Quantum Information Theory
Spring semester, 2017

Assignment 5

Assigned: Friday, April 21, 2017
Due: Friday, April 28, 2017 M. Skoglund

Problem 5.1: Explain and discuss the concept of (noisy) *quantum operation*, and state the operator–sum representation. Motivate the general properties required, based on interpreting "noise" as "interaction with the unknown environment."

Problem 5.2: The *depolarizing channel*, taking qubits as input, is described by

$$\mathcal{E}(\rho) = \frac{p}{2}I + (1-p)\rho$$

for $p \in (0,1)$. Show how $\mathcal{E}(\rho)$ can be implemented on operator–sum form in terms of the Pauli matrices.

Problem 5.3: The depolarizing channel can be generalized to d dimensions, as

$$\mathcal{E}(\rho) = \frac{p}{d}I + (1-p)\rho$$

Find an operator–sum representation for this general channel.

Problem 5.4: Assume $\mathcal{H}$ is a Hilbert space with basis $\{|x_i\rangle\}$ and let $|e\rangle$ be a vector that is orthogonal to all $|x_i\rangle$'s. For a state ρ in $\mathcal{H}$ and $p \in (0,1)$, let

$$\mathcal{E}(\rho) = (1-p)\rho + p|e\rangle\langle e|$$

be a noisy quantum operation/channel on ρ. Interpret the effect of this operation, and derive its Kraus operators.

Problem 5.5: Show that any operation $\mathcal{E}$ on a d-dimensional space ($d < \infty$) has an operator–sum representation with at most d^2 Kraus operators.

Problem 5.6: Consider the product $\mathcal{H} \otimes \mathcal{G}$, and let $\mathcal{E}$ be a quantum operation from $\mathcal{H}$ to $\mathcal{H}'$. The operation $\mathcal{E}$ is *entanglement breaking* if applying $\mathcal{E} \otimes I$ to entangled states in $\mathcal{H} \otimes \mathcal{G}$ results in states that are no longer entangled in $\mathcal{H}' \otimes \mathcal{G}$. Show that if $\mathcal{E}$ is entanglement breaking, then it has an operator–sum representation with Kraus operators that are unit rank.