

# Multihypothesis Motion-Compensated Prediction with Forward-Adaptive Hypothesis Switching

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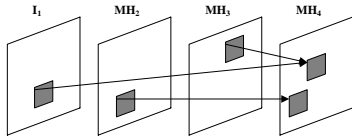
## Introduction

### Goal

- Improved motion-compensated prediction for efficient video compression

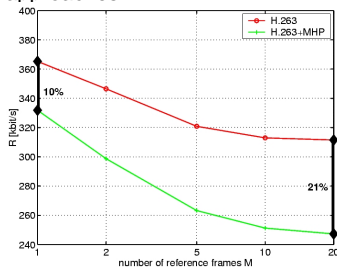
### Approaches

- Linear combination of motion-compensated signals
- Forward-adaptive selection of reference frames for motion-compensated prediction



### Advantage

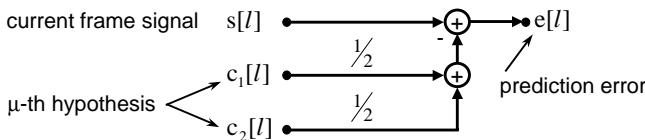
- More than "additive" gains can be achieved by combining both approaches



Mobile & Calendar  
QCIF, 10 fps  
@34 dB PSNR

## Multihypothesis Motion-Compensated Prediction

### Multihypothesis Motion Compensation



- All hypotheses  $c_\mu$  are shifted versions of the current frame signal
- The shift is determined by the displacement error

$$\Delta_\mu \sim \mathcal{N}(0, C_{\Delta\Delta})$$

- The ideal reconstruction of the band-limited signal  $s[L]$  is shifted by the continuous valued displacement error and re-sampled on the original orthogonal grid

$$\frac{\Phi_{ec}(\omega)}{\Phi_{ss}(\omega)} = \frac{N+1}{N} - 2P(\omega, \sigma_\Delta^2) + \frac{N-1}{N} P(\omega, 2\sigma_\Delta^2(1-\rho_\Delta))$$

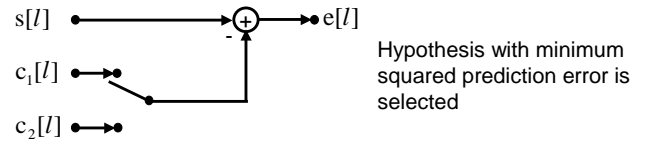
$$P(\omega, \sigma_\Delta^2) = \exp(-\frac{1}{2}\omega^T \omega \sigma_\Delta^2) \quad \text{with} \quad \omega = (\omega_x, \omega_y)$$

### Optimal Multihypothesis Motion Estimation

$$\min \sigma_c^2 \Rightarrow \rho_\Delta = \frac{1}{1-N}$$

## Forward-Adaptive Hypothesis Switching

### Selection of One Motion-Compensated Signal



### Minimizing the Radial Displacement Error

$$p_{\Delta_x \Delta_y}(x, y) = \frac{1}{2\pi\sigma_{\Delta_x}^2} \exp\left(-\frac{x^2 + y^2}{2\sigma_{\Delta_x}^2}\right) \xleftrightarrow{\Delta_r = \sqrt{\Delta_x^2 + \Delta_y^2}} p_{\Delta_r}(r) = \frac{r}{\sigma_{\Delta_x}^2} \exp\left(-\frac{r^2}{2\sigma_{\Delta_x}^2}\right)$$

$$\sigma_{\Delta_x}^2 = \frac{\sigma_{\Delta_x}^2}{M} \quad \Delta_r^M = \min\{\Delta_{r1}, \dots, \Delta_{rM}\}$$

$$p_{\Delta_x \Delta_y}^M(x, y) = \frac{1}{2\pi\sigma_{\Delta_x}^2} \exp\left(-\frac{x^2 + y^2}{2\sigma_{\Delta_x}^2}\right) \xleftrightarrow{\Delta_r^M} p_{\Delta_r}^M(r) = \frac{r}{\sigma_{\Delta_x}^2} \exp\left(-\frac{r^2}{2\sigma_{\Delta_x}^2}\right)$$

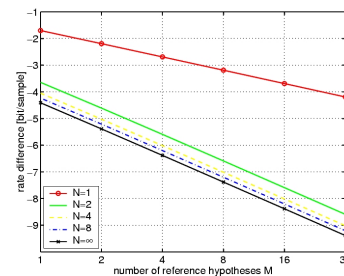
### Equivalent Predictor

- Individual and minimum radial displacement errors are Rayleigh distributed
- This suggests an equivalent predictor with reduced displacement error variance

## Multihypothesis Prediction with Forward-Adaptive Hypothesis Switching

Combine both predictors and superimpose  $N$  hypotheses where each hypothesis is obtained by switching among  $M$  motion-compensated signals

### Theoretical Result



Rate difference when compared to optimum intra-frame encoding:

$$\Delta R = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \log_2 \left( \frac{\Phi_{ec}(\omega)}{\Phi_{ss}(\omega)} \right) d\omega$$

### References

- B. Girod, "Efficiency Analysis of Multihypothesis Motion-Compensated Prediction for Video Coding," IEEE Tr. on Image Processing, vol. 9, no. 2, pp. 173-183, Feb. 2000.
- M. Flierl and B. Girod, "Multihypothesis Motion Estimation for Video Coding," in Proc. DCC, Snowbird, Utha, Mar. 2001.