A Delay-Based Spacing Policy for Vehicle Platooning: Analysis and Control

B. Besselink and K.H. Johansson

Abstract Reducing inter-vehicular distances and the formation of groups of closely spaced vehicles have the potential to increase traffic flow, reduce congestion, and reduce fuel consumption. In this chapter, such vehicle platoons subject to a delay-based spacing policy are considered and the design of distributed controllers is pursued. Specifically, it is shown that the use of the delay-based spacing policy ensures that all vehicles in the platoon track the same velocity profile in the spatial domain, which offers advantages as road properties such as hills, bends, or road speed limits are specified in this domain. The proposed controller exploits delayed information about the preceding vehicle to achieve string-stable platoon behavior. In addition, a relaxation of the delay-based spacing policy is presented that exploits more information about the preceding vehicle. This extended delay-based spacing policy is shown to lead to improved platoon behavior. The results are illustrated by means of simulations.

1 Introduction

Platooning amounts to the formation of automatically controlled groups of closely spaced vehicles, which has the potential to increase traffic flow and reduce congestion [1]. Moreover, platooning offers a reduced aerodynamic drag and therefore reduces fuel consumption and emissions, particularly for heavy-duty vehicles [2]. These clear economical and ecological advantages have led to a large interest in the efficient control of vehicle platoons, with an early work given by [3]. Many results have followed since, e.g., [4, 5].

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The so-called spacing policy, which specifies the desired inter-vehicular distance as a function of the velocity of two neighboring vehicles, largely determines the dynamics of a platoon and is therefore of crucial importance. The most common examples are the constant spacing [6] and constant headway [7, 8] policies. However, experiments with heavy-duty vehicles driving over hilly terrain in [2] have shown that these spacing policies might lead to undesirable platoon behaviors as they might require follower vehicles to accelerate while climbing a hill in an attempt to maintain the desired gap to their predecessor. Due to limited engine power, this might be infeasible. Fundamentally, these undesired dynamics are due to the fact that the constant spacing and constant headway policies do not guarantee that all vehicles in a platoon track the same velocity profile in the spatial domain. As the required velocity profile of a platoon is constrained by the spatial domain (due to, e.g., hills, bends, or road speed limits), this provides a fundamental limitation for the use of these spacing policies in practice.

These aspects have led to the analysis of the so-called delay-based spacing policy in [9] (see also [10] for an early discussion on spacing policies). In this policy, a vehicle tracks a time-delayed version of the trajectory of its predecessor, which can be shown to lead to velocity profiles that are equal in the spatial domain. In fact, the analysis and control design in [9] is performed using space (rather than time) as the independent variable.

This chapter builds on the work of [9], but takes a time-domain perspective. This leads to the following contributions.

First, distributed controller design for vehicle platooning in the time domain is presented, which leads to a controller that is easier to interpret and implement. To this end, an alternative characterization of the delay-based spacing policy is exploited. In addition, it is shown that a relaxation of the delay-based spacing policy on the basis of the velocity tracking error leads to string-stable platoon behavior, implying that perturbations do not amplify as they propagate through the platoon. Here, it is also observed that tracking of the delay-based spacing policy only requires time-delayed information about the preceding vehicle, making this approach inherently robust to (small) delays in wireless communication between vehicles.

Second, a further relaxation of the delay-based spacing policy is introduced that exploits more information about the preceding vehicle in the control design. In particular, rather than only using delayed information, a distributed-delay approach is taken in which also the current state of the predecessor is included. It is shown that the use of this additional information can lead to improved platoon performance in the sense that the propagation of perturbations through the platoon can be further suppressed.

Before developing the points introduced above, it is stressed that the role of delays is different than in other works on delays in vehicle systems. Namely, in these works, the delay is generally regarded as a detrimental effect as a result of wireless inter-vehicle communication (see, e.g., [11, 12]), whereas, in the current work, the delay specifies the desired inter-vehicle spacing.

The remainder of this chapter is organized as follows. Section 2 introduces the delay-based spacing policy that is used throughout this chapter, whereas control design and analysis for this spacing policy is presented in Sect. 3. A distributed-delay approach to improve platoon performance is presented in Sect. 4. Sections 5 and 6 present a numerical evaluation of the proposed control strategies and the conclusions, respectively.

2 A Delay-Based Spacing Policy for Platooning

The dynamics of a platoon of vehicles is largely determined by the spacing policy, which specifies the desired inter-vehicular distance within a platoon, as illustrated in Fig. 1. With s_i and $s_{\text{ref},i}$ the actual and desired position, respectively, of the i-th vehicle in the platoon, the delay-based spacing policy

$$s_{\text{ref},i}(t) = s_{i-1}(t - \Delta t), \tag{1}$$

is considered, with delay $\Delta t > 0$. In this spacing policy, which is discussed in [9] (see also [10]), a vehicle tracks a time-delayed version of the trajectory of its predecessor. When this spacing policy is perfectly tracked, each vehicle in the platoon achieves the same velocity on the same point on the road. This is formalized as follows [9].

Lemma 1 *Consider the kinematics $\dot{s}_i(t) = v_i(t)$ and assume $s_i(t) = s_{\text{ref},i}(t)$ and $v_i(t) > 0$ for all $t \in \mathbb{R}$. Then, (1) holds if and only if,*[1] *for all s,*

$$v_i(s) = v_{i-1}(s). \tag{2}$$

Lemma 1 provides the main motivation for the use of the delay-based spacing policy (1). Namely, the desired velocity of a vehicle is generally determined by spatial properties such as the road speed limit, but also bends and hills. It is therefore important to reflect this in the spacing policy, especially for large platoons. Existing spacing policies as the constant spacing [6] and constant headway [7, 8] policies do not share the property (2) and, as a result, might result in undesirable or even infeasible platoon behavior. An example is given by heavy-duty vehicles driving over a hilly road, where the last vehicle in a platoon does not always experience the same road grade as the first vehicle in the platoon. As a result, the last vehicle might not have sufficient engine power to maintain the platoon formation when, for example, a constant inter-vehicle spacing is required. This effect is observed in experiments in [2], whereas a more thorough motivation for the use of the delay-based spacing policy (1) can be found in [9].

In this work, the synthesis of distributed controllers for a platoon formation is pursued according to the delay-based spacing policy. Specifically, motivated by the above discussion, the control objectives are twofold. Namely, a control strategy is sought that, first, ensures that each vehicle in the platoon tracks a desired reference

[1] The slight abuse of notation $v_i(t)$ and $v_i(s)$ is used to indicate the velocity of vehicle i as a function of time and space, respectively.

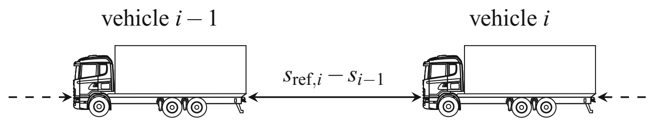

Fig. 1 Desired spacing policy $s_{\mathrm{ref},i}(t) - s_{i-1}(t)$ between vehicles in a platoon

velocity profile $v_{\mathrm{ref}}(\cdot)$ specified in the spatial domain, and, second, maintains a platoon formation by tracking the desired delay-based spacing policy (1).

Here, it is noted that these two objectives are aligned due to Lemma 1. Finally, the following assumption is made on the reference velocity profile.

Assumption 1 The reference velocity profile $v_{\mathrm{ref}}(\cdot)$ satisfies (for some $v_{\min}$) $v_{\mathrm{ref}}(s) \geq v_{\min} > 0$ for all $s \geq 0$ and is twice continuously differentiable.

3 Platoon Control Design and Stability Analysis

A platoon of $N+1$ vehicles is considered, where the index set $\mathscr{I}_0 := \{0, 1, \ldots, N\}$ represents the lead vehicle with index 0 and the follower vehicles with indices in the set $\mathscr{I} := \{1, 2, \ldots, N\}$. Following, e.g., [5, 13], each vehicle is modeled as

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \\ \tau \dot{a}_i(t) &= -a_i(t) + u_i(t). \end{aligned} \tag{3}$$

Here, $s_i(t) \in \mathbb{R}$, $v_i(t) \in \mathbb{R}$, and $a_i(t) \in \mathbb{R}$ denote the position, velocity, and acceleration of vehicle $i \in \mathscr{I}_0$, respectively. The control input $u_i(t) \in \mathbb{R}$ represents the desired acceleration, whereas the final equation in (3) can be regarded as the actuator dynamics of the vehicle with time constant $\tau > 0$. The dynamics (3) can be the result of applying feedback linearization to a more complex nonlinear vehicle model, see [5].

As stated in Sect. 2, the design of a controller is pursued that tracks, first, the desired velocity profile $v_{\mathrm{ref}}(\cdot)$ specified in the spatial domain, and, second, the delay-based spacing policy (1). In order to achieve the first objective, the relative velocity error e_i is introduced as

$$e_i(t) := \frac{v_i(t)}{v_{\mathrm{ref}}(s_i(t))} - 1, \tag{4}$$

for each $i \in \mathscr{I}_0$. It will be shown later that this choice for the velocity tracking error allows for obtaining relevant (string stability) properties of the controlled platoon. Next, it can be shown that choosing the input u_i in (3) as

$$
\begin{aligned}
u_i(t) &= a_i(t) + \tau v_{\mathrm{ref}}(s_i(t))\tilde{u}_i \\
&\quad - \tau v_{\mathrm{ref}}(s_i(t)) \left(3 \left. \frac{\mathrm{d}}{\mathrm{d}s} \frac{1}{v_{\mathrm{ref}}(s)} \right|_{s=s_i(t)} v_i(t) a_i(t) + \left. \frac{\mathrm{d}^2}{\mathrm{d}^2 s} \frac{1}{v_{\mathrm{ref}}(s)} \right|_{s=s_i(t)} v_i^3(t) \right)
\end{aligned} \tag{5}
$$

achieves feedback linearization of the final two equations in (3) with respect to the output (4). Namely, after applying the feedback linearizing control (5), the dynamics (3) can be written as

$$
\begin{aligned}
\dot{s}_i(t) &= v_{\mathrm{ref}}(s_i(t))\big(1 + e_i(t)\big), \\
\ddot{e}_i(t) &= \tilde{u}_i(t),
\end{aligned} \tag{6}
$$

for $i \in \mathscr{I}_0$. Here, the definition (4) is exploited and $\tilde{u}_i(t) \in \mathbb{R}$.

Whereas e_i in (4) provides a characterization of the velocity tracking error corresponding to the first control objective, a measure for the tracking error related to the desired spacing policy (1) is introduced next to target the second control objective. Specifically, the spacing tracking error $\varDelta_i$, $i \in \mathscr{I}$, is defined as

$$
\varDelta_i(t) := \int_{s_{i-1}(t-\varDelta t)}^{s_i(t)} \frac{1}{v_{\mathrm{ref}}(s)} \, \mathrm{d}s, \tag{7}
$$

where it is noted that the property $v_{\mathrm{ref}}(s) \geq v_{\min}$ (see Assumption 1) guarantees that $\varDelta_i(t) = 0$ if and only if $s_i(t) = s_{i-1}(t - \varDelta t)$, which corresponds to the desired spacing policy. The time-differentiation of $\varDelta_i$ in (7) yields

$$
\dot{\varDelta}_i(t) = \frac{v_i(t)}{v_{\mathrm{ref}}(s_i(t))} - \frac{v_{i-1}(t - \varDelta t)}{v_{\mathrm{ref}}(s_{i-1}(t - \varDelta t))} = e_i(t) - e_{i-1}(t - \varDelta t), \tag{8}
$$

such that the definition of the spacing tracking error $\varDelta_i$ is in agreement with the definition of the velocity tracking error e_i in (4).

However, rather than directly using $\varDelta_i$ as in (7) in the control design, controllers will be synthesized that aim at asymptotically achieving $\delta_i(t) = 0$, where δ_i is defined as

$$
\delta_i(t) := \varDelta_i(t) + h e_i(t), \tag{9}
$$

for $i \in \mathscr{I}$ and with $h > 0$. It is recalled that the two terms in (9) correspond to the spacing tracking error (7) and velocity tracking error (4), such that (9) can be regarded as a relaxation of the desired spacing policy. In particular, it allows the spacing between vehicles to increase if the follower vehicle drives at higher speed than the reference velocity. This will be shown to lead to a suppression of perturbations as they propagate through the string of vehicles, as is generally referred to as string stability. Even though the desired spacing is given by (1), the definition in (9) will be referred to as the delay-based spacing policy.

In order to obtain a controller that asymptotically achieves $\delta_i(t) = 0$, (9) is differentiated (three times) with respect to time to yield

$$\dddot{\delta}_i(t) = \tilde{u}_i(t) - \tilde{u}_{i-1}(t - \Delta t) + h\dot{\tilde{u}}(t). \tag{10}$$

Here, (8) as well as the dynamics in velocity tracking error coordinates (6) are exploited. Next, by regarding $\tilde{u}_i$ as a controller state, a virtual input $\xi_i \in \mathbb{R}$ can be introduced as

$$\xi_i(t) := h\dot{\tilde{u}}_i(t) + \tilde{u}_i(t), \tag{11}$$

for $i \in \mathscr{I}$. After choosing the virtual input ξ_i as

$$\xi_i(t) = -\left(k_0\delta_i(t) + k_1\dot{\delta}_i(t) + k_2\ddot{\delta}_i(t)\right) + \tilde{u}_{i-1}(t - \Delta t), \tag{12}$$

it is readily seen that the following closed-loop dynamics is obtained

$$\dddot{\delta}_i(t) + k_2\ddot{\delta}_i(t) + k_1\dot{\delta}_i(t) + k_0\delta_i(t) = 0 \tag{13}$$

for $i \in \mathscr{I}$. Consequently, the controller parameters k_j, $j \in \{0, 1, 2\}$ can be chosen (e.g., by the Routh–Hurwitz criterion or pole placement techniques) to achieve asymptotic stability of the dynamics (13), which asymptotically achieves $\delta_i(t) = 0$ for δ_i as in (9). The subspace for which $\delta_i = 0$ plays a similar role as the sliding surface in sliding mode control. In the final part of this section, it will be shown that asymptotically achieving $\delta_i = 0$ indeed reaches the objectives of tracking the desired reference velocity and delay-based spacing policy, characterized through e_i in (4) and Δ_i in (7), respectively.

At this point, it is worth noting that the total controller for the follower vehicles $i \in \mathscr{I}$ is given by the feedback linearizing part (5), the controller dynamics (11), and the feedback (12). This latter part relies on measurements of δ_i (obtained through (9)) and its time derivatives as well as the controller state of the preceding vehicle $\tilde{u}_{i-1}$, see (12).

Remark 1 The controller state $\tilde{u}_{i-1}$ in (12) can be obtained through (wireless) communication between vehicles. As only a delayed version of $\tilde{u}_{i-1}$ is required for the control of vehicle i, this control approach is inherently robust to (small) time delays in this communication. In Sect. 4, an approach will be discussed that also exploits information about the preceding vehicle in the interval $(t - \Delta t, t]$ in an attempt to improve platoon performance.

Remark 2 The controller presented above targets tracking of the desired delay-based spacing policy and only applies to the follower vehicles with indices in $\mathscr{I}$. The lead vehicle (with index $i = 0$) can be controlled to track the desired reference velocity profile v_{ref} by exploiting the same feedback linearizing controller (5) and choosing $\tilde{u}_0$ as

$$\tilde{u}_0(t) = -l_0 e_0(t) - l_1 \dot{e}_0(t). \tag{14}$$

It follows immediately from (6) with $\tilde{u}_0$ in (14) that the velocity error dynamics is asymptotically stable whenever $l_j > 0$ for $j \in \{0, 1\}$.

In the remainder of this section, stability properties of the controlled platoon are analyzed. Here, it is noted that asymptotic stability of the dynamics for δ_i in (13) does not directly imply that the desired delay-based spacing policy is achieved. This can be observed by considering the definition of δ_i in (9). Namely, after applying (8), it follows that (9) can be written as

$$h\dot{\Delta}_i(t) = -\Delta_i(t) + \delta_i(t) - he_{i-1}(t - \Delta t), \tag{15}$$

for $i \in \mathscr{I}$, such that the specific choice of δ_i in (9) induces the dynamics (15). Consequently, the total dynamics of the follower vehicles are given by the dynamics (13) and (15). Note that this dynamics follows from the vehicle dynamics (6) with the dynamic controller (11) and feedback (12) using the coordinates (7) and (9).

When the lead vehicle employs the controller discussed in Remark 2, the total platoon asymptotically achieves tracking of the desired velocity profile and the delay-based spacing policy. This is formalized as follows.

Theorem 1 *Consider the closed-loop platoon dynamics (13), (15) for* $i \in \mathscr{I}$ *and the lead vehicle controller (14). Then, the origin* $\Delta_i = \delta_i = \dot{\delta}_i = \ddot{\delta}_i = 0$, $i \in \mathscr{I}$ *and* $e_0 = \dot{e}_0 = 0$ *is asymptotically stable if and only if the controller parameters satisfy* $k_0 > 0$, $k_1 > 0$, $k_2 > 0$ *such that* $k_1 k_2 > k_0$ *and* $l_0 > 0$, $l_1 > 0$.

Proof The proof will be based on induction on the index of the follower vehicles, where the first vehicle is considered in the first step.

It is clear from the controller (14) and the dynamics $\ddot{e}_0 = \tilde{u}_0$ (see (6)) that the equilibrium $e_0 = \dot{e}_0 = 0$ is asymptotically stable under the conditions in the statement of the theorem. Consequently, the lead vehicle achieves tracking of the desired reference velocity $v_{\mathrm{ref}}(\cdot)$, as follows from the definition of e_i in (4).

A follower vehicle with index i satisfying the dynamics (9), (7) is considered in order to establish the inductive step. First, it is noted that the dynamics for δ_i in (9) is independent of the state Δ_i and is asymptotically stable under the conditions of the theorem. This follows from the Routh–Hurwitz criterion, e.g., [14]. Next, consider the dynamics for Δ_i in (7). Introduction of the function $V(\Delta_i) = \frac{1}{2} h \Delta_i^2$ and time-differentiation of V along the trajectories of (15) yields

$$\tfrac{\mathrm{d}}{\mathrm{d}t} V(\Delta_i(t)) \leq -(1-\alpha)|\Delta_i(t)|^2, \qquad \forall \alpha |\Delta_i(t)| > |\delta_i(t) - he_{i-1}(t - \Delta t)|, \tag{16}$$

for any α such that $0 < \alpha < 1$. This implies that the dynamics (15) is input-to-state stable (see [15]) with respect to the input $\delta_i(t) - he_{i-1}(t - \Delta t)$.

Taking $i = 1$, it is recalled that the relative velocity error dynamics of the lead vehicle is asymptotically stable, such that e_{i-1} vanishes (as it is generated by an asymptotically stable system). Similarly, δ_i vanishes due to asymptotic stability of

(13). Consequently, the dynamics of the first follower vehicle and the lead vehicle form a cascade interconnection of the input-to-state stable dynamics for Δ_i, whose total inputs result from the asymptotically stable dynamics for δ_i and the asymptotically stable dynamics for e_0. Due to the input-to-state stability property, this cascade it itself asymptotically stable (see [16]). Moreover, e_i can be regarded as an output of this asymptotically stable cascaded system (through (9)), which therefore vanishes as well. Induction on the vehicle index i then leads to the desired result. □

The result of Theorem 1 ensures the closed-loop stability of the desired delay-based spacing policy, regardless of the value of the delay Δt ($\Delta t \geq 0$). However, this does not guarantee that any disturbances caused by velocity tracking errors of the lead vehicle do not amplify through the platoon. The following result provides a bound on the amplification of velocity tracking errors.

Theorem 2 *Consider the spacing policy (9), let $\delta_i(t) = 0$ for all t, and assume that $\Delta_i(0) = 0$, $i \in \mathscr{I}$. Let the velocity tracking error of the lead vehicle be such that $e_0(t) = 0$ for all $t \leq 0$. Then, the velocity tracking errors e_i of the follower vehicles satisfy*

$$\|e_i\|_2 \leq \|e_{i-1}\|_2 \tag{17}$$

for all $i \in \mathscr{I}$. Here $\|e_i\|_2$ denotes the $\mathscr{L}_2$ signal norm, i.e., $\|e_i\|_2^2 = \int_0^\infty |e_i(t)|^2 \, \mathrm{d}t$.

Proof In order to prove the theorem, consider the dynamics (15) induced by the spacing policy (9). Setting $\delta_i = 0$, the transfer function from e_{i-1} to e_i can be obtained as

$$H_\delta(s) = \frac{1}{hs+1} e^{-s\Delta t}, \tag{18}$$

where the relation $he_i = \Delta_i$ (see (9) for $\delta_i = 0$) is used. It then follows that the $\mathscr{H}_\infty$-norm of H_δ, defined as $\|H_\delta\|_\infty = \sup_{\omega \in \mathbb{R}} |H_\delta(j\omega)|$, satisfies $\|H_\delta\|_\infty = 1$. This proves the result (17) through the Parseval identity. □

The result in Theorem 2 provides a so-called string-stability property, which ensures that velocity tracking errors do not amplify as they propagate through the platoon. An early reference on string stability is [17], whereas a recent overview is given in [13]. It is noted that the dynamics for δ_i in (13) is independent of the velocity tracking error, such that the assumption $\delta_i = 0$ is not restrictive. In fact, (17) holds for any controller that renders $\delta_i = 0$ in (9) invariant. As such, it is clear that the string-stability property in Theorem 2 is a direct result of the choice of the spacing policy (9) rather than the details of the controller design. Note that the controller parameters determine the speed of convergence towards the subspace on which $\delta_i = 0$, see (13).

Remark 3 The design of controllers achieving the delay-based spacing policy (1) was first pursued in [9]. However, rather than directly addressing the time delay Δt

in the time domain as pursued in the current work, the results in [9] rely on a formulation in which space (rather than time) is taken as the independent variable. By taking a time-domain approach, the results in this chapter allow for a more insightful interpretation. Moreover, this approach allows for considering more advanced spacing policies, as will be discussed in the next section.

4 Platoon Control Performance

In the previous section, control design on the basis of the delay-based spacing policy (9) was shown to achieve desirable string-stability properties (see Theorem 2). Here, it is recalled that the controller of a given vehicle relies on delayed information on its preceding vehicle, where the size of the delay corresponds to the desired time gap Δt in the spacing policy (1). This is particularly apparent from the definition of the spacing tracking error (7) and the feedback control part (12).

Consequently, information about the preceding vehicle in the time interval $(t - \Delta t, t]$, although available, is not exploited in the controller. In the current section, it will be shown that this additional information can be employed to improve the performance of the controlled platoon.

In particular, the delay-based spacing policy (9) is extended as

$$\eta_i(t) = \Delta_i(t) + he_i(t) - kp_{i-1}(t), \tag{19}$$

with $i \in \mathscr{I}$, $h > 0$, $k \geq 0$, and where p_{i-1} is defined as

$$p_{i-1}(t) = \int_{-\Delta t}^{0} e^{-\alpha(\Delta t + \theta)} e_{i-1}(t + \theta)\, \mathrm{d}\theta. \tag{20}$$

The influence of this additional term (20), with $\alpha \geq 0$, can be understood by considering the case $\eta_i = 0$ in (19) (this will be, similar to before, the control objective). Namely, the term p_{i-1} provides an additional relaxation of the desired delay-based spacing policy (7) by allowing a momentarily shorter inter-vehicular distance ($\Delta_i(t) > 0$) when the preceding vehicle drives faster than desired in the time interval $(t - \Delta t, t]$ ($p_{i-1}(t) > 0$). Note that the delay-based spacing policy (1) requires tracking of the trajectory of the preceding vehicle subject to the delay Δt. As such, information on e_{i-1} in the interval $(t - \Delta t, t]$, as characterized through p_{i-1} in (20), can be regarded as preview information of "future" behavior of the velocity of the preceding vehicle. This information is weighted using the kernel $e^{-\alpha(\Delta t + \theta)}$, which ensures (for $\alpha > 0$) that data in the near "future" (for θ close to $-\Delta t$) has the highest importance.

A controller on the basis of the extended delay-based spacing policy η_i in (19) can be synthesized using a similar approach as discussed in Sect. 3 for the spacing policy δ_i in (9). Therefore, in the remainder of this section, it will be assumed that $\eta_i(t) = 0$ in (19) and the performance of this extended spacing policy will be analyzed by

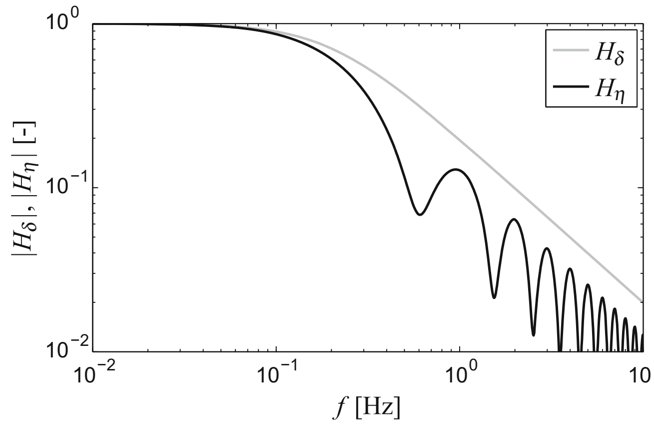

Fig. 2 Comparison of the frequency response functions $H_\delta(j\omega)$ in (18) and $H_\eta(j\omega)$ in (21), corresponding to the spacing policies (9) (for $\delta_i = 0$) and (19) (for $\eta_i = 0$), respectively. The parameter values are $\Delta t = 1$, $h = 0.8$, $k = 0.6$, and $\alpha = 0.9$

considering its transfer function, in analogy to (18) in Theorem 2. Specifically, the transfer function from e_{i-1} to e_i is obtained as

$$H_\eta(s) = \frac{1}{hs+1}e^{-s\Delta t} + \frac{ks}{hs+1}\frac{e^{-\alpha\Delta t} - e^{-s\Delta t}}{s-\alpha}. \tag{21}$$

Here, the expression (8) is used, whereas the right-most term in (21) is the result of the distributed delay in (20).

In order to compare the behavior of the extended spacing policy (19) with transfer function (21) to that of the spacing policy (9) with transfer function (18), the magnitude of their frequency response functions is depicted in Fig. 2. From this figure it is clear that the extended delay-based spacing policy characterized by H_η also ensures that perturbations in the velocity tracking error do not amplify to the string of vehicles, as was proven for the delay-based spacing policy δ_i in (9) in Theorem 2. In fact, the extended spacing policy η_i in (19) achieves a better suppression of disturbances, in particular for higher frequencies. Consequently, the use of the extended spacing policy (19) is expected to lead to increased performance of the controlled platoon. This will be verified through simulations in Sect. 5.

Remark 4 The implementation of the extended delay-based spacing policy (19) requires the evaluation of the integral in (20). However, the computation of this term cannot be obtained as the solution of a differential equation, as this involves an unstable pole-zero cancelation (at $s = \alpha$, see (21)). Instead, an online numerical computation of the integral term in $p_{i-1}(t)$ is required. It is remarked that these issues are also encountered in the finite spectrum assignment problem, which features a similar integral term. For an overview and computational approaches in this framework, see [18] and the references therein.

Table 1 Parameter values for the vehicle dynamics (3), spacing policy (9) and controller (5), (11) and (12). The lead vehicle is controlled through (14)

τ	1 (s)	l_0	2.00 (s^{-2})	k_0	7.92 (s^{-3})
Δt	1 (s)	l_1	2.82 (s^{-1})	k_1	11.96 (s^{-2})
h	0.8 (s)			k_2	6.00 (s^{-1})

5 Evaluation

The performance of the controller designed in Sect. 3 as well as the spacing policies δ_i in (9) and η_i in (19) is evaluated by means of simulations.

First, a platoon of eleven vehicles (i.e., $N = 10$ follower vehicles) is considered using the controller given by (5), (11), and (12). For the parameter values in Table 1, the tracking of the reference velocity profile $v_{\text{ref}}(\cdot)$ and delay-based spacing policy (9) is considered in Fig. 3. Herein, the initial conditions for the velocity profiles are randomly chosen and deviate from the equilibrium point. From this figure, it is clear that the equilibrium $e_i(t) = 0$, which corresponds to $v_i(t) = v_{\text{ref}}(s_i(t))$, is asymptotically stable. The same holds for the spacing tracking error Δ_i, indicating that the desired inter-vehicular spacing is obtained. To further illustrate the first point, the velocities v_i are depicted as a function of s_i in Fig. 4, from which it can be seen

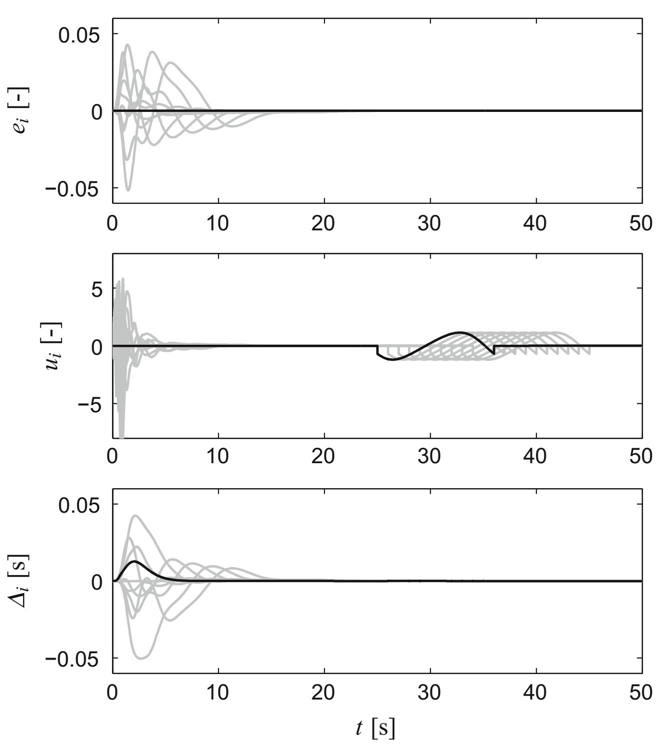

Fig. 3 Velocity tracking error e_i (*top*) and control input u_i (*middle*) for the lead vehicle (in *black*) and $N = 10$ follower vehicles (in *gray*) for non-equilibrium initial conditions and velocity reference $v_{\text{ref}}(s) = 20 - 1.75(1 - \cos(0.02\pi(s - 500)))$ for $500 \leq s \leq 700$ and $v_{\text{ref}}(s) = 20$ otherwise. Spacing errors Δ_i (*bottom*) are shown for the first follower vehicle (in *black*) and remaining follower vehicles (in *gray*)

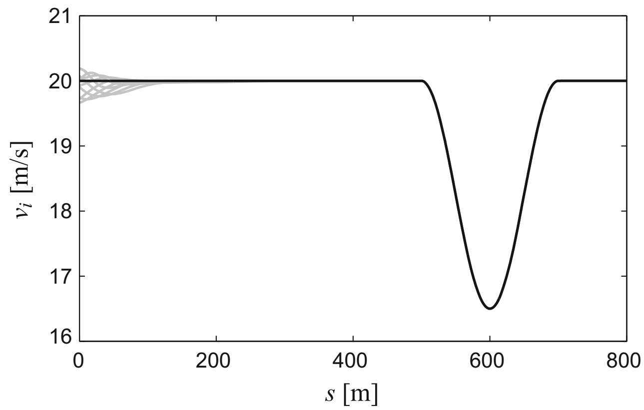

Fig. 4 Velocity v_i in the spatial domain for the lead vehicle (in *black*) and follower vehicles (in *gray*) corresponding to the simulation in Fig. 3

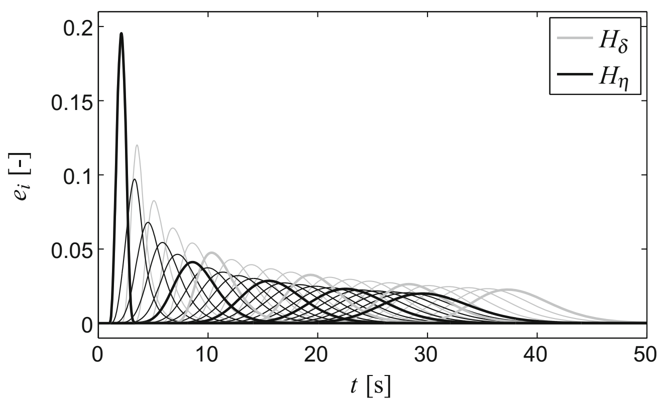

Fig. 5 Velocity tracking error e_i for the spacing policies (9) (in *gray*) and (19) (in *black*) for a platoon of $N = 20$ follower vehicles. The initial perturbation e_0 as well as the trajectories of vehicles 5, 10, 15, and 20 are shown in *thicker lines* for easy comparison. The parameter values correspond to those in Fig. 2

that all vehicles achieve the same velocity profile in the spatial domain as stated in Lemma 1.

Second, the spacing policy δ_i in (9) and extended spacing policy η_i in (19) are considered. Here, it is assumed that the spacing policies are tracked perfectly, i.e., $\delta_i(t) = 0$ and $\eta_i(t) = 0$ for all t and $i \in \mathscr{I}$, such that the design of controllers is not explicitly addressed. This case corresponds to the frequency response functions in Fig. 2. Figure 5 shows the velocity tracking errors of a platoon of $N = 20$ follower vehicles in case the lead vehicle experiences a perturbation e_0, which is considered as an input to the total platoon. It is clear that, for both spacing policies, the influence of this perturbation is decreased as it propagates through the platoon. In addition, as can also be seen in the frequency response function in Fig. 2, the extended delay-based spacing policy η_i in (19) achieves a larger suppression of disturbances and therefore a better performance. In fact, the disturbances are also handled earlier in time due to the distributed-delay term in (20). Here, it is recalled that the repeated application of (18) always leads to a delay of Δt seconds before a disturbance is propagated from a vehicle to its immediate follower. Finally, Fig. 6 shows the corresponding spacing tracking error, which confirms these observations and indicates an even more apparent benefit of the extended spacing policy (19).

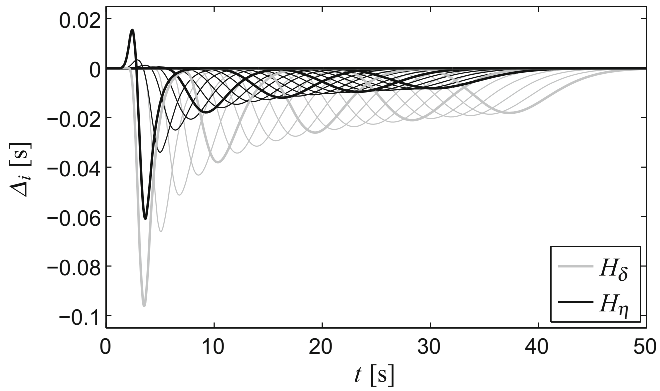

Fig. 6 Spacing tracking error Δ_i for the spacing policies (9) (in *gray*) and (19) (in *black*) for a platoon of $N = 20$ follower vehicles, corresponding to the velocity tracking errors in Fig. 5

6 Conclusions

The design of distributed platoon controllers for a delay-based spacing policy is discussed in this work, leading to platoon behavior in which all vehicles track the same velocity profile in the spatial domain. It is shown that a relaxation of this spacing policy leads to string-stable platoon behavior. Contrary to earlier results in [9], control design and analysis were done in the time domain (rather than the spatial domain), leading to a controller that is easier to interpret and implement. In addition, the time-domain approach allows for an extension of the delay-based policy. Herein, more information about the preceding vehicle is used by taking a distributed-delay approach, which is shown to lead to improved platoon behavior.

Future work will focus on further developing this distributed-delay approach, with particular emphasis on the implementation of this extended delay-based spacing policy and the resulting controller.

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References

1. Varaiya, P.: Smart cars on smart roads: problems of control. IEEE Trans. Autom. Control **38**(2), 195–207 (1993)
2. Alam, A., Besselink, B., Turri, V., Mårtensson, J., Johansson, K.H.: Heavy-duty vehicle platooning towards sustainable freight transportation: a cooperative method to enhance safety and efficiency. IEEE Control Syst. Mag. **35**(6), 34–56 (2015)
3. Levine, W., Athans, M.: On the optimal error regulation of a string of moving vehicles. IEEE Trans. Autom. Control, AC **11**(3), 355–361 (1966)
4. Peppard, L.E.: String stability of relative-motion PID vehicle control systems. IEEE Trans. Autom. Control **19**(5), 579–581 (1974)
5. Stanković, S.S., Stanojević, M.J., Šiljak, D.D.: Decentralized overlapping control of a platoon of vehicles. IEEE Trans. Control Syst. Technol. **8**(5), 816–832 (2000)

6. Swaroop, D., Hedrick, J.K.: Constant spacing strategies for platooning in automated highway systems. J. Dyn. Syst. Measur. Control **121**(3), 462–470 (1999)
7. Ioannou, P.A., Chien, C.C.: Autonomous intelligent cruise control. IEEE Trans. Veh. Technol. **42**(4), 657–672 (1993)
8. Swaroop, D., Hedrick, J.K., Chien, C.C., Ioannou, P.: A comparison of spacing and headway control laws for automatically controlled vehicles. Veh. Syst. Dyn. **23**(1), 597–625 (1994)
9. Besselink, B., Johansson, K.H.: Control of platoons of heavy-duty vehicles using a delaybased spacing policy. In: Proceedings of the 12th IFAC Workshop on Time Delay Systems, Ann Arbor, USA, pp. 364–369 (2015)
10. Newell, G.F.: A simplified car-following theory: a lower order model. Transp. Res. Part B: Methodol. **36**(3), 195–205 (2002)
11. Ge, J.I., Orosz, G.: Dynamics of connected vehicle systems with delayed acceleration feedback. Transp. Res. Part C: Emerg. Technol. **46**, 46–64 (2014)
12. Öncü, S., Ploeg, J., van de Wouw, N., Nijmeijer, H.: Cooperative adaptive cruise control: network-aware analysis of string stability. IEEE Trans. Intell. Transp. Syst. **15**(4), 1527–1537 (2014)
13. Ploeg, J., van de Wouw, N., Nijmeijer, H.: Lp string stability of cascaded systems: application to vehicle platooning. IEEE Trans. Control Syst. Technol. **22**(2), 786–793 (2014)
14. Antsaklis, P.J., Michel, A.N.: Linear Systems. Birkhäuser, Boston, USA (2006)
15. Sontag, E.D.: Smooth stabilization implies coprime factorization. IEEE Trans. Autom. Control **34**(4), 435–443 (1989)
16. Khalil, H.K.: Nonlinear Systems, 3rd edn. USA, Prentice Hall, Upper Saddle River (2002)
17. Fenton, R.E., Cosgriff, R.L., Olson, K., Blackwell, L.M.: One approach to highway automation. Proc. IEEE **56**(4), 556–566 (1968)
18. Michiels, W., Niculescu, S.-I.: Stability and Stabilization of Time-Delay Systems: An Eigenvalue-Based Approach. SIAM, Philadelphia, USA (2007)