Quantum Information Theory
Spring semester, 2017

Assignment 2
Assigned: Friday, March 24, 2017
Due: Friday, March 31, 2017 M. Skoglund

Problem 2.1: State and explain Postulates 1–3 of Hilbert space quantum mechanics

Problem 2.2: Explain and discuss the concepts *measurement* and *projective measurement*. Explain how outcomes of physical measurements, their probabilities and expected values are obtained

Problem 2.3: Define the concept of *qubit* and illustrate the corresponding Bloch sphere representation

Problem 2.4: For a qubit, the basis $\{|0\rangle, |1\rangle\}$ is the *computational* basis, this is the "standard" basis for qubits. Consider a qubit $|\psi\rangle$ in the computational basis, and the *Pauli matrices*

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(alternative notation $\sigma_0 \to I$, $\sigma_x \to \sigma_1 \to X$, $\sigma_y \to \sigma_2 \to Y$, $\sigma_z \to \sigma_3 \to Z$). When operating on $|\psi\rangle$, interpret the action performed by the Pauli matrices

Problem 2.5: Show that the operators defined by the Pauli matrices are self-adjoint, unitary, and have eigenvalues ± 1. Express their eigenvectors/states in the computational basis

Problem 2.6: For a qubit in the computational basis, define the $|\pm 1\rangle$-basis as

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

The operator represented by $H = |+\rangle\langle 0| + |-\rangle\langle 1|$ (in the computational basis) is the *Hadamard* transformation, or Hadamard gate. For $|\psi\rangle$ in the computational basis, express $H|\psi\rangle$ in the $|\pm\rangle$-basis and, vice-versa, for $|\psi\rangle$ in the $|\pm 1\rangle$-basis, express $H|\psi\rangle$ in the computational basis. What do your results say about the operator represented by H?

Problem 2.7: State, interpret and prove Heisenberg's uncertainty principle for projective measurements