Energy Consumption of Minimum Energy Coding in CDMA Wireless Sensor Networks*

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Abstract. A theoretical framework is proposed for accurate performance analysis of minimum energy coding schemes in Coded Division Multiple Access (CDMA) wireless sensor networks. Bit error rate and average energy consumption is analyzed for two coding schemes proposed in the literature: Minimum Energy coding (ME), and Modified Minimum Energy coding (MME). Since CDMA wireless systems are strongly limited by multi access interference, the system model includes all the relevant characteristics of the wireless propagation. Furthermore, a detailed model of the energy consumption is described as function of the coding schemes, the radio transmit powers, the characteristics of the transceivers, and the dynamics of the wireless channel. A distributed radio power minimization algorithm is also addressed. Numerical results show that ME and MME coding schemes exhibit similar bit error probabilities, whereas MME outperforms ME only in the case of low data rate and large coding codewords.

Keywords: Wireless Sensor Network (WSNs), Minimum Energy Coding, CDMA, OOK, Power Control, Outages.

1 Introduction

Despite the advancements in hardware and software technologies, one of the most relevant constraints to embody in the design of wireless sensor networks is the energy efficiency. The nodes are supposed to be deployed with reduced energy resources and without battery replacement. Motivating examples are found in areas such as industrial automation, environmental monitoring, and surveillance. Hence, the implementation of this technology has pushed the development

* Work done in the framework of the HYCON Network of Excellence, contract number FP6-IST-511368, and RUNES Integrated Project, contract number FP6-IST-004536. The work by C. Fischione and K. H. Johansson was partially funded also by the Swedish Foundation for Strategic Research through an Individual Grant for the Advancement of Research Leaders and by the Swedish Research Council. Corresponding author: C. Fischione.

K. Langendoen and T. Voigt (Eds.): EWSN 2007, LNCS 4373, pp. 212–227, 2007.

of techniques, protocols, and algorithms able to cope with the scarcity of communication and computation resources of the nodes.

Several techniques have been proposed to reduce the energy consumption of WSNs, while ensuring adequate performance. Often, cross-layer approaches are necessary in order to take into account several interacting techniques and protocols, as for example, distributed source coding [1] and sleep discipline [2]. In this paper, we restrict our attention to minimum energy coding schemes. Indeed, these schemes are easily implementable in real wireless sensor networks, and offer potentially several benefits.

Minimum Energy (ME) coding [3] is a technique to reduce the power consumption in digital transmitters when it is employed with On-Off Keying (OOK) modulation. The salient characteristic of ME coding and OOK is the reduction of information actually transmitted, with obvious energy savings. The advantages of ME coding have been investigated also in [4], where the authors proposed optimal ME coding along with channel coding. In [5], a closed-form expression of the bit error probability of ME coding and OOK modulation has been analyzed. In [6], ME coding has been applied to CDMA wireless systems. The authors have shown that, when ME coding technique is used with CDMA, multi access interference (MAI) is reduced. Therefore, ME enables better performance not only at the transmitter side, but also at the receiver, without the need of sophisticated correlation filters that increase the complexity and cost of the receiver node. In [7], the Modified Minimum Energy (MME) coding strategy has been proposed. MME is a variant of the ME coding strategy in that it aims at further reducing the energy consumption with sleep policies at the receiver.

In this paper we propose a detailed analysis of ME coding and MME coding in terms of energy consumption in CDMA WSNs. Since CDMA wireless systems are strongly limited in interference, we adopt a detailed model of the wireless channel, including path loss and shadowing. In the analysis, we accurately study the energy spent for coding, transmitting and receiving. The application of a decentralized radio power allocation strategy, which ensures transmit power consumption minimization, is addressed. Furthermore, by resorting to the extended Wilkinson moment matching method, we are able to characterize the bit error probability, which is a relevant factor of the total energy consumption of ME coding and MME coding.

With respect to existing relevant contributions [3], [4], [6] and [7], our approach is original because we provide a complete theoretical framework for characterization of the total energy consumption with respect to all the parameters of the system scenario, namely: ME, MME, CDMA, wireless channel, and actual energy consumption of the transceiver. For example, MME coding requires frequent startup of the radio, which cause severe influence on the average energy consumption for that scheme. We are able to accurately predict the performance of both ME coding and MME coding on realistic models of wireless sensor networks.

The remainder of the paper is organized as follows. In Section 2 the system model is described, and the main parameters and energy consumption of ME and MME are reported along with a description of the wireless scenario. The various components of the average energy consumption expressions are then investigated in the following sections. In Section 3, an algorithm for the minimization of the radio power consumption is discussed. A performance analysis of ME and MME in terms of radio power, bit error rate and total energy consumption is carried out in Sections 4 and 5. Numerical results are presented and discussed in Section 6. Finally, conclusions are given in Section 7.

2 System Description

Let us consider a scenario where there are K transmitter-receiver pairs of nodes (see Fig. 1). Data sensed by a node are firstly coded according to a Minimum Energy coding scheme. The corresponding bits are then handled by a OOK modulator: only the bits having value 1 are transmitted over the wireless interface after a DS-CDMA spreading operation. The power level per bit is denoted with P_i. The transmitted signal, after being attenuated by a flat fading wireless channel, is received corrupted by an additive Gaussian noise, having power spectral density $N_0/2$, and multi access interference caused by other transmitting nodes. At the receiver, the signal is de-spread, demodulated, and decoded in order to get the source data.

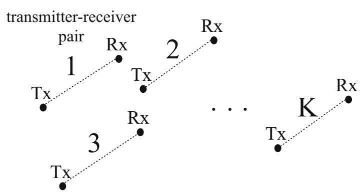

Fig. 1. System scenario: K pairs of nodes are simultaneously transmitting

2.1 ME Coding

With ME coding [3], each codeword of the source code-book is mapped into a new codeword having larger length but less number of 1 (or high) bits. Let us denote with L_0 the length of the source codeword, and with L_{ME} the length of the ME codeword. Thus, $L_0 < L_{ME}$, and the extra bits added are called redundant bits (see Fig. 2). The mapping is done such that source codewords having large probability of occurrence are associated to ME codewords with less high bits. Since only high bits are transmitted through the wireless interface, the transmission of ME codewords enable consistent energy savings. We denote with α_{ME} the probability of having high bits in a ME codeword. Note that, since there are several alternatives to associate source codewords to ME codewords, α_{ME} may assume different values.

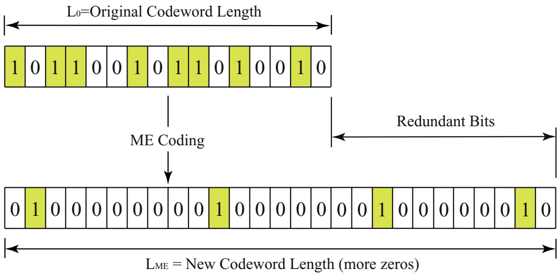

Fig. 2. Mapping of the source codewords into the codewords of the Minimum Energy Coding

Consider the link i between a transmitter and a receiver node. The energy consumption per ME codeword spent over the link can be expressed as follows:

$$\begin{aligned} E_i^{(ME)} &= E_i^{(tx)} + E_i^{(rx)} \\ &= P^{(tx,ckt)}\left(T^{(on,tx,ME)} + T_s\right) + \alpha_{ME} P_i T^{(on,tx,ME)} + \\ &\quad + P^{(rx,ckt)}\left(T^{(on,rx,ME)} + T_s\right) \ , \end{aligned} \tag{1}$$

where $E_i^{(tx)}$ and $E_i^{(rx)}$ are the average energy consumption of a node while transmitting and receiving, respectively; the power consumption of the electronic circuits, while transmitting and processing a codeword, is denoted with $P^{(tx,ckt)}$, and while receiving is denoted with $P^{(rx,ckt)}$; note that $P^{(tx,ckt)}$ and $P^{(rx,ckt)}$ do not include the radio power, which is P_i; $T^{(on,tx,ME)}$ is the transmitter activity time per ME codeword, and $T^{(on,rx,ME)}$ is the receiver activity time per codeword; finally, T_s is the start up time of the transceiver. In the above expressions, we have also modelled the major characteristic brought about by the ME and OOK modulation: the effective transmit time is only the fraction α_{ME} of the transmitter ontime.

With respect to the original codeword, the ME coding increases the value of two system parameters: the codeword length, L_{ME}, and the transmitter/receiver active time, $T^{(on,tx,ME)}$ and $T^{(on,rx,ME)}$. The increase of the transmitter active time is negligible with respect to the radio power consumption P_i ($P_i \gg P^{(tx,ckt,ME)}$). Increasing the receiver active time, however, may be harmful at the receiver itself, since the power spent to receive is approximately the same as that used to transmit. Furthermore, the larger codeword length of ME may increase the codeword error probability. These drawbacks of ME are compensated by the reduction of the multiple access interference caused by the decreased number of high bits.

2.2 MME Coding

The MME coding [7] exploits a structure of the codeword that allows the receiver to go in a sleep state, where the radio electronic circuitry is switched off[2]. In the MME coding technique, the ME codewords are partitioned into N_s sub-frames of length L_s, where each sub-frame starts with an indicator bit. When the indicator bit, which we denote with b_{ind}, is a high bit, it indicates that there are not high bits in that sub-frame, so there is no need for decoding, and the receiver can go to the sleep state. Conversely, if b_{ind} is a low bit, it indicates that there are high bits in the sub-frame, so the decoding operation must be performed, and the receiver cannot go to sleep. In Fig. 3, the MME codeword structure is reported. Note that the length of a MME codeword, L_{MME} is the same as L_{ME}. It should be noted that the MME coding may increase the length of the codeword of N_s bits with respect to ME coding, due to presence of the indicator bit. This drawback is, however, compensated by the potential energy savings that MME offers at the receiver.

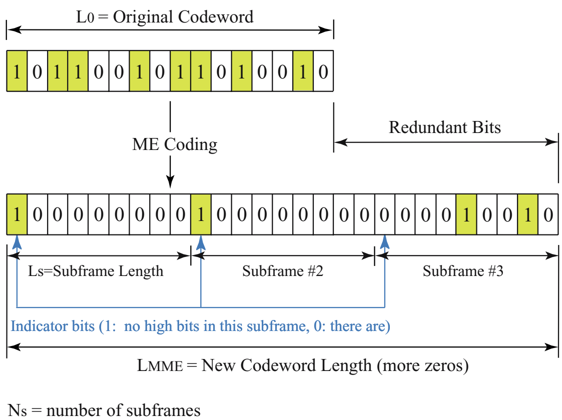

Fig. 3. MME codeword. The original codeword is mapped into a MME codeword partitioned in sub-frames. Each sub-frame starts with an indicator bit.

By adopting the same parameter definition employed in (1), the energy consumption per MME codeword can be modelled as follows:

$$\begin{aligned} E_i^{(MME)} &= E_i^{(tx)} + E_i^{(rx)} \\ &= P^{(tx,ckt)}\left[T^{(on,tx,MME)} + T_s\right] + \alpha_{MME} P_i T^{(on,tx,MME)} + \\ &\quad + P^{(rx,ckt)}\left[T^{(on,rx,MME)} + (N_i + 1)\, T_s\right] . \end{aligned} \tag{2}$$

In (2) we have introduced the average number of times, denoted with N_i, that the receiver has to awake from the sleep state. This term plays a fundamental role when evaluating the energy consumption: each time a radio receiver component is turned on, it spends an amount of energy given by $P^{(rx,ckt)}T_s$. Since $P^{(rx,ckt)}$ is the largest term among the powers at the receiver, and T_s is not negligible: when it is amplified by N_i, the resulting energy may be very large. The consequences are that MME may not offer an adequate performance improvement with respect to ME. Note that the term N_i has not been included in the energy model proposed in [7].

2.3 Wireless Channel

We consider an asynchronous DS-CDMA wireless access scheme, where the same fixed bandwidth W, and hence the same chip interval T_c, is allocated to each transmitter-receiver pair. The processing gain is denoted with $G = \frac{W}{R_b} = \frac{T_b}{T_c}$, where the bit interval is T_b.

Following an approach similar to that found in [9] and references therein, we can express the output of the correlation receiver of the link i as

$$Z_i(t) = D_i(t) + I_i(t) + N_g(t) \ , \tag{3}$$

where $D_i(t)$ is the desired signal for the pair i, $I_i(t)$ is the interference term due to the presence of multiple transmitting nodes (causing MAI) and $N_g(t)$ is a Gaussian random variable with zero mean and variance $\frac{N_0 T_b}{4}$. Specifically, it can be proved that

$$D_i(t) = \sqrt{\frac{P_i \Omega_{i,i}(t)}{2}} T_b b_i(t) \ . \tag{4}$$

and that the variance (correlation) of the MAI term is:

$$E\{I_i^2(t)\} = \sum_{\substack{j=1 \\ j \neq i}}^{K} \nu(t) P_j \Omega_{j,i}(t) \frac{T_b^2}{6G} \ . \tag{5}$$

In the expression (4) and (5), P_i, for $i = 1, \ldots, K$, denotes the radio power of the transmitter node in link i. We introduce the vector $\mathbf{P}$ for notational convenience:

$$\mathbf{P} = [P_1, \ldots, P_i, \ldots, P_K]^T \ . \tag{6}$$

The term $\nu(t)$ is a binary random variable that describes the transmission of a high bit ($\nu(t) = 1$) or low bits ($\nu(t) = 0$), with probability $\Pr(\nu(t) = 1) = \alpha$ and $\Pr(\nu(t) = 0) = 1 - \alpha$, respectively. In particular, $\alpha = \alpha_{ME}$ for ME coding, whereas $\alpha = \alpha_{MME}$ for MME coding. The wireless channel coefficient associated to the path from the transmitter of the link j to the receiver of the link i is denoted with $\Omega_{j,i}(t)$, and it is defined as

$$\Omega_{j,i}(t) = \mathrm{PL}_{j,i} e^{\xi_{j,i}(t)} \ . \tag{7}$$

where $\mathrm{PL}_{j,i}$ is the path loss, and $e^{\xi_{j,i}(t)}$ is the shadow fading component over the same path, with $\xi_{j,i}(t)$ being a Gaussian random variable having zero average and standard deviation $\sigma_{\xi_{j,i}}$. The path loss can be further written (in dBs) as follows

$$\mathrm{PL}_{j,i}|_{dB} = -P_l\left(d_r\right)|_{dB} - 10n \log_{10}\left(\frac{d_{j,i}}{d_r}\right) \ . \tag{8}$$

where d_r is the reference distance and $d_{j,i}$ is the distance between the transmitter in link j and the receiver of the link i, the term $P_l\left(d_r\right)|_{dB}$ denotes the path loss attenuation at the reference distance, and n is the path-loss decay constant. For notational convenience, we introduce the following vector to denote the wireless channel coefficients seen by the receiver of the pair i:

$$\mathbf{\Omega}_i(t) = \left[\Omega_{1,i}(t), \ldots, \Omega_{i,i}(t), \ldots \Omega_{K,i}(t)\right]^T . \tag{9}$$

When a high bit is transmitted, i.e. $b_i(t) = 1$, the quality of the received signal is measured by the Signal to Interference plus Noise Ratio. For the generic transmitter-receiver pair i, the SINR is defined as follows

$$SINR_i(t) = \frac{\sqrt{\frac{P_i \Omega_{i,i}(t)}{2}} T_b b_i(t)}{\sqrt{\frac{N_0 T_b}{4} + \sum_{\substack{j=1 \\ j \neq i}}^{K} \nu(t) P_j \Omega_{j,i}(t) \frac{T_b^2}{6G}}} . \tag{10}$$

Note that the SINR is a stochastic process, since it is dependent on the wireless channel coefficients $\mathbf{\Omega}_i(t)$, as well as on the binary on/off source activity $\nu(t)$. Moreover, it can be directly influenced by the transmit powers $\mathbf{P}$. When $b_i(t) = 0$, the SINR is obviously zero.

3 Optimal Transmission Power

In this section, we study the optimal value of the radio power P_i that appears in (1) and (2). In order to minimize the transmission power of the overall system, we adopt an optimization problem whose objective function is the sum of the powers of all the transmit nodes, while the constraints are expressed in terms of link outage probability [2], namely

$$\begin{aligned} &\min_{\mathbf{P}} \sum_{i=1}^{K} P_i \\ s.t.\ &P\left[SINR_i(t) \leq \gamma\right] \leq \bar{P}_{out}\ , \forall\, i = 1 \ldots K \\ &P_i > 0 \quad \forall\, i = 1, \ldots, K\ . \end{aligned} \tag{11}$$

In the optimization problem, note that γ is defined as the SINR threshold for the computation of the outage probability. In particular, the solution of the optimization problem ensures that the outage probability remains below the maximum value $\bar{P}_{out}$. Furthermore, note that the computation of the outage probability is performed with respect to the statistics of the wireless channel, and the distribution of high bits.

To solve the optimization problem (11), we have to model the constraints related to the outages of the links. Since the statistics of the SINR are in general unknown, we resort to the well know extended Wilkinson moment-matching method [9]. We approximate the SINR with an overall Log-normal distribution, thus obtaining that

$$SINR_i(t) = L_i(t)^{-\frac{1}{2}} \approx e^{-\frac{1}{2}X_i(t)} \ , \tag{12}$$

where $X_i(t)$ is a Gaussian process having average and standard deviation, respectively, given by μ_{X_i} and σ_{X_i}, i.e., $X_i \sim \mathcal{N}(\mu_{X_i}, \sigma_{X_i})$. The expression of the parameters of the Gaussian distribution are provided in the Appendix. The approximation is useful because allows for computing the outage probability while taking into account all the relevant aspects of the wireless propagation, the transmission power, and the distribution of high bit. It trivially results that

$$P\left[SINR_i(t) \le \gamma\right] \approx P\left[e^{-\frac{X_i(t)}{2}} \le \gamma\right] = Q\left(\frac{-2\ln\gamma - \mu_{X_i}}{\sigma_{X_i}}\right) \ . \tag{13}$$

The constraint on the outage probability can be easily rewritten in order to evidence the dependence on the transmission power coefficients. After some algebra, a relaxation of the program (11) can be rewritten as follows:

$$\begin{aligned} &\min_{\mathbf{P}} \sum_{i=1}^{K} P_i \\ &s.t.\ P_i \ge 2T_b^{-2}\mathrm{PL}_{i,i}^{-1}\left\{\beta_i^{(1)}[\mathbf{P}_{-i}]\right\}^{2(1-q_i)}\left\{\beta_i^{(2)}[\mathbf{P}_{-i}]\right\}^{-\frac{1}{2}+q_i}\gamma^2,\ i=1\dots K \\ &P_i > 0 \quad \forall\, i = 1,\dots,K \ , \end{aligned} \tag{14}$$

where $q_i = Q^{-1}\left(\bar{P}_{out}\right)$, and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$ is the complementary standard Gaussian distribution. The expressions for $\beta_i^{(1)}[\mathbf{P}_{-i}]$ and $\beta_i^{(2)}[\mathbf{P}_{-i}]$ are provided in the Appendix. The problem (14) is a relaxation since σ_{X_i} has been replaced with its square. This is equivalent to say that the outage constraints are tighter. It is possible to see that the relaxation reduces the computational burden, and that the solution is an upper bound of the solution of the original problem. The program (14) is a centralized problem, in the sense that, to compute the solution, a central node should be able to collect all information related to radio link coefficients, it should be able to solve the program, and finally it should broadcast the optimized powers to all other nodes. A centralized implementation exhibits clear disadvantages in terms of communication resources. Nevertheless, by following the same method proposed in [11], it can be proved that (14) can be solved with a fully distributed strategy. Each receiver node can find iteratively the optimal power as follows

$$P_i(n) = 2T_b^{-2}\mathrm{PL}_{i,i}^{-1}\left\{\beta_i^{(1)}[\mathbf{P}_{-i}(n-1)]\right\}^{2(1-q_i)}\left\{\beta_i^{(2)}\left[\mathbf{P}_{-i}(n-1)\right]\right\}^{-\frac{1}{2}+q_i}\gamma^2 \ , \tag{15}$$

where $\mathbf{P}_{-i}(n-1)$ are the powers that the node i sees at the iteration n. The power updating can be done asynchronously by each node, and it can be proved that for

$n \longrightarrow \infty$ the power converges to the optimal value of (14). The algorithm (15) is fully distributed, since the computation of the path loss parameter, as well as the other coefficients, is done locally by the nodes. In particular, note that the node i has just to compute the expectations $\beta_i^{(1)}[\mathbf{P}_{-i}(n-1)]$ and $\beta_i^{(2)}[\mathbf{P}_{-i}(n-1)]$, through (44) and (45).

4 Performance Analysis: Error Probability

In this section we characterize the average bit error probability for the ME coding, as well as for the MME coding. The characterization is useful to express the total energy consumption (2). With this goal in mind, we first derive the error probability for the decision variable (3). In particular, two cases are possible: the decision variable is decoded as a low bit, when a high bit was transmitted; or the decision variable is decoded as a high bit when a low bit was actually transmitted. We denote such probabilities with $p_{i|0}$ and $p_{i|1}$, respectively, where:

$$p_{i|0} = \Pr\left[Z_i(t) > \delta_i(t) | b_i(t) = 0, \boldsymbol{\Omega}_i(t), \nu(t)\right] \ , \tag{16}$$

$$p_{i|1} = \Pr\left[Z_i(t) < \delta_i(t) | b_i(t) = 1, \boldsymbol{\Omega}_i(t), \nu(t)\right] \ , \tag{17}$$

where $\delta_i(t)$ is the decision threshold for the variable $Z_i(t)$. The probabilities in (16) and (17) are computed adopting the usual standard Gaussian approximation [14], where $Z_i(t)$ is modelled as a Gaussian random variable conditioned to the distribution of the channel coefficients and coding. Specifically, it is assumed that:

$$Z_i(t) \sim \mathcal{N}\left(\mu_{Z_i(t)}, \sigma_{Z_i(t)}\right) \ , \tag{18}$$

where

$$\mu_{Z_i(t)} = \begin{cases} \mu_{Z_i(t)|0} = 0 & \text{if } b_i(t) = 0 \\ \mu_{Z_i(t)|1} = \sqrt{\frac{P_i \Omega_{i,i}(t)}{2}} T_b^2 & \text{if } b_i(t) = 1 \end{cases} , \tag{19}$$

and

$$\sigma_{Z_i(t)} = \sqrt{\frac{N_0 T_b}{4} + \sum_{\substack{j=1 \\ j \neq i}}^{K} \nu(t) P_j \Omega_{j,i}(t) \frac{T_b^2}{6G}} \ . \tag{20}$$

Hence, it is easy to compute the probabilities (16) and (17), which are given by

$$p_{i|0} = Q\left(\frac{\delta_i(t)}{\sigma_{Z_i(t)}}\right) \ ,$$

$$p_{i|1} = Q\left(\frac{\mu_{Z_i(t)|1} - \delta_i(t)}{\sigma_{Z_i(t)}}\right) \ .$$

The bit error probability, conditioned to the channel coefficients and coding, can be expressed as:

$$\begin{aligned} \Phi_i[e|\boldsymbol{\Omega}_i(t), \nu(t)] &= \Pr\left[b_i(t) = 0\right] p_{i|0} + \Pr\left[b_i(t) = 1\right] p_{i|1} \\ &= (1-\alpha)\, p_{i|0} + \alpha \cdot p_{i|1} \ . \end{aligned} \tag{21}$$

Finally, averaging with respect to the distribution of the channel coefficients and the distribution of the high bits,

$$\Phi_i = E_{\mathbf{\Omega}_i(t),\nu(t)}\left[(1-\alpha)\,Q\left(\frac{\delta_i(t)}{\sigma_{Z_i(t)}}\right) + \alpha \cdot Q\left(\frac{\mu_{Z_i(t)|1} - \delta_i(t)}{\sigma_{Z_i(t)}}\right)\right] \ . \quad (22)$$

The expression (22) should be minimized with respect to $\delta_i(t)$. However, it has to be remarked that it is hard to compute the expectation in (22), since the argument is non linear. Hence, no closed-form is available for Φ_i, and, as a consequence, it is difficult to find the optimal $\delta_i(t)$. Therefore, we resort to the heuristic

$$\delta_i^{opt}(t) = \frac{\mu_{Z_i(t)|1}}{2} \ . \quad (23)$$

Using (23), the probability of error turns out to be

$$\Phi_i = E_{\mathbf{\Omega}_i(t),\nu(t)}\left[Q\left(\frac{\mu_{Z_i(t)|1}}{2\sigma_{Z_i(t)}}\right)\right] = E_{\mathbf{\Omega}_i(t),\nu(t)}\left[Q\left(\frac{1}{2}e^{-\frac{1}{2}X_i(t)}\right)\right] \ . \quad (24)$$

Let us define $\zeta_i = \frac{1}{2}e^{-\frac{1}{2}X_i(t)}$. Then, using the Stirling approximation [14] for the computation of the expectation in (24), we obtain that

$$E_{\mathbf{\Omega}_i(t),\nu(t)}\left[Q\left(\zeta_i\right)\right] \approx \frac{2}{3}Q\left(\mu_{\zeta_i}\right) + \frac{1}{6}Q\left(\mu_{\zeta_i} + \sqrt{3}\sigma_{\zeta_i}\right) + \frac{1}{6}Q\left(\mu_{\zeta_i} - \sqrt{3}\sigma_{\zeta_i}\right) \ , \quad (25)$$

where, recalling the computation of the average and standard deviation of log-normal random variables (see also the Appendix), we have that

$$\begin{aligned} \mu_{\zeta_i} &= \tfrac{1}{2}e^{-\frac{1}{2}\mu_{X_i} + \frac{1}{8}\sigma^2_{X_i}} \ , \\ r_{\zeta_i} &= \tfrac{1}{4}e^{-\frac{1}{2}\mu_{X_i} + \frac{1}{8}\sigma^2_{X_i}} \ , \\ \sigma^2_{\zeta_i} &= r_{\zeta^i} - \mu^2_{\zeta^i} \ . \end{aligned} \quad (26)$$

4.1 Error Probability in ME Coding

The probability of bit error in the ME case, denoted with $\Phi_i^{(ME)}$, can be easily computed by using Eq. (25), along with (26), (42) and (43), where α takes the value α_{ME}.

4.2 Error Probability in MME Coding

An analysis on the MME performance regarding its probability of error demands a careful study which takes into account the special nature of the MME codeword. As done in [7], we compute the average equivalent bit error probability as the ratio between the average number of erroneous bits per MME codeword and the codeword length, namely

$$\Phi_i^{(MME)} = \frac{\overline{n}_{i,sf} N_s}{L_{MME}} \ , \quad (27)$$

where N_s is the number of sub-frames per codeword and $\overline{n}_{i,sf}$ is the average number of erroneous bits in a subframe transmitted over the link i:

$$\overline{n}_{i,sf} = \sum_{n=1}^{L_s-1} n p_i^{(n)} \ , \tag{28}$$

where $p_i^{(n)}$ stands for the probability of having n errors in the sub-frame, and, recalling that each sub-frame starts with an indicator bit, it can be computed as follows

$$p_i^{(n)} = \Pr\left[b_{ind} = 1\right] \Phi_i \Pr(\mathcal{A}_i) + \Pr\left[b_{ind} = 0\right] \left[(1 - \Phi_i) \Pr(\mathcal{A}_i) + \Phi_i \Pr(\mathcal{B}_i)\right] \ , \tag{29}$$

where the event $\mathcal{A}_i$ happens when there are n decoding errors in a sub-frame, and $\mathcal{B}_i$ happens when there are n high bits in the codeword. Note that in (29) we have actually considered that an error in decoding of an indicator bit has catastrophic consequences in the entire following sub-frame.

It is not difficult to see that [7]:

$$\Pr(\mathcal{A}_i) = \binom{L_s - 1}{n} \Phi_i^n \left(1 - \Phi_i\right)^{Ls-1-n} \ , \tag{30}$$

$$\Pr(\mathcal{B}_i) = \binom{L_s - 1}{n} \alpha_{MME}^n \left(1 - \alpha_{MME}\right)^{Ls-1-n} \ , \tag{31}$$

while

$$\Pr\left[b_{ind} = 0\right] = 1 - \left(1 - \alpha_{MME}\right)^{L_s-1} \ , \tag{32}$$

and

$$\Pr\left[b_{ind} = 1\right] = \left(1 - \alpha_{MME}\right)^{L_s-1} \ . \tag{33}$$

5 Performance Analysis: Energy Consumption

5.1 ME Coding

The energy consumption of the ME coding scheme is defined as the average of the energy consumption of all the sensor nodes, namely

$$E^{(ME)} = \frac{1}{K} \sum_{i=1}^{K} E_i^{(ME)} \ , \tag{34}$$

where the term $E_i^{(ME)}$ is defined as in (2), and it takes into account the radio power minimization discussed in section 3.

Looking at the energy model for a system using ME coding (1) it is easy to see that setting $\alpha_{ME} = 1$, one obtains the energy consumption of the BPSK case. The energy gain of the ME coding with respect to BPSK can be defined as the ratio of the energy used in a BPSK system and the energy used in a ME coding system:

$$\rho_{dB} = \left(\frac{E_{radio}^{BPSK}}{E_{radio}^{ME}} \right)_{dB} \ . \tag{35}$$

5.2 MME Coding

In order to investigate the energy of MME coding, it is necessary to characterize the values of $T^{(on,rx,MME)}$ and N_i in (2).

The value of $T^{(on,rx,MME)}$ is given by the average number of high bits in a MME codeword times the bit time, namely

$$T^{(on,rx,MME)} = \frac{\overline{n}_i}{R_b} \; . \tag{36}$$

In particular, having in mind the sub-frame structure, the average number of received high bits is given by

$$\overline{n}_i = N_s \left\{1 + \Pr\left[b_{ind} = 0\right]\left(1 - \Phi_i\right)\left(L_s - 1\right) + \Pr\left[b_{ind} = 1\right]\Phi_i\left(L_s - 1\right)\right\} \; . \tag{37}$$

In order to compute the average number of times that the radio module has to be active, first we have to consider that it has to be on for each indicator bit. Moreover, the radio is turned active for every sub-frame, with the exception of the case in which the previous sub-frame has been decoded (the radio is already on). This can be expressed as follows:

$$N_i = N_s\left[1 - \Pr\left(b_{ind} = 0\right)\left(1 - \Phi_i\right) - \Pr\left(b_{ind} = 1\right)\Phi_i\right] \; . \tag{38}$$

Eq. (36) and Eq. (38) can be plugged into (2) to obtain the energy consumption for the generic link i. Thus, averaging over all the links, the energy consumption for the MME case is:

$$E^{(MME)} = \frac{1}{K}\sum_{i=1}^{K} E_i^{(MME)} \; . \tag{39}$$

Finally, the receive energy gain is defined as:

$$\rho_{dB} = \left(\frac{E_{radio}^{ME}}{E_{radio}^{MME}}\right)_{dB} \; . \tag{40}$$

6 Numerical Results

In this section we provide numerical evaluation of the bit error probability, and the total energy consumption of ME and MME coding.

In the numerical results, we consider a system scenario with $K = 10$ pairs of nodes. Each pair is randomly placed, and the minimum distance among nodes is 3 m, while the maximum distance is 15 m. We have considered a homogeneous environment, where the path loss at the reference distance is set to $P_l\left(d_r\right)|_{\mathrm{dB}} =$ 55 dB, $d_r = 1$m, $n = 4$, and $\sigma_{\xi_{j,i}} = 5$dB, for $i, j = 1 \ldots K$. The source data rate is assumed to be $R_b = 1$Kbps. A spreading gain $G = 64$ has been used. We set the value of the power spectral density of the noise to $N_0/2|_{\mathrm{dB}} = -174$dBm, and the threshold for the SINR $\gamma = 3.1$ dB. Finally, we have taken as reference the

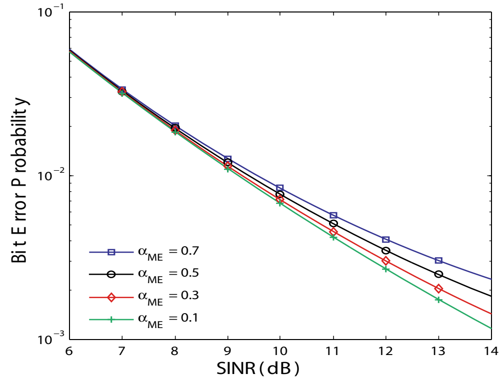

Fig. 4. Bit error probability for both the ME and MME cases. Note that they overlap.

CC2420 radio transceiver module by Chipcon [10], as is the one incorporated in the Telos motes, to set the values of the energy parameters. Namely: $P^{(tx,ckt)} = 36 \cdot 10^{-3}$ mW, $T_s = 0.58 \cdot 10^{-3}$ ms, $P^{(rx,ctk)} = 33.84$ mW.

In Fig. 4, the bit error probability is reported for both the case of ME and MME coding. Each curve is associated to a different value of $\alpha = \alpha_{ME} = \alpha_{MME}$. The probabilities are reported as function of the average received SINR, and are computed using $\Phi^{(ME)}$ and $\Phi^{(MME)}$ after the power minimization algorithm (15) provided the optimal powers. Details on the numerical results of the power minimization algorithm can be found in [16]. It is interesting to observe that there is not noticeable difference among the ME and MME cases. This can be explained by considering that the outage probability is the dominant term in the computation of the bit error probability, and the optimal powers ensure that the constraint of the outage probability is fulfilled in the same way for ME case and MME cases. A decreasing of the bit error probability can be evidenced when α decreases. This is obviously due to the fact that lower values of α decrease the multi access interference.

In Fig. 5, the energy gain is reported for MME case as computed with (40) for different values of the α coefficient and as function of the sub-frame length. As it can be observed, an optimum value can be found for the energy gain as function of L_s, for each value of α. The energy gain decreases as L_s increases since this causes the receive time to be longer. Low values of α determine good performance of MME. It is interesting to remark, however, that as α increases, there is a sharp decrease of the energy gain. When α increases more then 0.3, there is no advantage in using the MME coding with respect to the ME. This can be explained by observing that large probabilities of having high bits lead to large average number of wake-ups of the radio receiver.

Numerical values have been derived also with higher source data rate. Due to lack of space, we cannot report them in this paper. However, it could be possible to observe that as the source data rate increases, the MME receive energy gain quickly decreases.

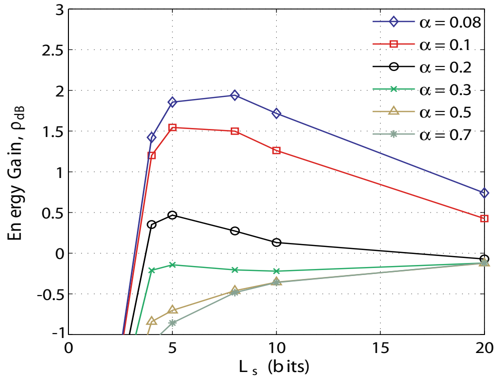

Fig. 5. MME Energy Gain as function of the sub-frame length for different values of α

7 Conclusions

In this paper, a general framework for accurate analysis of the performance of ME coding and MME coding in CDMA WSNs has been proposed. The analysis has been carried out in terms of radio power consumption, and energy consumption of the electronic circuit transceivers. Specifically, an accurate wireless propagation scenario has been taken into account, including path loss and shadowing. A distributed power minimization strategy has been described and implemented.

Numerical results show that, in the scenario considered, ME and MME codings do not have significant differences in terms of bit error probability. MME outperforms ME only for low bit rate data transmission and low probability of high bits. Therefore, MME is a good candidate for low rate applications. However, as technology evolves and smaller startup times might be reached, MME could be useful with higher data rates as well.

Future studies include extension of the analysis to optimizing the decision threshold, and investigating performance for a large set of channel conditions. Furthermore, experimental validation of the performance we have analytically predicted will be carried out.

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Appendix

By definition, we have that

$$L_i(t) = \frac{2}{P_i T_b^2 \mathrm{PL}_{ii}} \left(\sum_{\substack{j=1 \\ j \neq i}}^{K} \alpha_{ME} P_j \mathrm{PL}_{j,i} e^{\xi_{j,i} - \xi_{ii}} \frac{T_b^2}{6G} + \frac{N_0 T_b}{4} e^{-\xi_{ii}} \right) . \tag{41}$$

The extended Wilkinson moment matching method [9], allows for the computation of the average and standard deviation of X_i by matching the average and autocorrelation of $L_i(t)$ with those of $e^{X_i(t)}$, thus obtaining

$$\mu_{X_i} = 2 \ln M_i^{(1)} - \frac{1}{2} \ln M_i^{(2)} , \tag{42}$$

$$\sigma_{X_i}^2 = \ln M_i^{(2)} - 2 \ln M_i^{(1)} \ , \tag{43}$$

with

$$M_i^{(1)} \triangleq E_{\mathbf{\Omega}_i(t),\nu(t)}\{L_i\} \ , \tag{44}$$

$$M_i^{(2)} \triangleq E_{\mathbf{\Omega}_i(t),\nu(t)}\{L_i^2\} \ , \tag{45}$$

where the statistical expectation is taken with respect to the distributions of the channel coefficients and the high bit. Therefore, by recalling that $\mathbf{\Omega}_i(t)$ and $\nu(t)$ are statistically independent, and the definition of moments of log normal random variables [15], it is easy to show that

$$M_i^{(1)} = \frac{2}{P_i T_b^2 \mathrm{PL}_{i,i}} \beta_i^{(1)} \left[\mathbf{P_{-i}}\right] \ , \tag{46}$$

$$M_i^{(2)} = \frac{4}{P_i^2 T_b^4 \mathrm{PL}_{i,i}^2} \beta_i^{(2)} \left[\mathbf{P_{-i}}\right] \ , \tag{47}$$

where

$$\beta_1^i \left[\mathbf{P_{-i}}\right] = \sum_{\substack{j=1 \\ j \neq i}}^{K} \alpha P_j \mathrm{PL}_{j,i} e^{\frac{1}{2}\left(\sigma_{\xi_{j,i}}^2 + \sigma_{\xi_{i,i}}^2\right)} \frac{T_b^2}{6G} + \frac{N_0 T_b}{4} e^{\frac{1}{2}\sigma_{\xi_{i,i}}^2} \ , \tag{48}$$

$$\begin{aligned}
\beta_2^i \left[\mathbf{P_{-i}}\right] = & \sum_{\substack{j=1 \\ j \neq i}}^{K} \alpha^2 P_j^2 \mathrm{PL}_{j,i}^2 e^{2\left(\sigma_{\xi_{j,i}}^2 + \sigma_{\xi_{i,i}}^2\right)} \frac{T_b^4}{36G^2} + \frac{N_0^2 T_b^2}{16} e^{2\sigma_{\xi_{i,i}}^2} \\
& + \sum_{\substack{j=1 \\ j \neq i}}^{K} \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^{K} \alpha^2 P_j P_k \mathrm{PL}_{j,i} \mathrm{PL}_{k,i} e^{\frac{1}{2}\left(\sigma_{\xi_{j,i}}^2 + \sigma_{\xi_{k,i}}^2 + 4\sigma_{\xi_{i,i}}^2\right)} \frac{T_b^4}{36G^2} \\
& + \frac{N_0 T_b}{2} \sum_{\substack{j=1 \\ j \neq i}}^{K} \alpha P_j \mathrm{PL}_{j,i} e^{\frac{1}{2}\left(\sigma_{\xi_{j,i}}^2 + 4\sigma_{\xi_{i,i}}^2\right)} \frac{T_b^2}{6G} \ .
\end{aligned} \tag{49}$$

In previous expressions, we have defined

$$\mathbf{P_{-i}} = \left[P_1, P_2, \ldots, P_{(i-1)}, P_{(i+1)}, \ldots, P_K\right]^T \ , \tag{50}$$

with the purpose to evidence that nor in $\beta_i^{(1)}\left[\mathbf{P_{-i}}\right]$, nor in $\beta_i^{(2)}\left[\mathbf{P_{-i}}\right]$ there is dependence with the transmission power of the link i.