Probability and Random Processes
2024

Assignment 4
Assigned: Friday, February 9, 2024
Due: Thursday, February 15, 2024 M. Skoglund

Problem 4.1: Define the integral of a nonnegative measurable function.

Problem 4.2: Prove the general version of the DCT (you can use Fatou's lemma without proof).

Problem 4.3: For a given finite Borel measure μ on $(\mathbb{R}, \mathcal{B})$ define the distribution function F_μ. For a general function F, explain the requirements for F to be a distribution function (corresponding to some finite Borel measure).

Problem 4.4: Given a distribution function F, corresponding to μ on $(\mathbb{R}, \mathcal{B})$, and a Borel measurable function f, define the Lebesgue–Stieltjes integral of f w.r.t. F. Illustrate this concept using $\mu = \delta_b$ (Dirac measure concentrated at b).

Problem 4.5: Given a finite measure space $(\Omega, \mathcal{A}, \mu)$ and a real-valued measurable function $f : \Omega \to [0, 1]$, prove that

$$\lim_{n\to\infty} \int f^{1/n} d\mu = \mu\big(f^{-1}((0, 1])\big)$$

and

$$\lim_{n\to\infty} \int f^{n} d\mu = \mu\big(f^{-1}(\{1\})\big)$$