

*Video Coding with Motion  
Compensation for Groups of Pictures*

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# Motivation

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- Today's video coding schemes utilize DPCM with MCP.
- How about motion-compensated 3-d transform coding?

This talk . . .

provides an analysis based on a power spectral model.

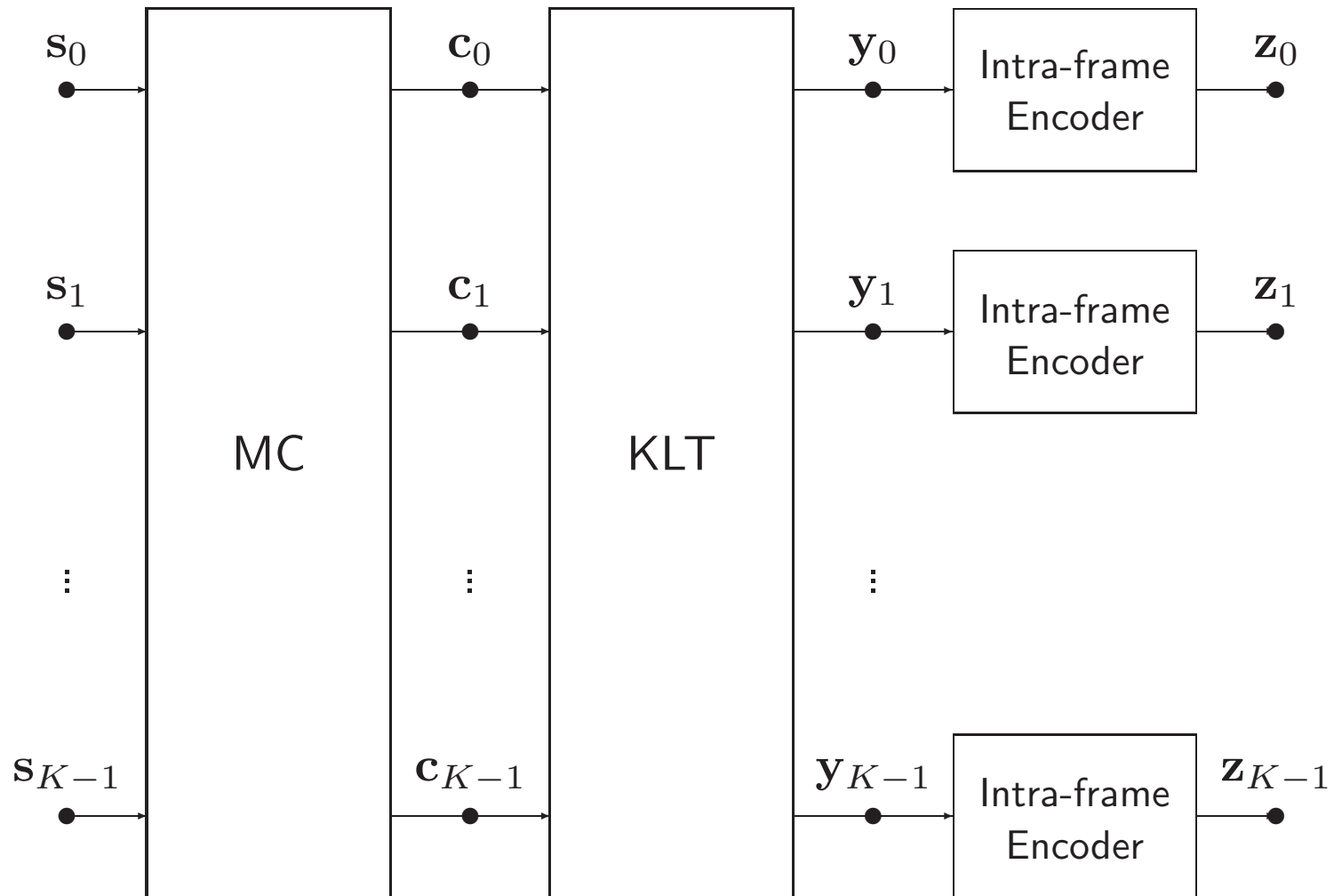
# Overview

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- Coding scheme for a group of pictures
- Model for a group of motion-compensated pictures
- Model assumptions
- Performance measure
- Performance and impact of residual noise
- Comparison to motion-compensated prediction

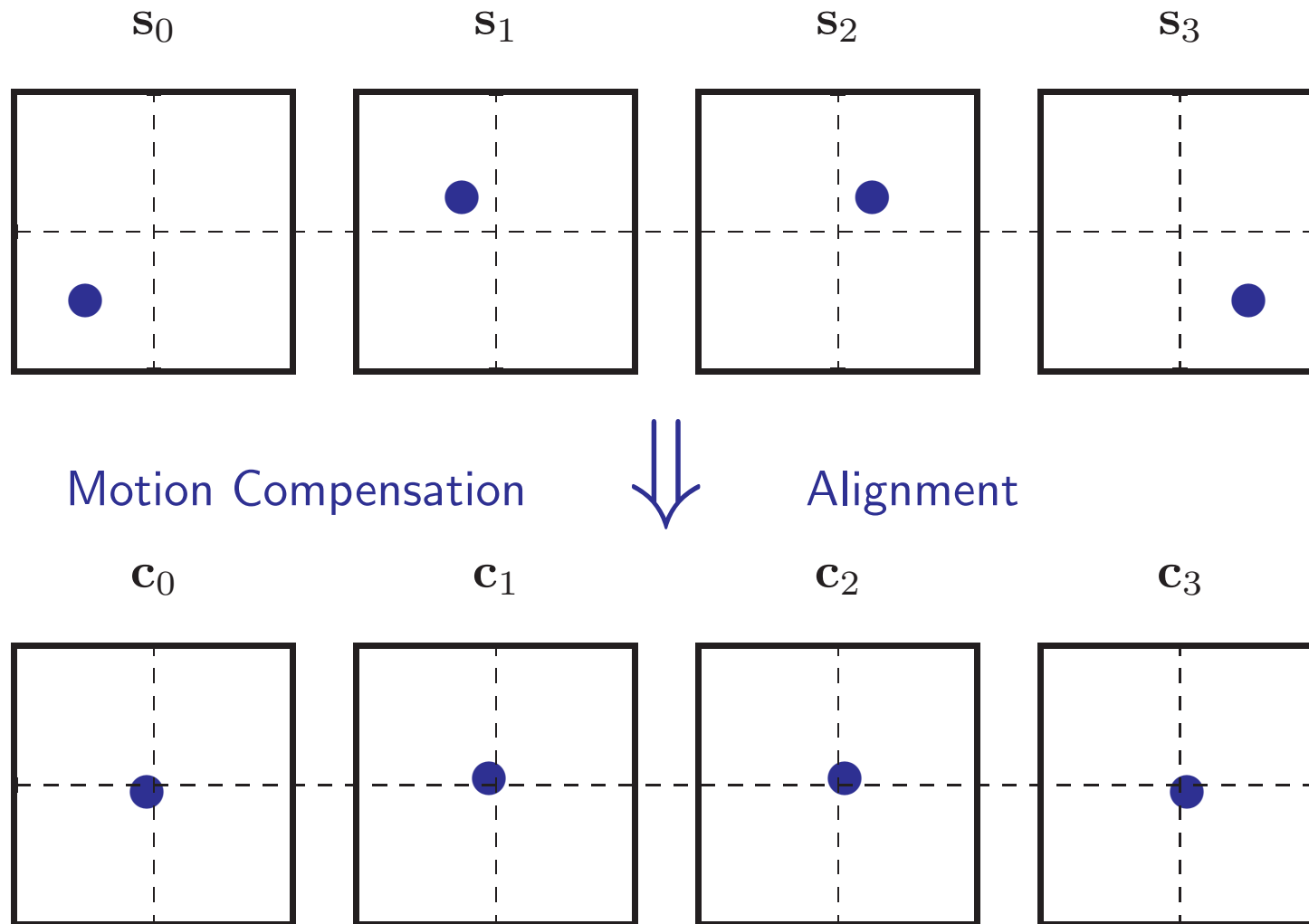
# Coding Scheme for a Group of Pictures

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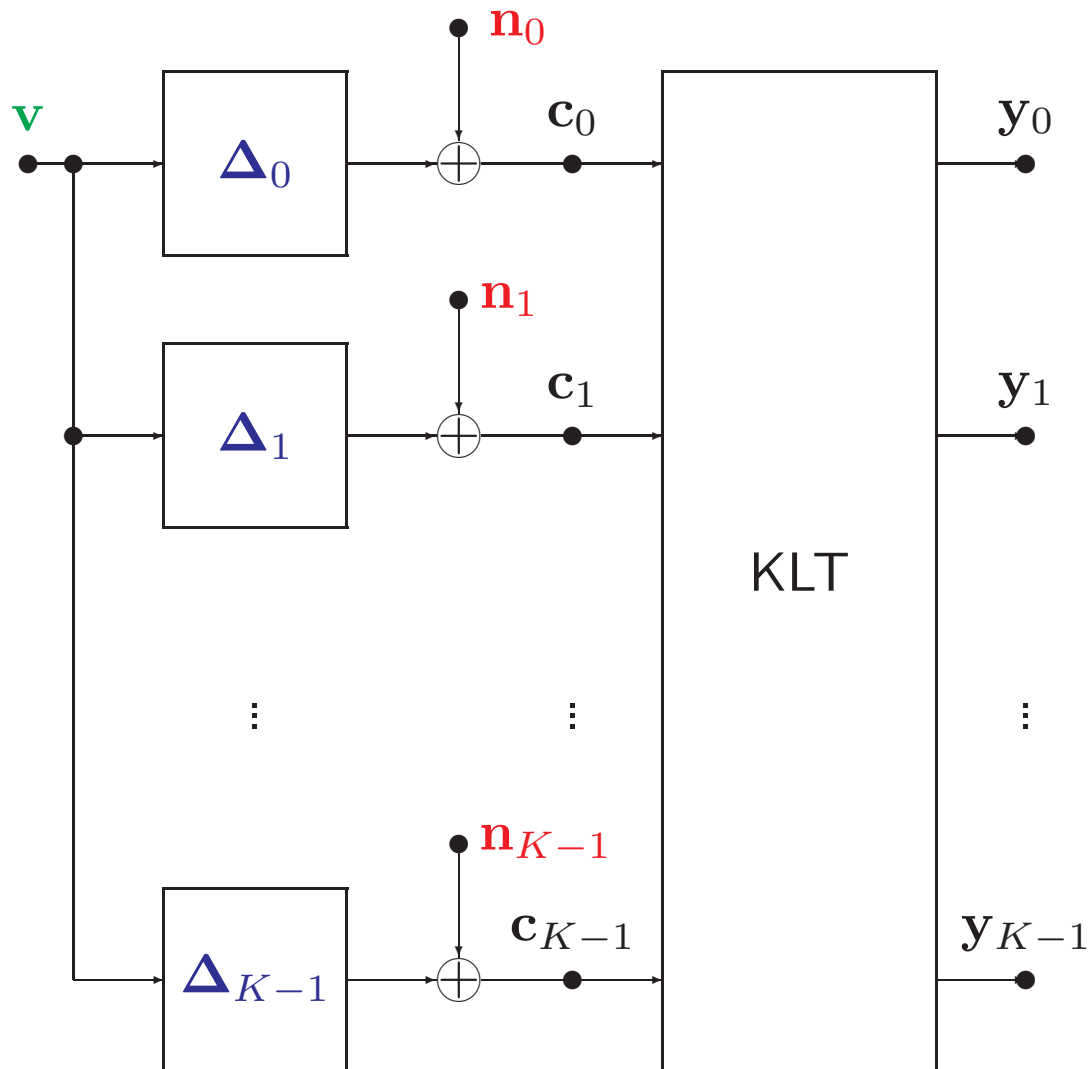


# Motion Compensation for a Group of Pictures

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# Model for a Group of Motion-Compensated Pictures



$v$  model picture

$\Delta_k$   $k$ -th displacement error

$n_k$   $k$ -th noise signal

$c_k$   $k$ -th motion-compensated signal

$y_k$   $k$ -th transform signal

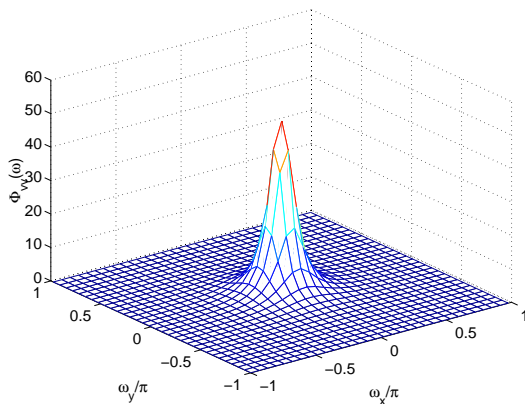
Codec has the freedom to determine its own reference picture!

# Basic Model Assumptions

## Model Picture

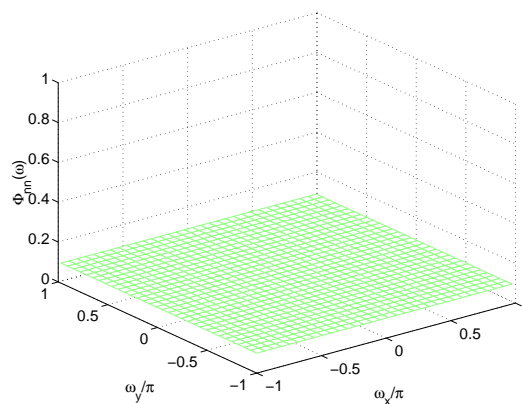
Bandlimited version of a 2-d signal with exponentially decaying and isotropic autocorrelation function.

Characterized by the PSD  $\Phi_{vv}(\omega)$  with variance  $\sigma_v^2 = 1$ .



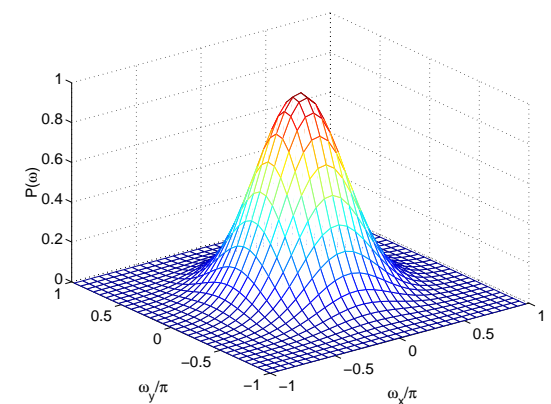
## Residual Noise

White noise with variance  $\sigma_n^2$  and PSD  $\Phi_{nn}(\omega)$ .



## Displacement Error

Normal distributed and isotropic with variance  $\sigma_\Delta^2$  and characteristic function  $P(\omega, \sigma_\Delta^2)$ .



# Assumptions about Displacement Errors

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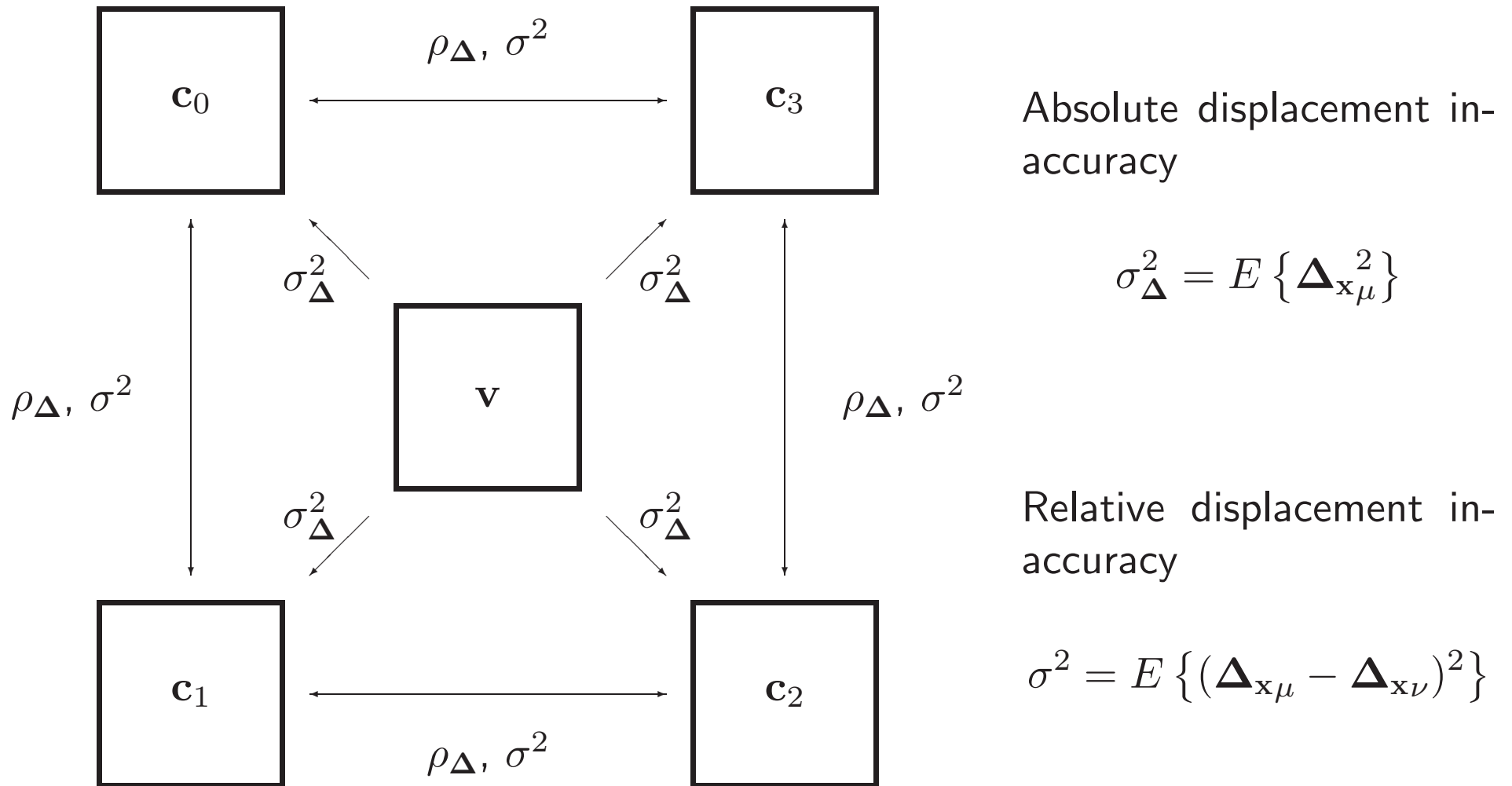
- 2-d stationary normal distribution with variance  $\sigma_{\Delta}^2$  and zero mean for each motion-compensated picture
- $x$ - and  $y$ -components are statistically independent
- Each displacement error pair is assumed to be **jointly Gaussian** with no preference among the  $K$  motion-compensated signals
- $K \times K$  covariance matrix of a displacement error component:

$$C_{\Delta_x \Delta_x} = C_{\Delta_y \Delta_y} = \sigma_{\Delta}^2 \begin{pmatrix} 1 & \rho_{\Delta} & \cdots & \rho_{\Delta} \\ \rho_{\Delta} & 1 & \cdots & \rho_{\Delta} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{\Delta} & \rho_{\Delta} & \cdots & 1 \end{pmatrix}$$

- Covariance matrix is nonnegative definite:  $\frac{1}{1-K} \leq \rho_{\Delta} \leq 1 \quad K > 1$



# Displacement Inaccuracy and Correlation



# Performance Measure

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- Rate difference for each picture  $k$

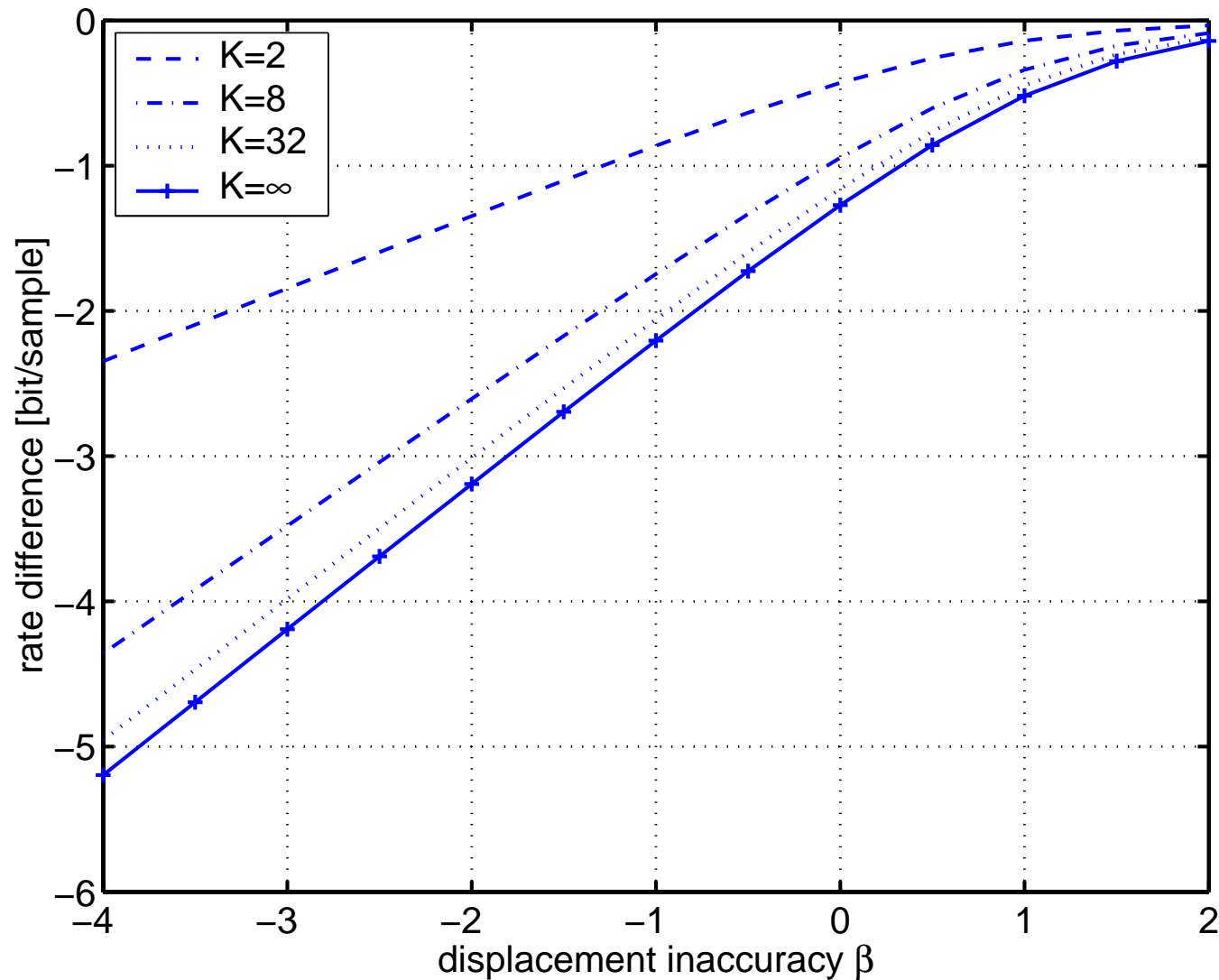
$$\Delta R_k = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{1}{2} \log_2 \left( \frac{\Phi_{y_k y_k}(\omega)}{\Phi_{c_k c_k}(\omega)} \right) d\omega$$

- Measures maximum bit-rate reduction
- Compared to optimum intra-frame encoding
- For the same mean squared reconstruction error
- For Gaussian signals

- Average rate difference

$$\Delta R = \frac{1}{K} \sum_{k=0}^{K-1} \Delta R_k$$

# Performance with Negligible Residual Noise



Calibration:

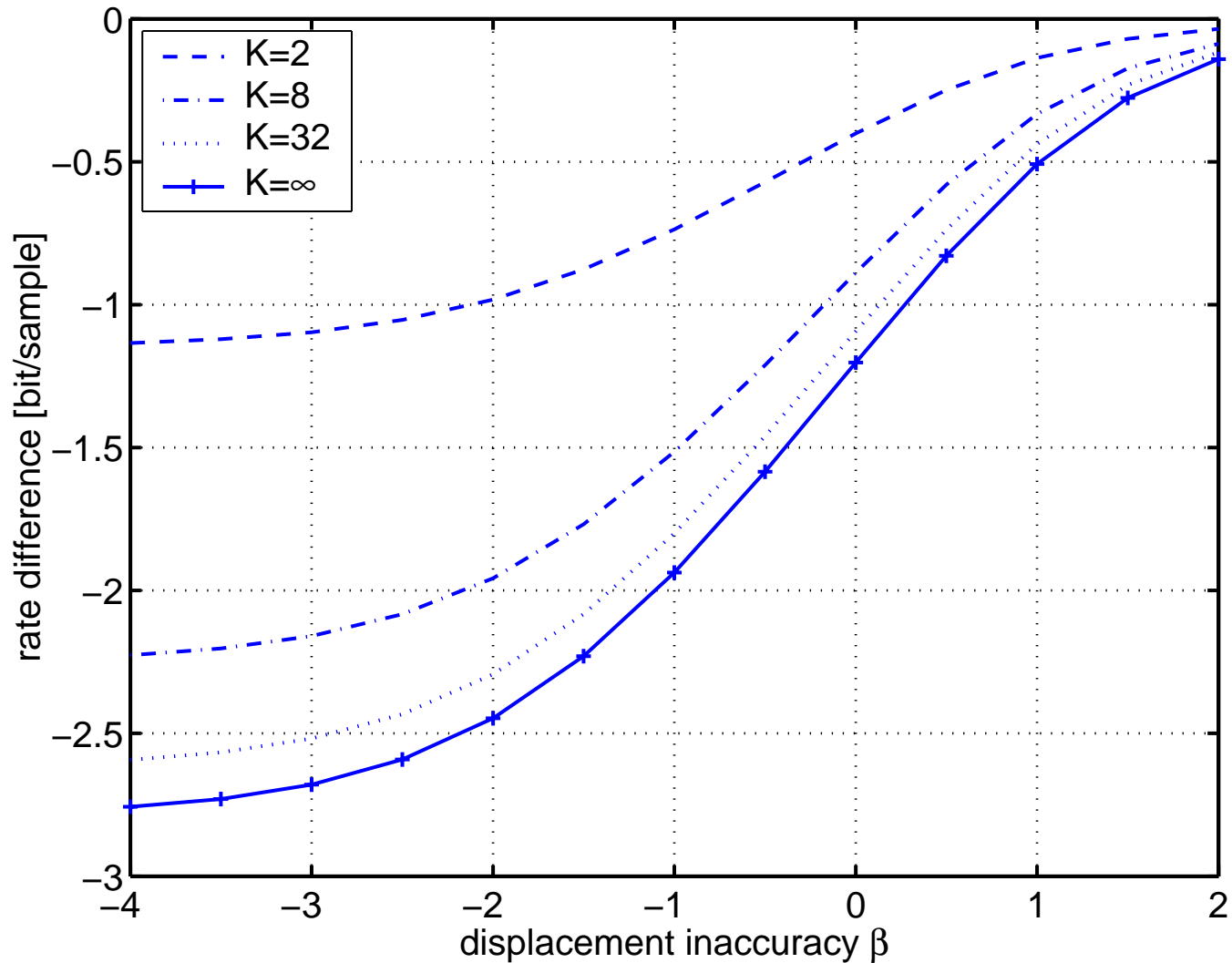
$$\beta = \frac{1}{2} \log_2(12\sigma_{\Delta}^2)$$

Accuracy:

$\beta = 0$  : IP  
 $\beta = -1$  : HP  
 $\beta = -2$  : QP

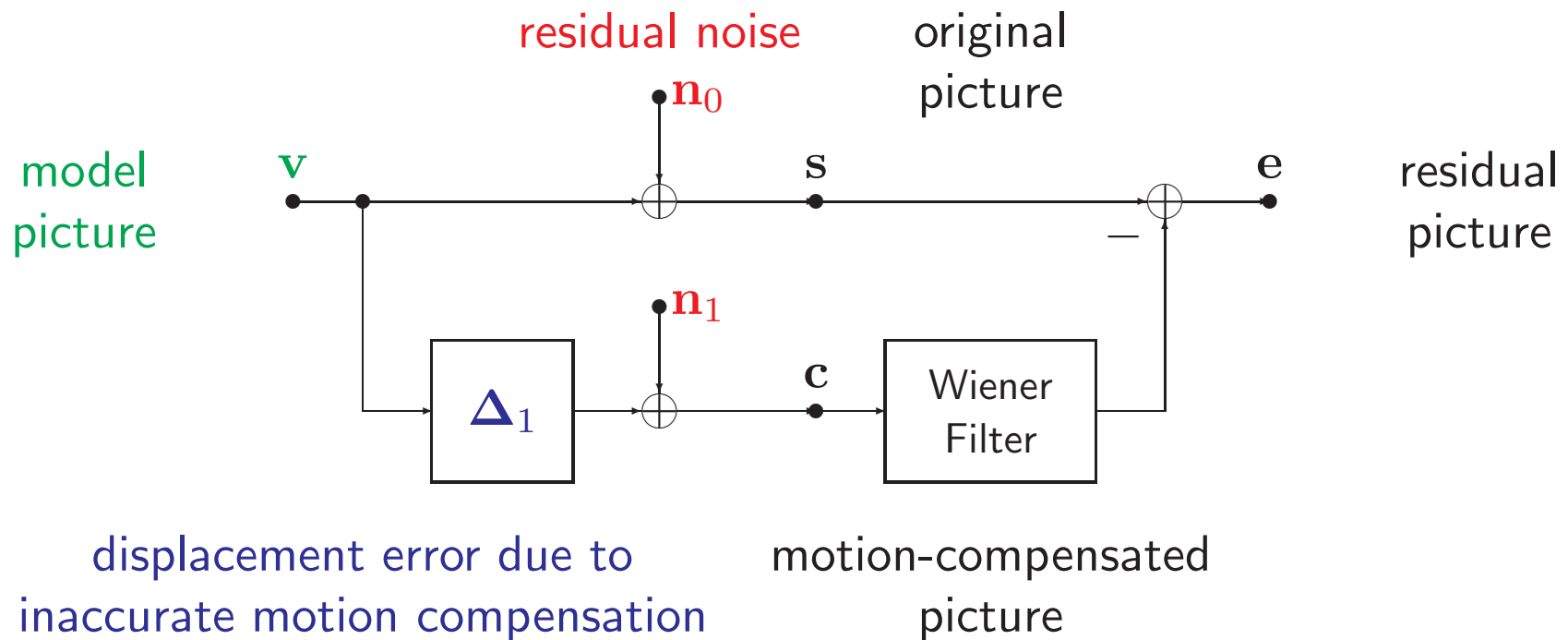
Identical absolute and relative displacement inaccuracy

# Performance with Residual Noise -30 dB

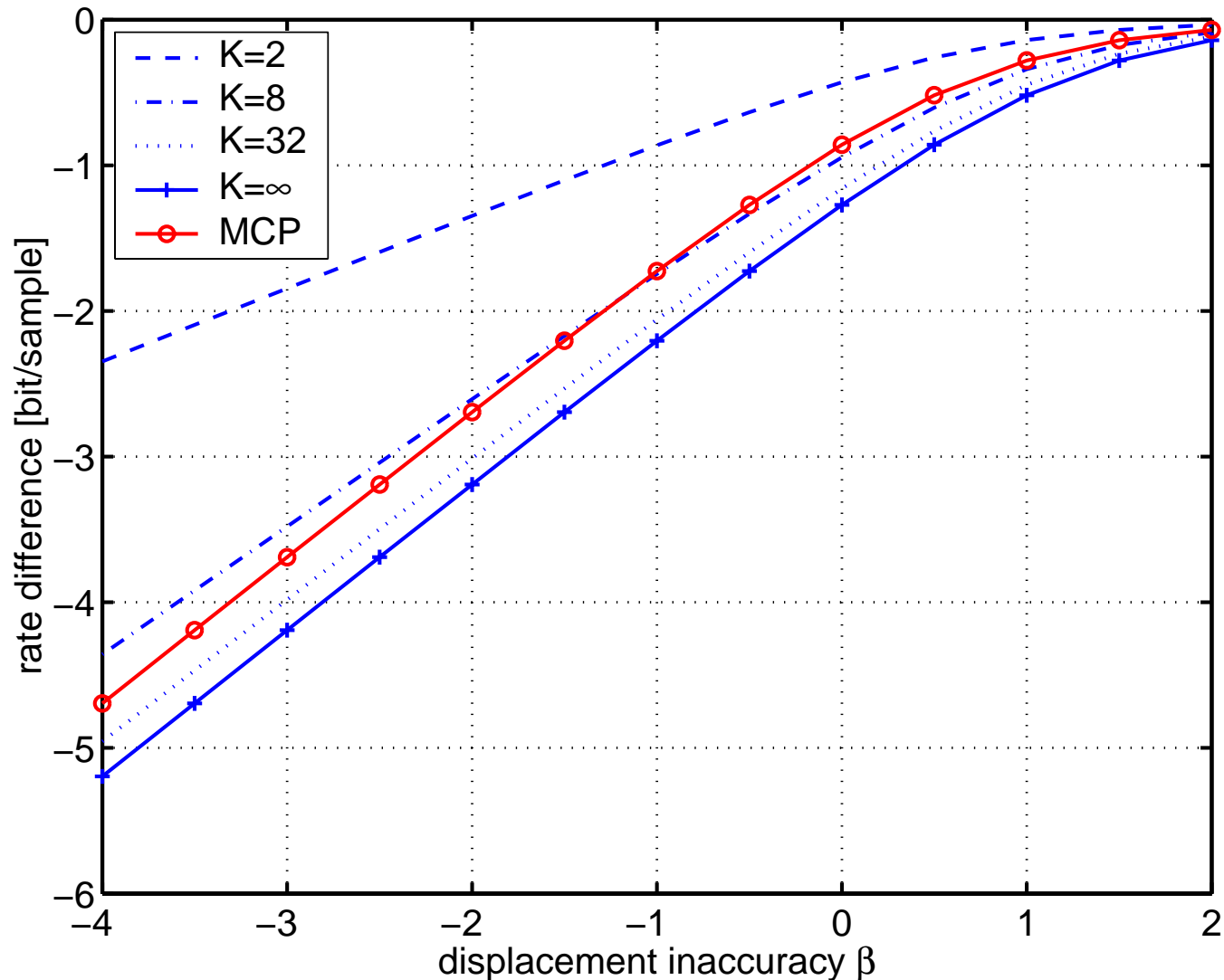


Identical absolute and relative displacement inaccuracy

# Motion-Compensated Prediction

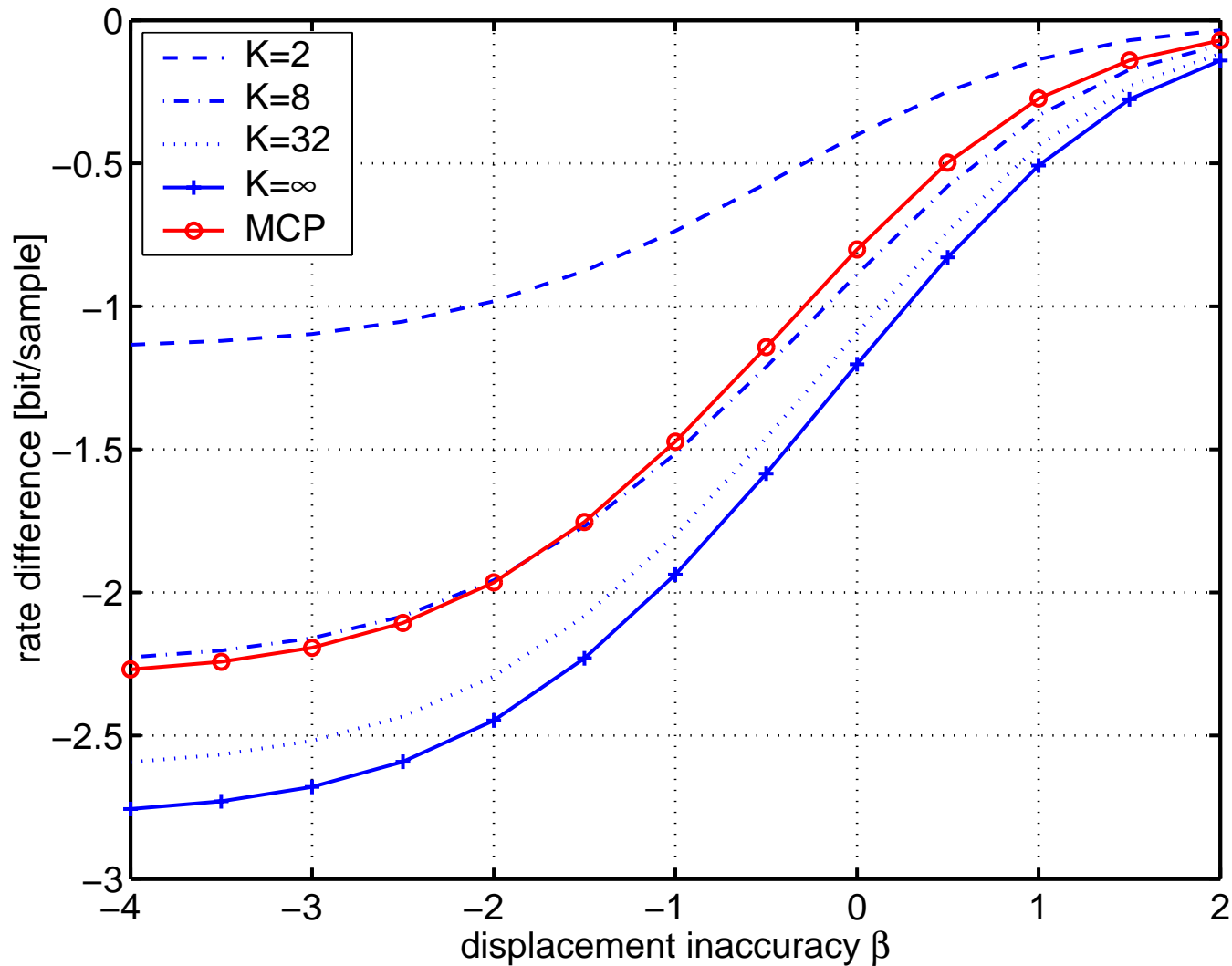


# Comparison to Motion-Compensated Prediction I



Absolute and relative displacement inaccuracy are identical,  
residual noise level -100 dB

# Comparison to Motion-Compensated Prediction II



Absolute and relative displacement inaccuracy are identical,  
residual noise level -30 dB

# Summary and Conclusions

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- Model for a group of motion-compensated pictures with correlated displacement error
- Motion-compensated pictures are decorrelated by the Karhunen-Loeve Transform
- Results of the analysis:
  1. Without residual noise, the slope of the rate difference achieves up to 1 bit/sample and displacement inaccuracy step
  2. Residual noise limits the gain by accurate motion compensation
- Comparison to motion-compensated prediction:
  1. The transform model outperforms MCP with optimum Wiener filter by at most 0.5 bit/sample
  2. The performance for a group of  $K = 8$  pictures is comparable to MCP