Quantum Information Theory
Spring semester, 2017

Assignment 1
Assigned: Friday, March 10, 2017
Due: Friday, March 24, 2017   M. Skoglund

Problem 1.1: Explain the concepts of a Hilbert space and its dual space

Problem 1.2: Explain the concept of compact self-adjoint operator and state the spectral theorem

Problem 1.3: In a finite-dimensional Hilbert space, relate a linear operator to its corresponding matrix representation. Demonstrate how to extend to infinite (countable) dimensions

Problem 1.4: For a compact self-adjoint operator T on a Hilbert space, show how $\log T$ can be computed in terms of logarithms of the eigenvalues of T. In finite dimensions, illustrate how this approach carries over to the eigenvalues of the matrix in any matrix representation

Problem 1.5: Similarly, prove that $\operatorname{Tr} T$ is equal to the sum of the series of eigenvalues of T and, in finite dimensions, illustrate how $\operatorname{Tr} T$ can be computed in terms of the eigenvalues of the corresponding matrix, or simply as the sum of its diagonal elements

Problem 1.6: Prove that a Hilbert space has a countable basis iff it is separable