A game theoretic approach to controller design for hybrid systems. *Proceedings of the IEEE*, 88(7).

Wolf-Heinrich, H. and Ahmed, S.R. (1998). *Aerodynamics of Road Vehicles*. Society of Automotive Engineers, Inc, Warrendale.