Information Theory
Spring semester, 2023

Assignment 11
Assigned: Friday, June 16, 2023
Due: After Summer, 2023 M. Skoglund

Problem 11.1: Consider the following memoryless continuous channel model

$$Y_n = A_n X_n + Z_n$$

where $X_n \in \mathbb{R}$ is the channel input, $Z_n \in \mathbb{R}$ a noise term and $A_n \in \mathbb{R}$ a time-varying channel amplitude, all at time n. Assume that Z_n is iid zero-mean Gaussian with $E[Z_n^2] = \sigma^2$ and that $\{A_n\}$ is iid. Assume also that the receiver knows the value of A_n but the transmitter does not.
Assuming that the formula proved in VerHan can be generalized in the obvious manner to amplitude-continuous channels subject to a power constraint, derive an expression for the capacity of the described channel subject to $E[X_n^2] \leq P$.

Problem 11.2: Consider again the channel model in Prob. 11.1 but assume instead that $A_n = A$ for all n, where A is a random variable drawn once according to a pdf $f(a)$. Derive an expression for the channel capacity subject to the power constraint $E[X_n^2] \leq P$.

Problem 11.3: Specialize Prob. 11.2 to the case

$$f(a) = a\, e^{-a^2/2}, \quad a \in [0, \infty]$$

That is, a *slowly fading Rayleigh* channel with $E[A_n^2] = 2$.

Problem 11.4: Consider a *mixed* DMC, described either by $p_1(y|x)$ or $p_2(y|x)$, with probabilities π and $1 - \pi$. That is, with probability π the channel is a DMC described by p_1 and with probability $1 - \pi$ described by p_2. What is the capacity of this channel?