Information Theory
Spring semester, 2023

Assignment 3
Assigned: Wednesday, April 5, 2023
Due: Friday, April 21, 2023 M. Skoglund

Problem 3.1: Cover & Thomas 5.28 (p. 150)

Problem 3.2:
Consider a coin with a fixed, but unkown, probability of turning up heads. Treat the unkown parameter as a random variable with a uniform prior. Given a sequence of independent flips of such a coin, derive a simple expression for the probability that the next flip will be a head.

Problem 3.3:
The binary sequence

$$s = 1111111110000001111111111111111100001 = 1^9 0^6 1^{16} 0^4 1$$

was generated by a stationary two-state Markov chain with transition probabilities $p_{1|0} = p_{0|1} = 0.2$. Encode s using

(a.1) a Huffman code for 3-bit symbols based on the source model,

(a.2) a Huffman code for 3-bit symbols based on relative frequencies in s;

(b.1) a Shannon-Fano-Elias code *or* an arithmetic code for 3-bit symbols based on the source model,

(b.2) a Shannon-Fano-Elias code *or* an arithmetic code for 3-bit symbols based on relative frequencies in s;

(c.1) an arithmetic stream code (as described in CT or class) based on the source model,

(c.2) an adaptive arithmetic stream code (as described in class; use Prob. 2), based on s;

(d.1) the "basic" Lempel-Ziv algorithm (CT),

(d.2) the "modified" Lempel-Ziv algorithm (class).

(e) Relate your answers to the entropy rate of the Markov source and the entropy of s based on relative frequencies.

You need not solve every problem by pen and paper. The arithmetic stream codes get quickly out of hand. If you choose to write a program, please turn in your "source code" with your solutions.