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IFAC-PapersOnLine 48-12 (2015) 364–369

Control of platoons of heavy-duty vehicles using a delay-based spacing policy ⋆

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Abstract: The formation of groups of heavy-duty vehicles driving at close inter-vehicular distances (known as a platoon) reduces the fuel consumption due to a decreased aerodynamic drag and has the potential to increase traffic flow. This paper motivates the use of a novel spacing policy, which specifies the desired distance between vehicles as a function of their states. Particularly, a delay-based spacing policy is introduced, which guarantees that all vehicles in the group follow the same velocity profile in space. It is shown that the proposed spacing policy offers advantages in the platoon control for non-constant reference velocities, which are common when driving over hilly terrain. A controller is designed to achieve this spacing policy, hereby exploiting an analysis of the vehicle dynamics in the spatial domain rather than time domain. Moreover, the controller is shown to attenuate the effect of disturbances as they propagate through the platoon. Simulations are used to illustrate the effectiveness of the proposed spacing policy and controller.

Keywords: Automotive control, co-operative control, delay, cruise control, stability analysis

1. INTRODUCTION

The operation of heavy-duty vehicles at small inter-vehicular distances has the potential to reduce fuel consumption through decreased aerodynamic drag. In fact, experimental results on groups of closely-spaced vehicles, known as platoons, have shown a reduction of fuel consumption of up to 10%, see Alam (2014) and Bonnet and Fritz (2010). Apart from this clear economical and ecological advantage, platooning also has the potential to increase traffic flow by a better utilization of the existing road infrastructure.

In order to safely achieve the desired small inter-vehicular distances, automation of the longitudinal dynamics is required, which is aided by the use of communication techniques to obtain information on the state of neighboring vehicles in a platoon. An early work on such control techniques is given by Levine and Athans (1966) and many results have appeared since, see, e.g., Peppard (1974); Stanković et al. (2000); Naus et al. (2010). A crucial aspect in controller design is given by the spacing policy, which defines the desired inter-vehicular distance as a function of the states of two neighboring vehicles. The constant spacing policy (see Swaroop and Hedrick (1999)) and the constant headway spacing policy (see Ioannou and Chien (1993)) are the most common examples, where the latter amounts to a relaxation of the constant spacing policy by accounting for the velocity of the follower vehicle. More advanced spacing policies are discussed in Yanakiev and Kanellakopoulos (1998).

However, in the controller design for such spacing policies, it is in general implicitly assumed that the platoon has a constant reference velocity. Any deviations from this reference (typically by the lead vehicle) are regarded as disturbances and the response of the follower vehicles is studied without explicitly considering their exact velocity profiles.Nonetheless, there are many cases in which a varying reference velocity is desirable, with the most notable example being given by heavy-duty vehicles driving over hilly terrain. Namely, the most fuel-efficient way of traversing a hilly road segment is generally given by a non-constant velocity profile (see Hellström et al. (2009)), where it is noted that heavy-duty vehicles might not be able to maintain speed during climbs due to limited engine power. However, the constant spacing and constant headway policies do not guarantee that all vehicles in a platoon follow the same velocity profile in the spatial domain (i.e., relative to the position on the road) and might for example require a follower vehicle to accelerate while climbing a hill. This might be infeasible due to limited engine power and lead to unsatisfactory platoon behavior, as has been recognized in experiments published in Alam (2014).

This paper therefore proposes the use of a spacing policy in which a vehicle tracks a time-delayed version of the trajectory of the preceding vehicle, which will be shown to guarantee that all vehicles track the same velocity profile in space. In particular, a controller that achieves this delay-based spacing policy will be developed, hereby following an approach in which space (rather than time) is taken as the independent variable. This approach enables a simple design procedure for such a controller, which will be shown to asymptotically stabilize the desired spacing policy as well as guarantees that perturbations from the

⋆ This research is funded by the FP7 Programme of the European Union through the project COMPANION.

10.1016/j.ifacol.2015.09.405

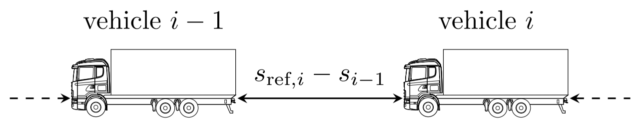

Fig. 1. Desired spacing policy $s_{\mathrm{ref},i}(t) - s_{i-1}(t)$ between automatically controlled vehicles in a platoon.

nominal velocity trajectory do not grow through the group of vehicles. Finally, the use of a delay-based spacing policy makes this approach inherently robust with respect to delays in the (wireless) inter-vehicle communication.

The remainder of this paper is organized as follows. First, in Section 2, properties of the constant spacing and constant headway spacing policies are analyzed, motivating the introduction of the constant time gap spacing policy. For the latter policy, controller design is pursued in Section 3 and its stability properties are analyzed in Section 4. The properties of this control strategy are illustrated by means of simulations in Section 5, before stating the conclusions of this work in Section 6.

2. SPACING POLICIES AND MOTIVATION

A crucial aspect in the dynamic behavior of platoons is given by the definition of the inter-vehicular spacing (see Figure 1). Various spacing policies have been proposed in the literature, of which the *constant spacing* policy and the *constant headway* policy are the most notable. These policies are shortly reviewed in this section, providing a motivation for a novel spacing policy as analyzed in the remainder of this paper: the *constant time gap* spacing.

In order to analyze the properties of these spacing policies, let s_i denote the longitudinal position of vehicle i and v_i its velocity. Naturally, they satisfy the kinematic relation

$$\dot{s}_i(t) := \frac{\mathrm{d}s_i}{\mathrm{d}t}(t) = v_i(t). \tag{1}$$

A spacing policy describes the desired behavior $s_{\mathrm{ref},i}(t)$ of vehicle i on the basis of its predecessor with index $i-1$. Figure 2 depicts the velocity of vehicles in a platoon for various spacing policies, where it is assumed that the velocity of the lead vehicle $v_0(t)$ is prescribed and all follower vehicles track the desired behavior perfectly, i.e., $s_i(t) = s_{\mathrm{ref},i}(t)$.

The constant spacing policy (see, e.g., Swaroop and Hedrick (1999)) takes the form

$$s_{\mathrm{ref},i}(t) = s_{i-1}(t) - d, \tag{2}$$

where $d \geq 0$ is the desired inter-vehicular distance. By using the assumption $s_i(t) = s_{\mathrm{ref},i}(t)$ and (1), the policy (2) implies that changes in velocity occur simultaneously in time (i.e., $v_i(t) = v_{i-1}(t)$). This is also apparent from the top left graph in Figure 2. If the change in velocity of the lead vehicle is the result of a disturbance, it is clear that the effect of this disturbance is not mitigated. In fact, is has been shown in Seiler et al. (2004) that disturbance attenuation cannot be obtained for any controller that only uses measurements of the preceding vehicle $i-1$ for the control of vehicle i.

An alternative spacing policy that inherently attenuates the effect of disturbances is given by the constant headway policy (see, e.g., Swaroop et al. (1994); Ioannou and Chien (1993)), which reads

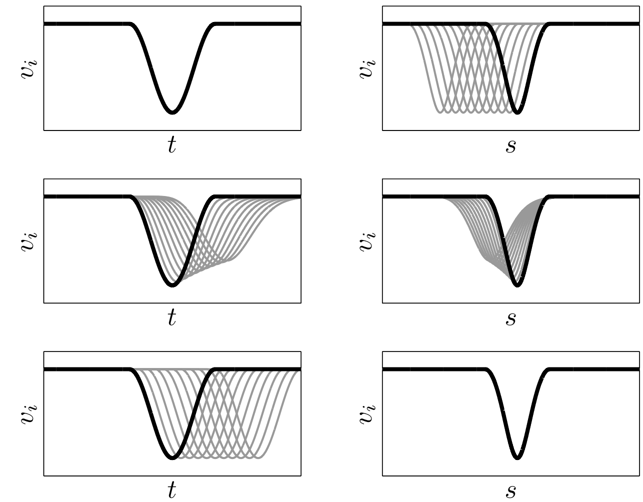

Fig. 2. Velocities v_i of ten follower vehicles (gray) as a result of a predefined velocity profile of the lead vehicle (black) for a constant spacing policy (top row), constant headway policy (middle row), and constant time gap policy (bottom row). The left column shows the velocity as a function of time t, whereas the right column gives the velocity as a function of space s.

$$s_{\mathrm{ref},i}(t) = s_{i-1}(t) - (d + hv_i(t)), \tag{3}$$

for some $h > 0$. By again using $s_i(t) = s_{\mathrm{ref},i}(t)$ and (1), this can be written as

$$h\dot{s}_i(t) = -s_i(t) + s_{i-1}(t) - d, \tag{4}$$

which shows that the desired reference position is essentially obtained by application of a first-order filter to the position of the preceding vehicle. It is this filtering, which is also apparent from the center left graph in Figure 2, that is responsible for the inherent attenuation of disturbances.

However, it is clear from the graphs in the right column of Figure 2 that, for the constant spacing and constant headway spacing policies, the changes in velocity occur on different positions in space for successive vehicles in the platoon. If the velocity change of the first vehicle was due to road properties such as hills rather than small undesired disturbances, this is potentially a large disadvantage. To illustrate this, consider a platoon of heavy-duty vehicles climbing a hill. Due to limited engine power, a large gradient can cause the lead vehicle of the platoon to decrease velocity as in Figure 2. However, it is clear that follower vehicles might be required to have a higher velocity on this hill (i.e., at the same location in space) when they are subject to a constant spacing or constant headway policy. This might be infeasible due to limited engine power and leads to undesired platoon behavior, as recognized in Alam et al. (2013) and Turri et al. (2014).

In this paper, a spacing policy is introduced that guarantees that vehicles track the same velocity profile *in space*, which avoids the aforementioned disadvantages. In particular, this spacing policy is given by

$$s_{\mathrm{ref},i}(t) = s_{i-1}(t - \Delta t), \tag{5}$$

where a time-delayed version of the trajectory of the preceding vehicle is tracked with time gap $\Delta t > 0$. It indeed achieves equal velocity profiles in space, as stated next.

Lemma 1. Consider the kinematics (1) and assume $s_i(t) = s_{\mathrm{ref},i}(t)$ and $v_i(t) > 0$ for all $t \in \mathbb{R}$. Then, (5) holds if and only if[1], for some function $v_{\mathrm{ref}}(\cdot)$,

$$v_i(s) = v_{i-1}(s) = v_{\mathrm{ref}}(s). \tag{6}$$

Proof. In order to prove the lemma, let s be a point in space and let $t_i(s)$ be the time instance when vehicle i passes that point. Note that the assumption $v_i(t) > 0$ for all $t \in \mathbb{R}$ guarantees that $t_i(s)$ is uniquely defined. Then, using $s_i(t) = s_{\mathrm{ref},i}(t)$, (5) can equivalently written as

$$t_i(s) = t_{i-1}(s) + \Delta t, \tag{7}$$

for all $s \in \mathbb{R}$. Next, the expression of the kinematic relation (1) in spatial domain leads to

$$\frac{\mathrm{d}t_i}{\mathrm{d}s}(s) = \frac{1}{v_i(s)}, \tag{8}$$

after which integration yields

$$t_i(s_1) - t_i(s_0) = \int_{s_0}^{s_1} \frac{1}{v_i(s)}\,\mathrm{d}s, \tag{9}$$

for some initial position s_0. When considering (9) for vehicles i and $i-1$, the subtraction of both results and use of (7) leads to

$$\int_{s_0}^{s_1} \frac{1}{v_i(s)} - \frac{1}{v_{i-1}(s)}\,\mathrm{d}s = \Delta t - \Delta t = 0. \tag{10}$$

As (10) holds for all $s_0, s_1 \in \mathbb{R}$ such that $s_1 \geq s_0$, it is clear that $v_i(s) = v_{i-1}(s) =: v_{\mathrm{ref}}(s)$ for all s, proving the first part of the lemma.

To prove the converse, assume that $v_i(s) = v_{i-1}(s) = v_{\mathrm{ref}}(s)$. Subsitution of this in the left-hand term in (10) gives $t_i(s_1) - t_{i-1}(s_1) = t_i(s_0) - t_{i-1}(s_0) =: \Delta t$, finalizing the proof of the lemma.

The objective of this paper can now be stated as the synthesis of a controller that, first, asymptotically stabilizes the desired constant time gap policy (5) and, second, attenuates perturbations on the lead vehicle's velocity.

3. PLATOON CONTROLLER DESIGN

Consider a platoon of $N+1$ vehicles, where $\mathcal{I} := \{1, 2, \ldots, N\}$ is the set of indices for the follower vehicles. The lead vehicle is denoted by index 0 and $\mathcal{I}_0 := \{0, 1, \ldots, N\}$. Then, following, e.g., Stanković et al. (2000) and Ploeg et al. (2014), each vehicle is modeled as

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \\ \tau \dot{a}_i(t) &= -a_i(t) + u_i(t), \end{aligned} \tag{11}$$

where $i \in \mathcal{I}_0$. In (11), $s_i(t) \in \mathbb{R}$, $v_i(t) \in \mathbb{R}$, and $a_i(t) \in \mathbb{R}$ represent the position, velocity, and acceleration of vehicle i, respectively. The final equation in (11) describes the actuator dynamics with time scale τ (with $\tau > 0$), where $u_i(t) \in \mathbb{R}$ is the external input. Here, it is remarked that the model (11) might be the result of the feedback linearization of a more complex nonlinear model (see Stanković et al. (2000)).

[1] The slight abuse of notation $v_i(t)$ and $v_i(s)$ will be used to indicate the velocity of vehicle i as a function of time and space, respectively.

Motivated by the discussion in Section 2, a controller will be designed that, first, guarantees that all vehicles track the same velocity profile $v_{\mathrm{ref}}(\cdot)$ in space, and, second, achieves an inter-vehicular spacing according to the constant time gap spacing policy (5). Here, it is noted that these objectives are aligned by Lemma 1. Because of the first objective, it is convenient to consider the spatial domain rather than time domain. Thereto, let the space s be the independent variable and denote $t_i(s)$ as the time instance at which vehicle i passes s. Then, after introducing

$$\Delta_i(s) := t_i(s) - t_{i-1}(s) - \Delta t, \tag{12}$$

the spacing policy (5) is equivalent characterized as $\Delta_i(s) = 0$, where it is recalled that the constant time gap spacing policy is only well-defined for positive velocities. This leads to the following assumption.

Assumption 2. The reference $v_{\mathrm{ref}}(\cdot)$ satisfies $v_{\mathrm{ref}}(s) > 0$ for all $s \geq 0$ and is twice continuously differentiable.

Using the kinematic relation (1), the vehicle dynamics (11) can be written in the spatial domain as

$$\begin{aligned} \frac{\mathrm{d}t_i}{\mathrm{d}s}(s) &= \frac{1}{v_i(s)}, \\ \frac{\mathrm{d}v_i}{\mathrm{d}s}(s) &= \frac{a_i(s)}{v_i(s)}, \\ \tau\frac{\mathrm{d}a_i}{\mathrm{d}s}(s) &= -\frac{a_i(s)}{v_i(s)} + \frac{u_i(s)}{v_i(s)}, \end{aligned} \tag{13}$$

when $v_i(s) > 0$ for all $s \geq 0$ and for $i \in \mathcal{I}_0$.

In order to achieve tracking of the reference velocity v_{ref}, the tracking error e_i is defined as

$$e_i(s) = \frac{1}{v_i(s)} - \frac{1}{v_{\mathrm{ref}}(s)}, \tag{14}$$

where the nonlinear form is chosen in order to match the dynamics of t_i in (13). Next, the input u_i is introduced as

$$\begin{aligned} u_i(s) &= a_i(s) + 3\tau\frac{a_i^2(s)}{v_i(s)} \\ &\quad - \tau v_i^4(s)\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left(\frac{1}{v_{\mathrm{ref}}(s)}\right) + \tilde{u}_i(s)\right). \end{aligned} \tag{15}$$

The choice of u_i in (15) can be shown to achieve feedback linearization (see Khalil (2002) for details on feedback linearization) of the final two equations in (13) with respect to the output e_i. In particular, the application of (15) in (13) leads to

$$\begin{aligned} \frac{\mathrm{d}t_i}{\mathrm{d}s}(s) &= e_i(s) + \frac{1}{v_{\mathrm{ref}}(s)}, \\ \frac{\mathrm{d}^2 e_i}{\mathrm{d}s^2}(s) &= \tilde{u}_i(s), \end{aligned} \tag{16}$$

with $\tilde{u}_i$ a virtual input and $i \in \mathcal{I}_0$.

As the lead vehicle in the platoon (with index $i = 0$) has no predecessor, its only objective is the tracking of the reference velocity $v_{\mathrm{ref}}(\cdot)$, which by (14) corresponds to the stabilization of the equilibrium point $e_i = \frac{\mathrm{d}e_i}{\mathrm{d}s} = 0$ of the second equation in (16). As this dynamics is linear, it follows directly that the feedback controller

$$\tilde{u}_0(s) = -p_0 e_0(s) - p_1\frac{\mathrm{d}e_0}{\mathrm{d}s}(s) \tag{17}$$

achieves the desired behavior when $p_0 > 0$ and $p_1 > 0$.

Whereas the control of the lead vehicle is targeted at tracking the reference velocity, follower vehicles will be controlled to achieve the desired constant time gap spacing policy (12), which, by Lemma 1, also guarantees tracking of the reference velocity profile. Thereto, the variable δ_i is introduced as

$$\delta_i(s) = t_i(s) - t_{i-1}(s) - \Delta t + he_i(s), \tag{18}$$

with $h > 0$, $i \in \mathcal{I}$. In (18), the first part corresponds to the desired spacing Δ_i as in (12), whereas the second part (i.e., he_i) allows relaxation of the spacing policy when the velocity profile is not tracked perfectly. It is noted that the inclusion of the additional term he_i in (18) has a similar effect as the inclusion of the term hv_i in the constant headway spacing policy (3). In particular, it will be shown that the inclusion of he_i achieves string stability with respect to perturbations on the lead vehicle's velocity.

In order to design a controller that asymptotically achieves $\delta_i(s) = 0$, (18) is differentiated to obtain

$$\frac{\mathrm{d}^3\delta_i}{\mathrm{d}s^3}(s) = \tilde{u}_i(s) - \tilde{u}_{i-1}(s) + h\frac{\mathrm{d}\tilde{u}_i}{\mathrm{d}s}(s), \tag{19}$$

as follows from the dynamics (16). Then, by introduction of the virtual input ξ_i as

$$h\frac{\mathrm{d}\tilde{u}_i}{\mathrm{d}s}(s) + \tilde{u}_i(s) = \xi_i(s) \tag{20}$$

and the selection of ξ_i as

$$\xi_i(s) = -\left(k_0\delta_i(s) + k_1\frac{\mathrm{d}\delta_i}{\mathrm{d}s}(s) + k_2\frac{\mathrm{d}^2\delta_i}{\mathrm{d}s^2}(s)\right) + \tilde{u}_{i-1}(s), \tag{21}$$

it can be observed that the controller parameters k_i, $j \in \{0,1,2\}$, can be chosen (e.g., by using the Routh Hurwitz criterion or pole placement techniques) to achieve asymptotic stability of the controlled dynamics for δ_i given by (19)–(21). It is noted that (20) amounts to a dynamic controller prescribing the control input $\tilde{u}_i$ (as used in (16) for $i \in \mathcal{I}$) on the basis of measurements of δ_i and its derivatives in space as well as the control input of the preceding vehicle $\tilde{u}_{i-1}$ (see (21)). To obtain the latter, (wireless) communication techniques are required.

Remark 3. The control input $\tilde{u}_{i-1}$ of the preceding vehicle is required for a given position s in (21), which it past some time before vehicle i. Consequently, this control approach is inherently robust to (small) time-delays.

4. CLOSED-LOOP STABILITY ANALYSIS

The properties of the closed-loop system given by the platoon velocity error dynamics (16) and controllers (17) for the lead vehicle ($i = 0$) and (20) with (21) and (18) for the follower vehicles ($i \in \mathcal{I}$) are analyzed in this section. Here, it is noted that the design of the controllers, that stabilize $e_0 = 0$ and $\delta_i = 0$, $i \in \mathcal{I}$, respectively, does not directly imply that the desired constant time gap spacing policy is achieved. This is due to the definition of δ_i in (18) which, apart from the desired spacing policy Δ_i, contains the term he_i, such that stabilization of $\delta_i = 0$ does not directly guarantee the stabilization of $\Delta_i = 0$. To further illustrate this, it is noted that (18) can be written as

$$\delta_i(s) = \Delta_i(s) + h\frac{\mathrm{d}\Delta_i}{\mathrm{d}s}(s) + he_{i-1}(s), \tag{22}$$

which clearly shows that the choice of δ_i induces dynamics on the spacing error Δ_i (Note that $\frac{\mathrm{d}\Delta_i}{\mathrm{d}s} = e_i - e_{i-1}$).

In the analysis of the controlled platoon behavior, interest is on the dynamics of the inter-vehicular spacings $\Delta_i(s)$ rather than the absolute position (given through the times $t_i(s)$). Thus, the closed-loop platoon dynamics is given by the velocity tracking dynamics of the lead vehicle

$$\frac{\mathrm{d}^2e_0}{\mathrm{d}s^2}(s) = -p_0e_0(s) - p_1\frac{\mathrm{d}e_0}{\mathrm{d}s}(s), \tag{23}$$

as follows from substitution of (17) in (16), as well as the spacing dynamics of the follower vehicles given by

$$h\frac{\mathrm{d}\Delta_i}{\mathrm{d}s}(s) = -\Delta_i(s) + \delta_i(s) - he_{i-1}(s), \tag{24}$$

$$\frac{\mathrm{d}^3\delta_i}{\mathrm{d}s^3}(s) = -k_0\delta_i(s) - k_1\frac{\mathrm{d}\delta_i}{\mathrm{d}s}(s) - k_2\frac{\mathrm{d}^2\delta_i}{\mathrm{d}s^2}(s), \tag{25}$$

$$he_i(s) = -\Delta_i(s) + \delta_i(s), \tag{26}$$

with $i \in \mathcal{I}$. Here, (24)–(25) follow from the dynamics (16) with the dynamic controller (20)–(21) and the coordinate transformation (12), (18). The corresponding velocity tracking error e_i is obtained as the output equation (26).

For the full closed-loop dynamics (23)–(25) with (26), the following result will be useful in showing that asymptotic stabilization of the desired constant time gap spacing policy is indeed achieved.

Lemma 4. Consider the dynamics (24). Then, there exists functions[2] β of class $\mathcal{KL}$ and γ of class $\mathcal{K}_\infty$ such that

$$|\Delta_i(s)| \leq \beta\big(|\Delta_i(0)|, s\big) + \gamma\left(\sup_{s\geq 0}\big|\delta_i(s) - he_{i-1}(s)\big|\right) \tag{27}$$

for any initial condition $\Delta_i(0)$ and trajectories $\delta_i(\cdot)$, $e_{i-1}(\cdot)$.

Proof. Differentiation (with respect to space) of the function $V(\Delta_i) = h\Delta_i^2$ along trajectories of (24) yields

$$\tfrac{\mathrm{d}}{\mathrm{d}s}V(\Delta_i) \leq -(1-\alpha)|\Delta_i|^2, \quad \forall \alpha|\Delta_i| > |\delta_i - he_{i-1}|, \tag{28}$$

for any α satisfying $0 < \alpha < 1$, which proves the result (27) by the theory of input-to-state stability, see, e.g., Sontag (1989); Khalil (2002).

Now, the following result can be proven.

Theorem 5. Consider the closed-loop platoon dynamics (23)–(25) with (26) for $i \in \mathcal{I}$. Then, the origin is the unique equilibrium point, which is asymptotically stable if and only if the controller parameters are chosen as $p_0 > 0$, $p_1 > 0$ and $k_0 > 0$, $k_1 > 0$, $k_2 > 0$ such that $k_1k_2 > k_0$.

Proof. The proof will be based on induction on the index of the follower vehicles, whereas the lead vehicle is considered in the first step.

The lead vehicle satisfies the closed-loop dynamics (23), from which it is directly observed that the equilibrium $e_0 = \frac{\mathrm{d}e_0}{\mathrm{d}s} = 0$ is asymptotically stable for the parameter values $p_0, p_1 > 0$. Consequently, there exists a function β_0 of class $\mathcal{KL}$ such that $|e_0(s)| < \beta_0(|\bar{e}_0(0)|, s)$ where $\bar{e}_0 = [\, e_0 \ \frac{\mathrm{d}e_0}{\mathrm{d}s} \,]^{\mathrm{T}}$.

To establish the inductive step, consider vehicle i satisfying the dynamics (24)–(25). The equilibrium point for (25)

[2] A continuous function $\alpha : [0,a) \to [0,\infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. If, in addition, $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$, it is of class $\mathcal{K}_\infty$. A continuous function $\beta : [0,a) \times [0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{KL}$ if, for each fixed s, the function $\beta(\cdot, s)$ is of class $\mathcal{K}$ and, for each fixed r, $\beta(r, \cdot)$ is decreasing and satisfies $\beta(r,s) \to 0$ as $s \to \infty$.

is unique (as $k_0 > 0$) and satisfies $\delta_i = \frac{\mathrm{d}\delta_i}{\mathrm{d}s} = \frac{\mathrm{d}^2\delta_i}{\mathrm{d}s^2} = 0$. Then, assuming that the equilibrium corresponding to the preceding vehicle satisfies $e_{i-1} = 0$, it follows from (24) that $\Delta_i = 0$ is the unique equilibrium point of (24), which corresponds to $e_i = 0$ through (26). Consequently, by induction, the origin is the unique equilibrium point of the controlled platoon (23)–(25) with (26) and $i \in \mathcal{I}$.

Asymptotic stability of this equilibrium for the follower vehicles will again be proven by induction. By the Routh-Hurwitz criterion (see, e.g., Antsaklis and Michel (2006)), the equilibrium of (25) is asymptotically stable, such that $|\delta_i(s)| \leq \kappa_i(|\bar{\delta}_i(0)|, s)$ for some function κ_i of class $\mathcal{KL}$ and $\bar{\delta}_i = [\, \delta_i \; \frac{\mathrm{d}\delta_i}{\mathrm{d}s} \; \frac{\mathrm{d}^2\delta_i}{\mathrm{d}s^2} \,]^{\mathrm{T}}$. Then, assuming that $|e_{i-1}(s)| < \beta_{i-1}(|\bar{e}_{i-1}(0)|, s)$, it is clear that the term $\delta_i - he_{i-1}$ in (24) vanishes when $s \to \infty$. By the input-to-state stability property of Lemma 4, it now follows that $\Delta_i(s)$ vanishes as well (see, e.g., Lemma 4.7 in Khalil (2002)), proving asymptotic stability of the interconnection (24)–(25) and the velocity error dynamics of the preceding vehicle. As a result, it is clear from (26) that there exists a function β_i of class $\mathcal{KL}$ such that $|e_i(s)| < \beta_i(|\bar{e}_i(0)|, s)$ with $\bar{e}_i = [\, \bar{e}_{i-1}^{\mathrm{T}} \; \Delta_i \; \bar{\delta}_i^{\mathrm{T}} \,]^{\mathrm{T}}$, such that asymptotic stability of the origin follows by induction on the vehicle index i. This finalizes the proof of the theorem.

Theorem 5 thus ensures the closed-loop stability of the desired constant time gap spacing policy, hereby also guaranteeing that all vehicles in the platoon track the same velocity reference profile (in space). However, even though asymptotic stability is clearly a requirement for desirable platoon behavior, it does not guarantee that disturbances caused by velocity tracking errors of the lead vehicle do not amplify through the string of vehicles. Nonetheless, the following theorem shows that such errors remain bounded.

Theorem 6. Consider the closed-loop follower dynamics (24)–(26) for $i \in \mathcal{I}$ and controller parameters satisfying $k_0 > 0$, $k_1 > 0$, $k_2 > 0$ such that $k_1 k_2 > k_0$. For initial conditions satisfying $\Delta_i(0) = 0$ and $\delta_i(0) = \frac{\mathrm{d}\delta_i}{\mathrm{d}s}(0) = \frac{\mathrm{d}^2\delta_i}{\mathrm{d}s^2}(0) = 0$ and velocity tracking error $e_0(\cdot)$ for the lead vehicle, the errors $e_i(\cdot)$ for the follower vehicles satisfy, with $i \in \mathcal{I}$ and for all $s \geq 0$,

$$\int_0^s |e_i(\bar{s})|^2 \,\mathrm{d}\bar{s} \leq \int_0^s |e_{i-1}(\bar{s})|^2 \,\mathrm{d}\bar{s}. \qquad (29)$$

Proof. In order to proof the theorem, it is first noted that the structure of (24), (25) implies that the manifold characterized by $\delta_i = \frac{\mathrm{d}\delta_i}{\mathrm{d}s} = \frac{\mathrm{d}^2\delta_i}{\mathrm{d}s^2} = 0$ is positively invariant. As a result $\delta_i(s) = 0$ for all $s \geq 0$, such that differentiation (with respect to space) of a function $V(\Delta_i) = \frac{1}{2}h\Delta_i^2$ along trajectories of (24) yields

$$\frac{\mathrm{d}}{\mathrm{d}s}V(\Delta_i) \leq -\Delta_i^2 - \Delta_i h e_{i-1}, \qquad (30)$$

$$= -\tfrac{1}{2}|he_i|^2 + \tfrac{1}{2}|he_{i-1}|^2 - \tfrac{1}{2}|he_{i-1} + \Delta_i|^2, \qquad (31)$$

where the equality (26) is used (for $\delta_i = 0$). Then, the integration of (31) gives

$$\tfrac{1}{2}\int_0^s |he_i(\bar{s})|^2 \,\mathrm{d}\bar{s} \leq \tfrac{1}{2}\int_0^s |he_{i-1}(\bar{s})|^2 \,\mathrm{d}\bar{s} - V(\Delta_i(s)), \qquad (32)$$

such that the result (29) follows by noting that $V(\Delta_i(s)) \geq 0$ and by scaling with $2h^{-2}$. It is remarked that (29) essentially represents an $\mathcal{L}_2$-gain (see van der Schaft (2000)), albeit in the spatial domain.

Table 1. Parameter values for the vehicle dynamics (11), spacing policy (18) and controllers (17), (20)–(21).

τ	1 [s]	p_0	0.09 [m^{-2}]	k_0	0.064 [m^{-3}]
Δt	1 [s]	p_1	0.60 [m^{-1}]	k_1	0.480 [m^{-2}]
h	10 [m]			k_2	1.200 [m^{-1}]

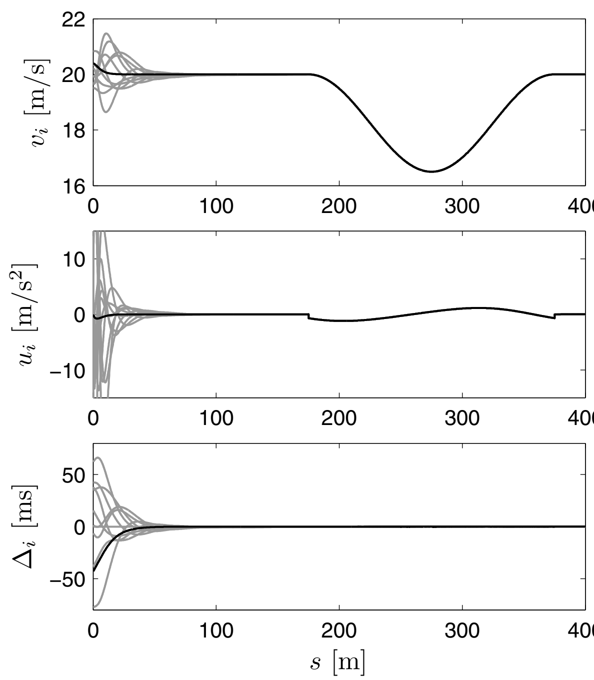

Fig. 3. Velocity (top) and control input (middle) for the lead vehicle (in black) and $N = 10$ follower vehicles (in gray) for non-equilibrium initial conditions and velocity profile $v_{\mathrm{ref}}(s) = 20 - 1.75\big(1 - \cos(10^{-2}\pi(s - 175))\big)$ for $175 \leq s \leq 375$ and $v_{\mathrm{ref}}(s) = 20$ otherwise. The spacing errors (bottom) are depicted for the first follower vehicle (in black) and remaining follower vehicle (in gray).

Condition (29) guarantees that deviations from the reference velocity do not amplify through the platoon. Such a stability notion (albeit with time as the independent variable) is commonly referred to as string stability (see Fenton et al. (1968) for an early definition and Ploeg et al. (2014) for a recent overview) and it is noted that an absence of string stability can lead to traffic jams or collisions. From the proof of Theorem 6, it is clear that the string stability property (29) is a direct consequence of the choice of δ_i in (18) rather than of the specific controller. In fact, any controller that renders a manifold on which $\delta_i = 0$ invariant achieves this property.

5. EVALUATION

In order to evaluate the performance of the controller as designed in Section 3, simulations are performed for an eleven-vehicle platoon (i.e., $N = 10$), hereby using the parameter values of Table 1.

First, the tracking of a velocity profile is considered in Figure 3, where initial conditions $t_i(0)$ and $v_i(0)$ in (12) are randomly chosen. From this figure, it is clear that the equilibrium $e_i(s) = 0$, which corresponds to $v_i(s) = v_{\mathrm{ref}}(s)$, is asymptotically stable. Similarly, it can be observed that

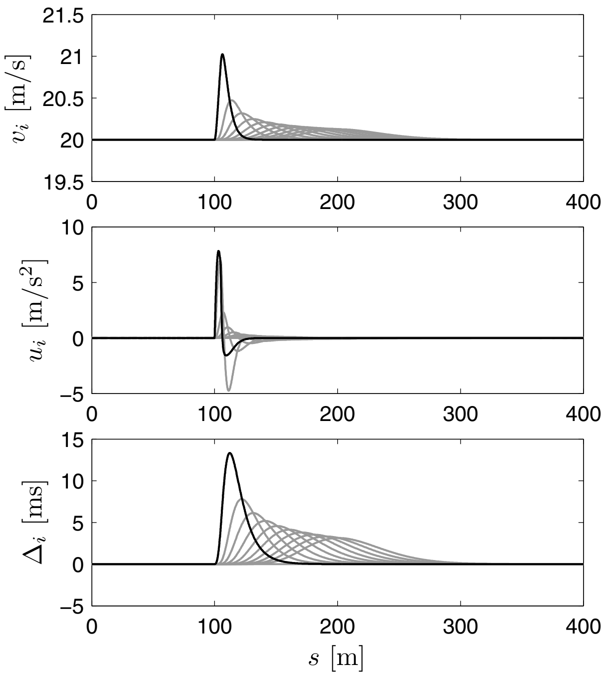

Fig. 4. Velocity (top) and control input (middle) for the lead vehicle (in black) and $N = 10$ follower vehicles (in gray) for $v_{\rm ref} = 20$ and where the lead vehicle is subject to an input disturbance $w_0(s) = 75$, $s \in [100, 105]$ and $w_0(s) = 0$ otherwise. The spacing errors (bottom) are depicted for the first follower vehicle (in black) and remaining follower vehicle (in gray).

the corresponding inter-vehicular spacing (given as a time-delay) is achieved as well.

Second, the propagation of disturbances through the platoon is analyzed. Thereto, an input disturbance is applied to the first vehicle, leading to a nonzero velocity error $e_0(s)$. Figure 4 shows that this disturbance does not amplify through the string of vehicles, as guaranteed by Theorem 6. Instead, the perturbations decrease for increasing vehicle index, indicating string stable behavior.

6. CONCLUSIONS

A novel delay-based spacing policy was analyzed in this paper, which has the property that all heavy-duty vehicles in a platoon exhibit the same velocity profile in space. For this spacing policy, an analysis in spatial domain leads to a controller that tracks a reference velocity profile and maintains the desired inter-vehicular distances as well as guarantees string stability for the platoon.

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