# Probability and Random Processes
## 2024

**Assignment 6**
Assigned: Thursday, February 22, 2024
Due: Thursday, March 7, 2024 M. Skoglund

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**Problem 6.1:** Define and explain the concepts total variation and bounded variation.

**Problem 6.2:** Define and explain the concept of an absolutely continuous real-valued function.

**Problem 6.3:** Define and explain the concepts discrete, continuous and absolutely continuous random variable.

**Problem 6.4:** Define the Radon–Nikodym derivative and relate it to the concept of an absolutely continuous random variable.

**Problem 6.5:** Assume that $\{f_n\}$ is a sequence of functions that converges pointwise to $f < \infty$. Prove that $V_a^b f \le \liminf_n V_a^b f_n$.

**Problem 6.6:** Give an example of a function $f$ that is absolutely continuous on $[0, 1]$ but is such that $f'$ is not Riemann integrable on $[0, 1]$. That is, for such a function

$$f(x) = f(0) + \int_0^x f'(t)dt$$

does not hold when the integral is a Riemann integral.

**Problem 6.7:** Let $X$ be a discrete random variable on $(\Omega, \mathcal{A}, P)$ with probability mass function $p_X(x) = P(\{\omega : X = x\})$. Let $\mu$ be counting measure on $(\mathbb{R}, \mathcal{B})$, that is $\mu(B) = |B|$ (cardinality) for $B \in \mathcal{B}$. Show that $\mu_X \ll \mu$ and that

$$p_X = \frac{d\mu_X}{d\mu}$$