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A Game-Theoretic Framework for Studying Truck Platooning Incentives

Farhad Farokhi and Karl H. Johansson

***Abstract*— An atomic congestion game with two types of agents, cars and trucks, is used to model the traffic flow on a road over certain time intervals. In this game, the drivers make a trade-off between the time they choose to use the road, the average velocity of the flow at that time, and the dynamic congestion tax that they are paying to use the road. The trucks have platooning capabilities and therefore, have an incentive for using the road at the same time as their peers. The dynamics and equilibria of this game-theoretic model for the interaction between car traffic and truck platooning incentives are investigated. We use traffic data from Stockholm to validate the modeling assumptions and extract reasonable parameters for the simulations. We perform a comprehensive simulation study to understand the influence of various factors, such as the percentage of the trucks that are equipped with platooning devices on the properties of the pure strategy Nash equilibrium that is learned using a joint strategy fictitious play.**

I. INTRODUCTION

Urban traffic congestion creates many problems, such as, increased transportation delays and fuel consumption, air pollution, and dampened economic growth in heavily congested areas [1]–[3]. To circumvent part of these issues, the local governments in some urban areas introduced congestion taxes to manage the traffic congestion over existing infrastructures. For instance, Stockholm implemented a congestion taxing system in August, 2007 after a seven-month trial period between January–July, 2006. A survey of the influence of the congestion taxes over the trial period can be found in [4], which technically shows significant improvements in travel times as well as favorable economic and environmental effects. Behavioral aspects and other influences of the Stockholm congestion taxing system is discussed in [5]–[8].

In parallel to reducing the congestion, we can employ other means to improve the fuel efficiency and decrease the carbon emission [1]. Trucks or heavy-duty vehicles can improve their fuel efficiency by platooning with their peers. In [9], the authors report 4.7%-7.7% reduction in the fuel consumption (depending on the distance between the vehicles among other factors) when two identical trucks move close to each other at $70\,\mathrm{km/h}$. The fuel saving is mainly due to the reduced air drag force on the vehicles when they form platoons. In a futuristic scenario where several trucks are equipped with platooning devices, they are able to save fuel by cooperating with each other. Implementing truck platooning in a large-scale setup is not easy since a global decision-maker might become complex and the vehicles can belong to competing entities. It is interesting to instead study if a desirable behavior can emerge from simple local strategies. In this paper, we consider such a case where the traffic flow can be modeled as a congestion game and the desired behavior corresponds to an equilibrium of this game.

In this paper, we model the traffic flow at non-overlapping intervals of the day using an atomic[1] congestion game with two types of agents. The first type of agents are cars or trucks that do not have platooning equipments. They optimize their utility, which is a sum of the penalty for deviating from their preferred time interval for using the road, the average velocity of the traffic flow along the road, and the congestion tax that they pay for using the road at that time interval. The second type of agents are trucks equipped with platooning devices. In addition to the above mentioned term, they have an incentive for using the road with other second-type agents. We model the average velocity of the flow at each time interval as a linear function of the number of the vehicles that are using the road at that time interval. We use real traffic data from the northbound E4 highway from Lilla Essingen to the end of Fredhällstunneln in Stockholm, Sweden (see Figure 1) to validate this modeling assumption. We show that the atomic congestion game admits at least one pure strategy Nash equilibrium under an appropriate congestion tax policy for the first type of agents. Then, we use joint strategy fictitious play to learn a Nash equilibrium. Intuitively, we interpret the learning algorithm as the way drivers decide on a daily basis to choose the time interval on which they are using the road by optimizing their utility given the history of their actions. Iterating over days, the drivers' decisions (i.e., the profile of the learning algorithm) converges almost surely to a pure strategy Nash equilibrium. Using the parameters extracted from the real congestion data, we construct a simulation setup to study the performance of the joint strategy fictitious play as well as the properties of the captured Nash equilibrium. For instance, we study the robustness to perturbations of the learning algorithm, e.g., accidents along the road, sudden weather changes in a day, or temporary road constructions.

Modeling the traffic flow using congestion games and studying their efficiency are well-known problems (see [11]–[17] among many other studies). However, contrary to all these studies, we employ an atomic congestion game with two types of agents to study the possibility of platooning and its incentives when dealing with strategic drivers. A model that inspired our traffic flow modeling was introduced in [13]. In the current paper, we expand this model by adding

The authors are with ACCESS Linnaeus Center, School of Electrical Engineering, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden. E-mails: {farakhi,kallej}@kth.se

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[1]We use the term atomic to emphasize the fact that we are not dealing with a continuum of players or fractional flows when modeling the traffic flow as a congestion game [10], [11].

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a second type of agents that benefit from using the road with their peers to study the truck platooning incentives.

The rest of the paper is organized as follows. In Section II, we formulate the described congestion game. We study the properties of this congestion game and introduce the joint strategy fictitious play for learning a Nash equilibrium in Section III. Finally, we present the simulations in Section IV and conclude the paper in Section V.

A. Notation

The sets of real and integer numbers are denoted by $\mathbb{R}$ and $\mathbb{Z}$, respectively. Throughout the paper, we use the notation $\mathbb{Z}_{\geq a}^{\leq b} = \{n \in \mathbb{Z} \,|\, a \leq n \leq b\}$ for any $a, b \in \mathbb{Z} \cup \{\pm\infty\}$. Whenever $b = +\infty$, we just write $\mathbb{Z}_{\geq a}$. We use calligraphic letters, such as $\mathcal{R}$, to denote all other sets. The cardinality of any set $\mathcal{R}$ is denoted by $|\mathcal{R}|$. Finally, we define the characteristic function $\mathbf{1}_{x=y}$ to be equal to one if the condition $x = y$ holds true and equal to zero otherwise.

II. Game-Theoretic Modeling

We model the traffic flow at certain time intervals of the day on a given road using an atomic congestion game. The agents in this congestion game are the vehicles (or, rather the drivers of these vehicles) and their actions are the time intervals they choose to use the road at each day. Let us divide the time of the day into R non-overlapping intervals and denote each interval by r_i, $i = 1, \dots, R$. The set of all these intervals (i.e., agents' actions) is denoted by $\mathcal{R} = \{r_1, r_2, \dots, r_R\}$. We consider the case where the underlying congestion game is composed of two types of agents. For the sake of brevity, we name the first type of agents cars and the second type of agents trucks throughout the paper. We assume N cars and M trucks are playing in this congestion game. We denote the actions of the cars and the trucks by $z = \{z_i\}_{i=1}^N$ and $x = \{x_i\}_{i=1}^M$, respectively. Let us describe the utilities of the cars and the trucks in the following subsections.

A. Cars

Assume car $i \in \mathbb{Z}_{\geq 1}^{\leq N}$ is maximizing its utility given by

$$U_i(z_i, z_{-i}, x) = \xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) + v_{z_i}(z, x) + p_i^{\mathrm{c}}(z, x), \qquad (1)$$

where the mapping $\xi_i^{\mathrm{c}} : \mathcal{R} \times \mathcal{R} \to \mathbb{R}$ describes the penalty for deviating from the preferred time interval for using the road denoted by $T_i^{\mathrm{c}} \in \mathcal{R}$ (e.g., due to being late for work or delivering goods), $v_{z_i}(z, x)$ is the average velocity of the traffic flow at time interval z_i, and $p_i^{\mathrm{c}}(z, x)$ is a potential congestion tax for using the road on a specific time interval.

Following [13], [18], [19], we assume that $v_r(z, x)$ (i.e., the average velocity at time interval $r \in \mathcal{R}$) is linearly dependent on the road congestion

$$n_r(z, x) = \sum_{\ell=1}^{N} \mathbf{1}_{\{z_\ell = r\}} + \sum_{\ell=1}^{M} \mathbf{1}_{\{x_\ell = r\}},$$

which is the total number of vehicles (both cars and trucks) that are using the road at $r \in \mathcal{R}$.

Fig. 1: The dashed black curve shows the segment of northbound E4 highway between Lilla Essingen and Fredhällstunneln in Stockholm where we are using to validate the model and extract reasonable parameters.

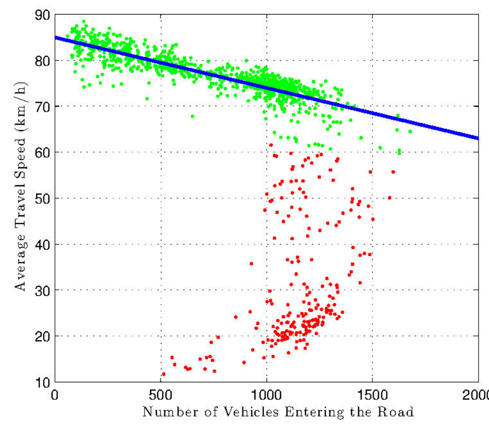

Fig. 2: Average velocity of the traffic flow as a function of the number of vehicles that are entering the segment of northbound E4 highway between Lilla Essingen and Fredhällstunneln for 15 min time intervals.

Let us use the real traffic data from the measurements on the northbound E4 highway in Stockholm from Lilla Essingen to the end of Fredhällstunneln to validate this assumption, see Figure 1. The measurements are extracted during October 1–15, 2012. Figure 2 illustrates the average velocity of the flow as a function of the number of the vehicles. As we can see, for up to 1000 vehicles, a linear relationship $v_r(z, x) = a n_r(z, x) + b$ with $a = -0.0110$ and $b = 84.9696$ describes the data well. However, for higher numbers of the vehicles, it fails to capture the behavior of around 20% of the data (shown by the red dots in Figure 2). Note that part of these outlier measurements can be caused by traffic accidents, sudden weather changes during the day, or temporary road constructions.

The choice of the penalty mappings ξ_i^{c}, $i \in \mathbb{Z}_{\geq 1}^{\leq N}$, does not change the theoretical results presented in the paper, but can capture various models of the drivers. For instance, following [13], we can use $\xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) = \alpha_i^{\mathrm{c}} |z_i - T_i^{\mathrm{c}}|$, with scalar $\alpha_i^{\mathrm{c}} < 0$, to describe the case where the driver of car i get penalized by deviating from the preferred time interval. With this function, the driver get penalized symmetrically no matter if she uses the road sooner or later than T_i^{c}. By increasing $|\alpha_i^{\mathrm{c}}|$, she becomes less flexible. Another penalty function is $\xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) = \alpha_i^{\mathrm{c}} \max(z_i - T_i^{\mathrm{c}}, 0)$, which penalizes the driver of car i only for being late. For the simulations in the paper, we assume that all vehicles use the first penalty mapping.

B. Trucks

We assume truck $j \in \mathbb{Z}_{\geq 1}^{\leq M}$ is maximizing the utility

$$\begin{aligned} V_j(x_j, x_{-j}, z) =& \xi_j^{\mathrm{t}}(x_j, T_j^{\mathrm{t}}) + v_{x_j}(z, x) \\ &+ \beta v_{x_j}(z, x) g(m_{x_j}(x)) + p_i^{\mathrm{t}}(z, x), \end{aligned} \tag{2}$$

where, similar to the utilities of the cars, $\xi_j^{\mathrm{t}}(x_j, T_j^{\mathrm{t}})$ is the penalty for deviating from the preferred time T_j^{t} for using the road, $v_{x_j}(z, x)$ is the average velocity of the traffic flow, and $p_i^{\mathrm{t}}(z, x)$ is a potential congestion tax for using the road at time interval x_j. Trucks have an extra term $\beta v_{x_j}(z, x) g(m_{x_j}(x))$ in their utility because of their benefit in using the road at the same time as the other trucks. Here, $g : \mathbb{Z}_{\geq 1}^{\leq M} \to \mathbb{R}$ is a nondecreasing function and $m_r(x) = \sum_{\ell=1}^{M} \mathbf{1}_{\{x_\ell = r\}}$ is the number of trucks that are using the road at time interval $r \in \mathcal{R}$. The increased utility can be justified by the fact that whenever there are many trucks on the road at the same time interval, they can potentially form platoons to increase the fuel efficiency. It should be noted that this extra utility is a function of the average velocity of the flow since the trucks cannot save a significant amount of fuel through platooning whenever traveling at low velocities (see [9], [20] and the references therein for a discussion on this matter). The function $g : \mathbb{Z}_{\geq 1}^{\leq M} \to \mathbb{R}$ describes the dependency of the platooning incentive on the number of trucks that are using the road at that time interval. Again, the choice of this function would not change the mathematical results presented in this paper, but it can help us to capture the relationship between the fuel saving and the number of the trucks on the road in a more realistic fashion. For instance, $g(m_{x_j}(x)) = m_{x_j}(x)$ shows that the vehicles can even benefit from a low number of trucks but $g(m_{x_j}(x)) = m_{x_j}(x) \mathbf{1}_{m_{x_j}(x) \geq \tau}$ describe the case where the trucks do not benefit until they reach a critical number $\tau \in \mathbb{Z}_{\geq 1}$. For the simulations in this paper, we assume that all trucks use the first mapping.

Notice that in the utilities (1) and (2), we introduced potential congestion taxes for the cars and the trucks. They are used to ensure that the described game is a potential game. Such a game admits at least one pure strategy Nash equilibrium and we can use joint strategy fictitious play to learn an equilibrium.

III. Learning a Pure Strategy Nash Equilibrium

In this section, we introduce the joint strategy fictitious play [21] to learn a pure strategy Nash equilibrium of the congestion game introduced in Section II. In the reminder of this paper, we assume that the trucks do not pay any congestion tax; i.e., $p_j^{\mathrm{t}}(z, x) = 0$ for all $j \in \mathbb{Z}_{\geq 1}^{\leq M}$. Let us start by proving that the congestion game is a potential game under an appropriate congestion tax policy for cars.

Lemma 1: Let each car $i \in \mathbb{Z}_{\geq 1}^{\leq N}$ pay the congestion tax

$$p_i^{\mathrm{c}}(z, x) = a\beta \sum_{\ell=1}^{m_{z_i}(x)} g(\ell), \tag{3}$$

for using the road at time interval $z_i \in \mathcal{R}$. Then, the atomic congestion game with the utilities introduced in (1) and (2) is a potential game[2] with the potential function $\Phi(x, z) = \sum_{k=1}^{4} \Phi_k(x, z)$ where

$$\Phi_1(x, z) = \sum_{i=1}^{N} \xi_i^{\mathrm{c}}(z_i, T_i^{\mathrm{c}}) + \sum_{j=1}^{M} \xi_j^{\mathrm{t}}(x_j, T_j^{\mathrm{t}}),$$

$$\Phi_2(x, z) = \sum_{r=1}^{R} \sum_{k=1}^{n_r(x,z)} (ak + b),$$

$$\Phi_3(x, z) = \sum_{r=1}^{R} \beta(a n_r(x, z) + b) \sum_{\ell=1}^{m_r(x)} g(\ell),$$

$$\Phi_4(x, z) = -a\beta \sum_{r=1}^{R} \sum_{\ell=1}^{m_r(x)} \sum_{k=1}^{\ell-1} g(k).$$

Furthermore, this game admits at least one pure strategy Nash equilibrium[3].

Proof: The proof of this lemma follows the same line of reasoning as in the proof of Proposition 4.1 in [13]. Let us start by analyzing the trucks. If $x_j = x'_j$, the result trivially holds. Consequently, we consider the case where $x_j \neq x'_j$, which results in

$$\Phi(x_j, x_{-j}, z) - \Phi(x'_j, x_{-j}, z) = \sum_{k=1}^{4} \Phi_k(x_j, x_{-j}, z) - \Phi_k(x'_j, x_{-j}, z).$$

We continue the proof by considering each term of this summation separately. For the first term, clearly, we have

$$\Phi_1(x_j, x_{-j}, z) - \Phi_1(x'_j, x_{-j}, z) = \xi_j^{\mathrm{t}}(x_j, T_j^{\mathrm{t}}) - \xi_j^{\mathrm{t}}(x'_j, T_j^{\mathrm{t}}).$$

Let us define $x' = (x'_j, x_{-j})$. For the second term, we have

$$\begin{aligned} &\Phi_2(x_j, x_{-j}, z) - \Phi_2(x'_j, x_{-j}, z) \\ &\quad = \sum_{r=1}^{R} \sum_{k=1}^{n_r(x,z)} (ak + b) - \sum_{r=1}^{R} \sum_{k=1}^{n_r(x',z)} (ak + b) \\ &\quad = \sum_{k=1}^{n_{x_j}(x,z)} (ak + b) + \sum_{k=1}^{n_{x'_j}(x,z)} (ak + b) \\ &\quad\quad - \sum_{k=1}^{n_{x_j}(x',z)} (ak + b) - \sum_{k=1}^{n_{x'_j}(x',z)} (ak + b), \end{aligned}$$

[2]The introduced congestion game is a potential game with potential function $\Phi : \mathcal{R}^N \times \mathcal{R}^M \to \mathbb{R}$ if $\Phi(x, z_i, z_{-i}) - \Phi(x, z'_i, z_{-i}) = U_i(z_i, z_{-i}, x) - U_i(z'_i, z_{-i}, x)$ and $\Phi(x_j, x_{-j}, z) - \Phi(x'_j, x_{-j}, z) = V_j(x_j, x_{-j}, z) - V_j(x'_j, x_{-j}, z)$ for all $i \in \mathbb{Z}_{\geq 1}^{\leq N}$ and $j \in \mathbb{Z}_{\geq 1}^{\leq M}$. We would like to refer the interested readers to [22] to study potential games.

[3]For the introduced congestion game, a pure strategy Nash equilibrium is a pair $(z, x) \in \mathcal{R}^N \times \mathcal{R}^M$ such that $U_i(z_i, z_{-i}, x) \geq U_i(z'_i, z_{-i}, x)$ for all $z'_i \in \mathcal{R}$ and $i \in \mathbb{Z}_{\geq 1}^{\leq N}$ and $V_j(x_j, x_{-j}, z) \geq V_j(x'_j, x_{-j}, z)$ for all $x'_j \in \mathcal{R}$ and $j \in \mathbb{Z}_{\geq 1}^{\leq M}$. We would like to refer the interested readers to [23] for a comprehensive review of the game theoretic concepts.

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where the second equality holds because of the fact that $n_r(x, z) = n_r(x', z)$ for all $r \neq x_j, x'_j$. Note that

$$n_{x_j}(x', z) = n_{x_j}(x, z) - 1,\ n_{x'_j}(x, z) = n_{x'_j}(x', z) - 1, \quad (4)$$

and as a result,

$$\Phi_2(x_j, x_{-j}, z) - \Phi_2(x'_j, x_{-j}, z) = (an_{x_j}(z, x) + b) - (an_{x'_j}(z, x') + b).$$

For the third term, we get the identity in (5), on top of the next page, where the last equality follows from using (4) and the fact that $m_{x_j}(x') = m_{x_j}(x) - 1$ and $m_{x'_j}(x) = m_{x'_j}(x') - 1$. Finally, using the same argument as in the case of the second term and the third term, we get

$$\Phi_4(x_j, x_{-j}, z) - \Phi_4(x'_j, x_{-j}, z) = -a\beta \sum_{\ell=1}^{m_{x_j}(x)-1} g(\ell) + a\beta \sum_{\ell=1}^{m_{x'_j}(x')-1} g(\ell).$$

Combining all these differences, we get

$$\begin{aligned}\Phi(x_j, x_{-j}, z) - \Phi(x'_j, x_{-j}, z) =& \beta(an_{x_j}(x, z) + b)g(m_{x_j}(x)) \\ &- \beta(an_{x'_j}(x', z) + b)g(m_{x'_j}(x')) \\ &+ \xi^{\mathrm{t}}_j(x_j, T^{\mathrm{t}}_j) - \xi^{\mathrm{t}}_j(x'_j, T^{\mathrm{t}}_j) \\ &+ (an_{x_j}(z, x) + b) - (an_{x'_j}(z, x') + b) \\ =& V_j(x_j, x_{-j}, z) - V_j(x'_j, x_{-j}, z).\end{aligned}$$

Now, let us prove this fact for the cars as well. If $z_i = z'_i$, the result trivially holds. Thus, we investigate the case where $z_i \neq z'_i$. Similarly, we consider each term of the summation separately. For the first term, we have

$$\Phi_1(x, z_i, z_{-i}) - \Phi_1(x, z'_i, z_{-i}) = \xi^{\mathrm{c}}_i(z_i, T^{\mathrm{c}}_i) - \xi^{\mathrm{c}}_i(z'_i, T^{\mathrm{c}}_i).$$

We define the notation $z' = (z'_i, z_{-i})$. Following a similar reasoning as in the case of the trucks, for the second and the third terms, we get

$$\Phi_2(x, z_i, z_{-i}) - \Phi_2(x, z'_i, z_{-i}) = (an_{z_i}(z, x) + b) - (an_{z'_i}(z', x) + b),$$

and

$$\Phi_3(x, z_i, z_{-i}) - \Phi_3(x, z'_i, z_{-i}) = a\beta \sum_{\ell=1}^{m_{z_i}(x)} g(\ell) - a\beta \sum_{\ell=1}^{m_{z'_i}(x)} g(\ell).$$

For the forth term, we get $\Phi_4(x, z_i, z_{-i}) - \Phi_4(x, z'_i, z_{-i}) = 0$ since this term is only a function of x which is not changed. Again, combining all these differences, we get

$$\begin{aligned}\Phi(x, z_i, z_{-i}) - \Phi(x, z'_i, z_{-i}) =& (an_{z_i}(z, x) + b) - (an_{z'_i}(z', x) + b) \\ &+ \xi^{\mathrm{c}}_i(z_i, T^{\mathrm{c}}_i) - \xi^{\mathrm{c}}_i(z'_i, T^{\mathrm{c}}_i) \\ &+ a\beta \sum_{\ell=1}^{m_{z_i}(x)} g(\ell) - a\beta \sum_{\ell=1}^{m_{z'_i}(x)} g(\ell) \\ =& U_i(z_i, z_{-i}, x) - U_i(z'_i, z_{-i}, x).\end{aligned}$$

Procedure 1 The joint strategy fictitious play for learning a Nash equilibrium of the introduced congestion game.

Input: $p \in (0, 1)$
Output: (x^*, z^*)
for $t = 0, 1, \ldots$ **do**
  for $i = 1, \ldots, N$ **do**
    Calculate $z'_i \in \arg\max_{r \in \mathcal{R}} \hat{U}_i(r; t-1)$
    if $U_i(z'_i, z_{-i}(t-1), x(t-1)) \leq U_i(z_i(t-1), z_{-i}(t-1), x(t-1))$ **then**
      $z_i(t) \leftarrow z_i(t-1)$
    else
      With probability $1-p$, $z_i(t) \leftarrow z_i(t-1)$, otherwise $z_i(t) \leftarrow z'_i$
    end if
    for $j = 1, \ldots, M$ **do**
      Calculate $x'_j \in \arg\max_{r \in \mathcal{R}} \hat{V}_j(r; t-1)$
      if $V_j(z(t-1), x'_j, x_{-j}(t-1)) \leq V_j(z(t-1), x_j(t-1), x_{-j}(t-1))$ **then**
        $x_j(t) \leftarrow x_j(t-1)$
      else
        With probability $1-p$, $x_j(t) \leftarrow x_j(t-1)$, otherwise $x_j(t) \leftarrow x'_j$
      end if
    end for
  end for
end for

Finally, note that every potential game admits at least one pure strategy Nash equilibrium [22]. ■

Recall that a game that is a potential game admits at least a pure strategy Nash equilibrium and we can use joint strategy fictitious play to learn a Nash equilibrium [21], [22].

Note that when $g : \mathbb{Z}^{\leq M}_{\geq 1} \to \mathbb{R}$ is linear, then $p^{\mathrm{c}}_i(z, x)$ at each time interval grows quadratically with the number of the trucks that are using the road at that interval. Therefore, the cars avoid the time intervals that the trucks use to travel.

Now, we briefly introduce the joint strategy fictitious play and analyze its convergence for the introduced congestion game. The agents calculate an average utility given the history of the actions. At time step $t \in \mathbb{Z}_{\geq 0}$, car $i \in \mathbb{Z}^{\leq N}_{\geq 1}$ computes $\hat{U}_i(r; t)$ using the recursive equation

$$\hat{U}_i(r; t) = (1 - \lambda_t)\hat{U}_i(r; t-1) + \lambda_t U_i(r, z_{-i}(t), x(t)), \quad (6)$$

with the initial condition $\hat{U}_i(r; -1) = \xi^{\mathrm{c}}_i(r, T^{\mathrm{c}}_i)$ for all $r \in \mathcal{R}$. In (6), $\lambda_t \in (0, 1]$ is a forgetting factor which captures the extent that the agents forget the actions from the past. If $\lambda_t = 1$, the agents are myopic (i.e., only consider the actions from the previous time step) while if $\lambda_t = 1/t$, the agents value the whole history at the same level. Following the same approach, truck $j \in \mathbb{Z}^{\leq M}_{\geq 1}$ calculates $\hat{V}_j(r; t)$ using the recursive equation

$$\hat{V}_j(r; t) = (1 - \lambda_t)\hat{V}_j(r; t-1) + \lambda_t V_j(r, x_{-j}(t), z(t)),$$

with $\hat{V}_j(r; -1) = \xi^{\mathrm{t}}_j(r, T^{\mathrm{t}}_j)$ for all $r \in \mathcal{R}$. Procedure 1 shows the joint strategy fictitious play for the introduced congestion game. This numerical procedure converges to a Nash equilibrium of the congestion game under the congestion tax policy (3).

THEOREM 2: Let the actions of the agents be generated by the joint strategy fictitious play in Procedure 1. Then,

$$
\begin{aligned}
\Phi_3(x_j, x_{-j}, z) - \Phi_3(x_j', x_{-j}, z) &= \sum_{r=1}^{R} \beta(an_r(x,z)+b) \sum_{\ell=1}^{m_r(x)} g(\ell) - \sum_{r=1}^{R} \beta(an_r(x',z)+b) \sum_{\ell=1}^{m_r(x')} g(\ell) \\
&= \beta(an_{x_j}(x,z)+b) \sum_{\ell=1}^{m_{x_j}(x)} g(\ell) + \beta(an_{x_j'}(x,z)+b) \sum_{\ell=1}^{m_{x_j'}(x)} g(\ell) \\
&\quad - \beta(an_{x_j}(x',z)+b) \sum_{\ell=1}^{m_{x_j}(x')} g(\ell) - \beta(an_{x_j'}(x',z)+b) \sum_{\ell=1}^{m_{x_j'}(x')} g(\ell) \\
&= \beta(an_{x_j}(x,z)+b) g(m_{x_j}(x)) - \beta(an_{x_j'}(x',z)+b) g(m_{x_j'}(x')) \\
&\quad + a\beta \sum_{\ell=1}^{m_{x_j}(x)-1} g(\ell) - a\beta \sum_{\ell=1}^{m_{x_j'}(x')-1} g(\ell),
\end{aligned} \tag{5}
$$

these actions converge almost surely to a pure strategy Nash equilibrium of the atomic congestion game if the cars pay the congestion tax (3).

Proof: The proof is a consequence of combining Theorem 3.1 in [21] with Lemma 1. ■

IV. Numerical Example

Let us assume that $N = 10000$ cars and $M = 100$ trucks are using the segment of the highway illustrated in Figure 1 from 7:00am to 9:00am on a daily basis. We divide the time horizon into eight equal non-overlapping intervals. Hence, we fix the action set as $\mathcal{R} = \{1, \dots, 8\}$, where each number represents an interval of 15 min. Let T_i^{c}, $i \in \mathbb{Z}_{\geq 1}^{\leq N}$, be randomly chosen from the set $\mathcal{R}$ using the discrete distribution

$$
\mathbb{P}\{T_i^{\mathrm{c}} = n\} = \begin{cases} 1/6, & n = 2, 4, \\ 1/4, & n = 3, \\ 1/12, & \text{otherwise.} \end{cases}
$$

Let us also use a similar probability distribution to extract T_j^{t}, $j \in \mathbb{Z}_{\geq 1}^{\leq M}$. Hence, we consider the case where the drivers statistically prefer to use the road at $r = 3$ which corresponds to 7:30am to 7:45am. Let α_i^{c}, $i \in \mathbb{Z}_{\geq 1}^{\leq N}$, and α_j^{t}, $j \in \mathbb{Z}_{\geq 1}^{\leq M}$, be randomly generated following a uniform distribution within the interval $[-7.5, -2.5]$. Finally, let $a = -0.0110$ and $b = 84.9696$ as discussed in Section II.

A. Learning Algorithm Performance

In this subsection, we start by simulating the joint strategy fictitious play in Procedure 1. Let us fix $\beta = 10^{-3}$, $p = 0.4$, and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{Z}_{\geq 0}$. Figure 3 illustrates the number of the vehicles (both cars and trucks) that are using a specific time interval to commute $n_r(x(t), z(t))$, $r \in \mathcal{R}$, as a function of the iteration number. As can be seen in this figure, the learning algorithm converges to a pure strategy Nash equilibrium in this example relatively fast (recall that there are $|\mathcal{R}|^{M+N}$ possible action combinations). Figure 4 shows the evolution of the traffic distribution. Figure 5 shows the number of trucks $m_r(x(t))$, $r \in \mathcal{R}$, that are using the road on various time intervals. For instance, at the learned Nash equilibrium, thirty trucks use the time interval 7:45am to 8:00am while at the same time, most of them avoid using

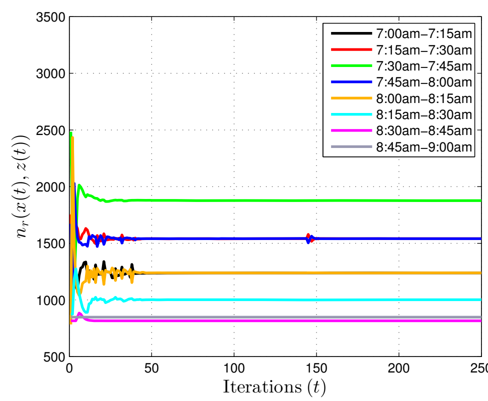

Fig. 3: $n_r(x(t), z(t))$, $r \in \mathcal{R}$, versus the iteration number for $\beta = 10^{-3}$ when using the joint strategy fictitious play in Procedure 1 with $p = 0.4$ and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{Z}_{\geq 0}$.

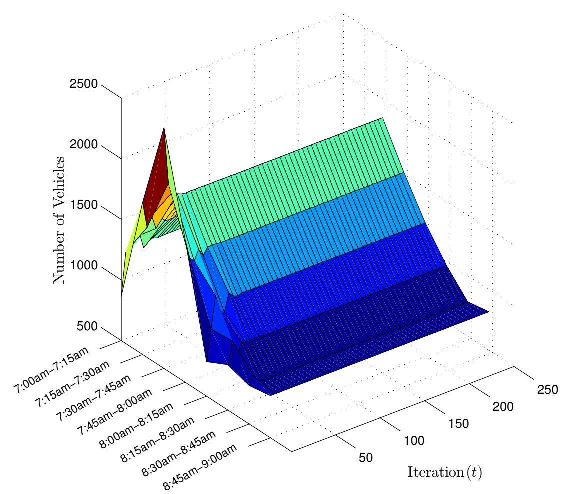

Fig. 4: Number of the vehicles in each time interval for $\beta = 10^{-3}$ when using the joint strategy fictitious play in Procedure 1 with $p = 0.4$ and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{Z}_{\geq 0}$.

7:15am to 7:30am because it is highly congested (and they would not save much fuel if they commute at this time).

B. Nash Equilibrium Efficiency

Figure 6 shows the number of the vehicles in each time interval and the corresponding average velocity in that time

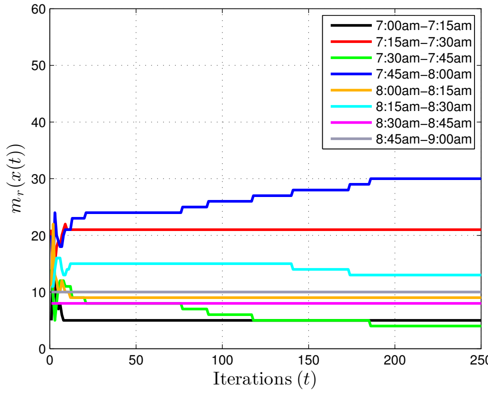

Fig. 5: $m_r(x(t))$, $r \in \mathcal{R}$, versus the iteration number for $\beta = 10^{-3}$ when using the joint strategy fictitious play in Procedure 1 with $p = 0.4$ and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{Z}_{>0}$.

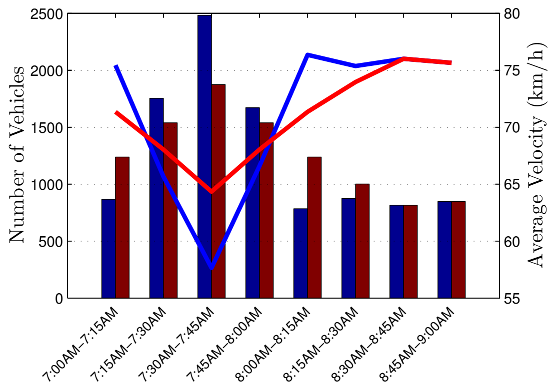

Fig. 6: Number of the vehicles and the average velocity of the traffic flow in each time interval for the case where the drivers neglect the congestion in their decision making (blue) and for the learned pure strategy Nash equilibrium (red).

interval. The blue color denotes the case where the drivers do not consider the congestion in their decision making; i.e., they commute whenever pleases them, $z_i = T_i^{\mathrm{c}}$ for all $i \in \mathbb{Z}_{\geq 1}^{\leq N}$ and $x_j = T_j^{\mathrm{t}}$ for all $j \in \mathbb{Z}_{\geq 1}^{\leq M}$. The red color denotes the case where the drivers implement the pure strategy Nash equilibrium that they have learned using Procedure 1. As we can see in this figure, the proposed congestion game reduces the average commuting time (increases the average velocity). Following [17], [24], we can define the social welfare

$$S(x, z) = \min_{r \in \mathcal{R}} v_r(z, x) = \min_{r \in \mathcal{R}} a n_r(x, z) + b.$$

This social welfare is the worst-case average velocity of the traffic flow. Another definition of social welfare could be the total fuel consumption or the overall carbon emission. In a utopia, the government should be able to implement a global solution of the optimization problem

$$(x^\bullet, z^\bullet) \in \operatorname*{arg\,max}_{(z,x) \in \mathcal{R}^N \times \mathcal{R}^M} S(x, z),$$

to achieve the lowest congestion at all time intervals. However, this solution cannot be implemented in a society with strategic (selfish) agents since they have no incentive in following a socially optimal decision $(x^\bullet, z^\bullet)$. Note that

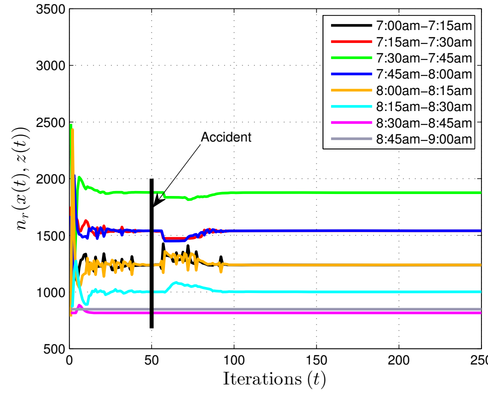

Fig. 7: $n_r(x(t), z(t))$, $r \in \mathcal{R}$, versus the iteration number when an unexpected behavior (e.g., an accident) disrupt the traffic flow on the fiftieth day of learning.

since $a < 0$, we have

$$\begin{aligned}(x^\bullet, z^\bullet) &\in \operatorname*{arg\,max}_{(z,x) \in \mathcal{R}^N \times \mathcal{R}^M} \min_{r \in \mathcal{R}} a n_r(x, z) + b \\ &\in \operatorname*{arg\,min}_{(z,x) \in \mathcal{R}^N \times \mathcal{R}^M} \max_{r \in \mathcal{R}} n_r(x, z),\end{aligned}$$

and as a result, we get

$$S(x^\bullet, z^\bullet) = a \left\lceil \frac{N + M}{|\mathcal{R}|} \right\rceil + b = 71.0766\,\mathrm{km/h}.$$

Therefore, we have

$$\frac{S(x^\bullet, z^\bullet)}{S(x^*, z^*)} = 1.1048,$$

which shows that the acquired pure strategy Nash equilibrium (x^*, z^*) is not efficient with respect to the introduced welfare function[4]. However, it is somewhat better than the case where the drivers do not consider the congestion in their decision making (i.e. they travel whenever pleases them) as

$$\frac{S(x^\bullet, z^\bullet)}{S(\{T_j^{\mathrm{t}}\}_{j=1}^M, \{T_i^{\mathrm{c}}\}_{i=1}^N)} = 1.2330.$$

C. Robustness of the Learning Algorithm

Let us now consider the case where on the fiftieth day of learning (i.e., iteration $t = 50$) an unexpected behavior (e.g., a traffic accident) significantly decreases the average velocity of the traffic flow during 7:15am and 8:00am (i.e., for $r = 2, 3, 4$). To reflect this matter in the simulations, we assume that on the fiftieth iteration, the average velocity for $r = 2, 3, 4$ is given by $(a n_r(x(t), z(t)) + b)/10$. Figure 7 illustrates the number of vehicles that are using a specific time interval to commute $n_r(x(t), z(t))$, $r \in \mathcal{R}$, as a function of the iteration numbers. Note that there is a sudden drop in the number of the vehicles that are using the time intervals corresponding to $r = 2, 3, 4$ for a while (around twenty iterations) after the accident. However, the learning process recovers the Nash equilibrium after another fifty iterations.

[4]It is worth mentioning that if we choose the potential function Φ in Lemma 1 as the social welfare function, the learned Nash equilibrium is indeed efficient since Procedure 1 results in a local maximizer of this potential function.

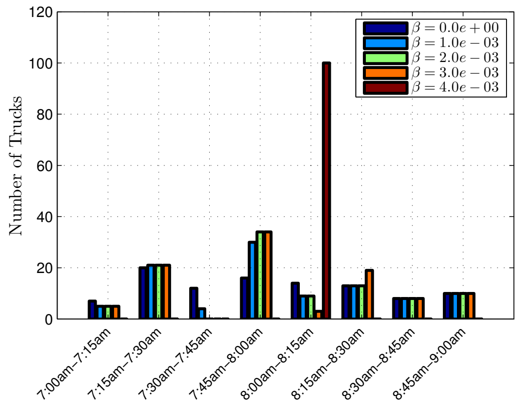

Fig. 8: Number of the trucks in each time interval for various choices of the coefficient β.

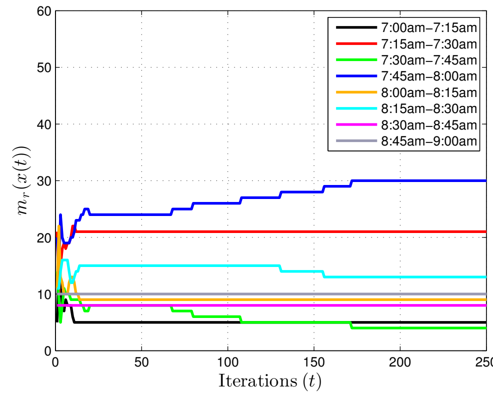

Fig. 9: $m_r(x^*)$, $r \in \mathcal{R}$, versus the iteration number for the case where the drivers value their time differently.

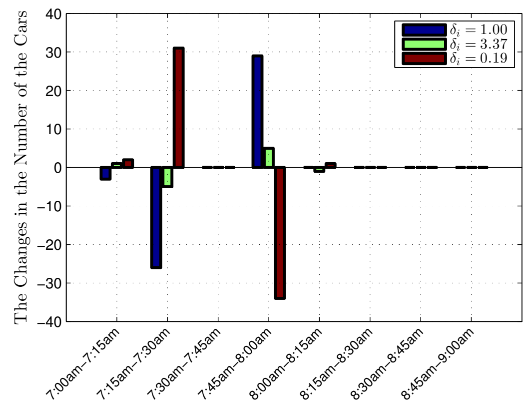

Fig. 10: Number of the cars in each time interval for the case where the drivers value their time differently subtracted by number of the cars in each time interval for the case where their drivers value the time equally.

D. Effect of the Fuel-Saving Coefficient

In this subsection, we aim at illustrating the effect of the fuel-saving coefficient β on the behavior of the trucks. We perform all the simulations using the joint strategy fictitious play introduced in Procedure 1 with $p = 0.4$ and $\lambda_t = 3 \times 10^{-2}$ for all $t \in \mathbb{Z}_{\geq 0}$. Figure 8 illustrates the number of trucks for the learned Nash equilibrium at different time intervals for various choices of the coefficient β. As we expect, when $\beta = 0$, the trucks are reluctant to platoon (but instead stick to the time that favors them the most). However, as we increase the coefficient β, a higher number of trucks drive at the same time interval. Note that for $\beta = 4\times10^{-3}$, all hundred trucks use the road during exactly one time interval (i.e, 8:00am to 8:15am).

E. Drivers Having Different Time Values

In 2001, the consulting firm Inregia in Sweden, by the request of Swedish Institute for Transport and Communications Analysis, performed a survey to estimate the value of time for the road user in Stockholm [25], [26]. This study showed that various groups of people value their time differently. According to the study, drivers valued time as 0.98, 3.30, and $0.19\,\mathrm{SEK/min}$ for work and school commuting trips, business trips, and other trips, respectively [25], [26]. Let us include this effect in the introduced congestion game setup. Assume that in the utility of car $i \in \mathbb{Z}_{\geq 1}^{\leq N}$, we set the term $p_i^{\mathrm{c}}(z, x) = \delta_i^{-1}\Big(a\beta \sum_{\ell=1}^{m_{z_i}(x)} g(\ell)\Big)$, where $\delta_i \in \mathbb{R}_{\geq 0}$ is the value of time for the driver of car i. For work and school commuting trips, we scale the value of time to $\delta_i = 1.00$. Therefore, we get $\delta_i = 3.37$ and $\delta_i = 0.19$ for business trips and other trips, respectively. Now, allow us to randomly distribute the cars into three groups of work and school trips, business trips, and other trips with probabilities 0.754, 0.036, 0.210, respectively, as suggested in [26]. Figure 9 shows the number of trucks in each time interval as a function of the iteration number in this case. Comparing with Figure 5, we can clearly see that in this example, the difference in the value of time has not changed the trucks behavior (certainly in the Nash equilibrium, but the transient response is different). Figure 10 shows the number of the cars in each time interval for the case where the drivers value their time differently subtracted by number of the cars in each time interval for the case where the drivers value their time equally. Clearly, the cars that value their time the most, or equivalently, the ones that are willing to pay higher congestion taxes (i.e., $\delta_i = 1.00, 3.37$), can move to the time interval where thirty trucks are traveling. However, the cars that do not value their time much (i.e., $\delta_i = 0.19$) switch to a less expensive alternative.

F. Trucks with and Without Platooning Equipment

Few trucks are currently fitted with platooning equipments. In this subsection, we try to understand the influence of this matter on the properties of the learned Nash equilibrium. To illustrate the effect of trucks without platooning equipment, let us consider two types of trucks where the first type can indeed participate in platoons and the second type do not have the necessary equipments for doing so. We count the second type of trucks as ordinary cars since they do not benefit from traveling at the same time interval as the other trucks. Hence, N shows the number of ordinary cars together with the trucks without platooning equipment and M denotes the number of trucks that can potentially participate in forming the platoons. We fix $N+M = 10000$. Figure 11 illustrates the number of the trucks that have

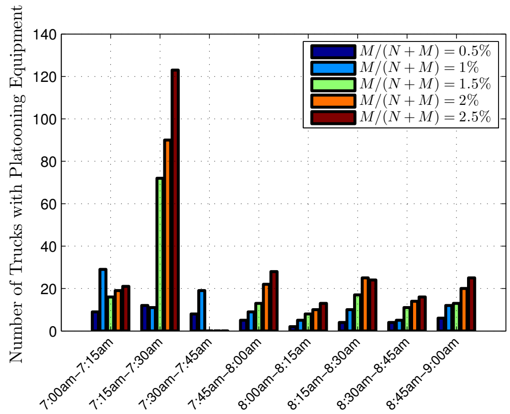

Fig. 11: Number of the vehicles in each time interval for the learned pure strategy Nash equilibrium for various choices of $M/(M+N)$.

platooning equipment in each time interval for various ratios of $M/(M+N)$. Evidently, the number of the trucks (with platooning equipment) in most of the time intervals grows linearly with $M/(M+N)$ (as we expect since there are more trucks). However, some of the intervals, such as, 7:30am to 7:45am become less favorable (as they are highly congested) and the trucks in these intervals completely move to their neighboring intervals as $M/(M+N)$ increases.

V. Conclusions and Future Work

We introduced a model for traffic flow on a specific road at various time intervals per day using an atomic congestion game with two types of agents (namely, cars and trucks). Cars only optimize their trade-off between using the road at the time they prefer, the average velocity of the traffic flow, and the congestion tax they are paying. However, trucks benefit from using the road at the same time as the other trucks. We motivated this extra utility using an increased possibility of platooning with the other trucks and as a result, saving fuel. We used congestion data from Stockholm to validate some of the assumptions in the modeling (i.e., linear relationship between the average velocity of commuting and the number of the vehicles that are using the road at that time). Then, we used the joint strategy fictitious play to learn a pure strategy Nash equilibrium of this game. We conducted a comprehensive simulation study to analyze the effect of different factors on the properties of the learned Nash equilibrium. As a future work, we can consider using mechanism design tools to enforce a socially optimal solution, such as, an optimal carbon emission profile, through appropriate congestion tax policy.

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